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REINFORCED CONCRETE STRUCTURES

VOLUME

1

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ЖЕЛЕЗОБЕТОННЫЕ КОНСТРУКЦИИ

«СТРОЙИЗДАТ», МОСКВА

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REINFORCED CONCRETE STRUCTURES

VOLUME 1

REINFORCED CONCRETE STRENGTH AND MEMBERS

Translated from the Russian
by
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MIR PUBLISHERS · MOSCOW

First published 1981

Revised from the 1978 Russian third edition

The Russian Alphabet and Transliteration

А а	a	К к	k	Х х	kh
Б б	b	Л л	l	Ц ц	ts
В в	v	М м	m	Ч ч	ch
Г г	g	Н н	n	Ш ш	sh
Д д	d	О о	o	Щ щ	shch
Е е	e	П п	p	ъ	’’
Ё ё	e	Р р	r	ы	y
Ж ж	zh	С с	s	ь	’
З з	z	Т т	t	Э э	e
И и	i	У у	u	Ю ю	yu
Й й	y	Ф ф	f	Я я	ya

The Greek Alphabet

Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ ϑ θ	Theta	Π π	Pi	Ω ω	Omega

На английском языке

CONTENTS

PREFACE

This is the English translation of the third Russian edition, revised and enlarged in comparison with the first (1962) and second (1976-1977) editions. The book has mainly been revised to bring it abreast with the requirements of a new curriculum on reinforced concrete structures, and also to embody the experience accumulate in the use of the previous editions.

The book is based on the lectures the authors have been reading at the Moscow Civil Engineering Institute for a long time. The material is presented with emphasis on highly industrialized precast reinforced concrete. A good deal of attention is given to prestressed structures which are discussed along with ordinary reinforced concrete structures, rather than in isolated sections.

The book is in two volumes. Volume One, devoted to a basic theory of reinforced concrete, describes the main properties of concrete, reinforcing steel and reinforced concrete, and sets forth the design principles of reinforced concrete members. Volume Two, set aside for Soviet practice in the design of reinforced concrete buildings and structures, introduces the reader to the design of bearing-wall and skeleton types of construction, floors, frames, foundations and other structures.

The theoretical basis and design principles for reinforced concrete structures have been brought in line with new standards which were put in force in the Soviet Union since January, 1977 and are in agreement with the International Recommendations for the Design and Construction of Concrete Structures prepared by the Comité

Européen du Béton—Fédération Internationale de la Précontrainte. The International System of Units (SI) is used throughout.

Intended as a textbook for students majoring in civil engineering, the book will also be of real value to students in other departments and practical engineers.

V. Baikov
E. Sigalov

LIST OF SYMBOLS

Forces and moments due to external loads and prestressing force in a cross section of a member:

M = bending moment
N = longitudinal force
Q = shearing force
T = torque
N_0 = resultant of the forces in the prestressed steel before the transfer of the prestress to the concrete, or in the prestressed and nonprestressed steel with zero stress in the concrete

Characteristics of materials:

R_{pr} and R_{pr}^b = design and basic axial compressive strength of concrete (prism-crushing strength), respectively
R_{ten} and R_{ten}^b = design and basic tensile strength of concrete, respectively
R_s and R_s^b = design and basic strength of tensile longitudinal and transverse steel in the inclined-section analysis of bending members
$R_{s,tr}$ = design strength of transverse steel in shear-strength analysis
$R_{s,com}$ = design strength of compressive steel
E_c = tangent modulus of elasticity of concrete in compression and tension
E_s = modulus of elasticity of steel
$n = E_s/E_c$

Characteristics of cross sections:

b = width of a rectangular section; width of the rib of a T- or I-section; double thickness of the wall of a circular or box section
h = depth of a rectangular, T- or I-section
b_f and h_f = width and depth of the flange in the tension zone of a T- or I-section, respectively

b_f and h_f' = width and depth of the flange in the compression zone of a T- or I-section
F = total cross-sectional area of concrete
F_c = cross-sectional area of concrete in compression
F_{tr} = cross-sectional area of a transformed section
I_{tr} = moment of inertia of a transformed section
F_s = cross-sectional area of longitudinal steel: all steel in axially loaded members; tensile steel in bending members; steel at the tensile or less compressed face in eccentrically compressed members; steel nearest to the longitudinal force in members subjected to eccentrical tension
F_{pr} = same for prestressed steel
F_s' = cross-sectional area of longitudinal steel: compressive steel in bending members; steel at the most compressed face in eccentrically compressed members; steel most distant from the longitudinal force in members subjected to eccentrical tension
F_{pr}' = same for prestressed steel
a and a' = distance from the tensile and compressive resultant in the steel to the nearest side of a cross section, respectively
h_0 or h_0' = effective depth of a cross section, equal to h-a or h-a', respectively
x = depth of the compression zone of a cross section
ξ = relative depth of the compression zone of a cross section
z_1 = distance from the compressive to tensile resultant in a section (the arm of the internal couple)
z_c = distance from the centroid of concrete in compression to the resultant of the forces in the steel of area F_s or F_{pr}
e and e' = distance from the point of application of the longitudinal force to the resultant of the forces in the steel of area F_s or F_s', respectively
e_0 = eccentricity of the longitudinal force, N, with respect to the centroid of a transformed section
e_{0pr} = eccentricity of the prestressing force, N_0, with respect to the centroid of a transformed section
e_{0tot} = eccentricity of the resultant of the longitudinal force, N, and prestressing force, N_0, with respect to the centroid of a transformed section
$e_{s,tot}$ = eccentricity of the resultant of the longitudinal force, N, and prestressing force, N_0, with respect to the centroid of the steel of area F_s
$e_{s,pr}$ = eccentricity of the prestressing force, N_0, with respect to the centroid of the steel of area F_s

INTRODUCTION

1. Reinforced Concrete as a Construction Material

Numerous tests have shown that concrete is strong in compression and weak in tension. A simply supported plain concrete beam subjected to bending has to resist tension in the zone lying below the neutral axis (Fig. 1*a*); accordingly, its load-bearing capacity is poor.

The same beam containing reinforcing steel in the tension zone (Fig. 1*b*) has a load-bearing capacity which may be as high as 20 times that of the plain concrete beam.

Reinforced concrete members (such as columns) intended to support compressive loads are also reinforced with steel bars (Fig. 1*c*). Since steel has high compressive and tensile strength, it considerably improves the load-bearing capacity of compression members.

That concrete works well together with reinforcing steel results from a happy combination of their physical and mechanical properties:

(1) the strong bond forming between the hardened concrete and steel makes them deform together under load;

(2) the dense concrete (having a high cement content) protects the encased steel against corrosion and direct exposure to fire;

(3) the coefficients of linear thermal expansion of steel and concrete are about the same; so, when the ambient temperature varies within ± 100 C, the initial stresses in the materials are insignificant, and the steel does not slip from the concrete.

Reinforced concrete has found extensive application in construction owing to its durability, fire resistance resistance to environmental effects, high dynamic resistance, low maintenance costs of buildings and structures, etc. Also, fine and coarse aggregates are available in large quantities almost everywhere, so reinforced concrete can be produced practically all over the world.

Reinforced concrete is more durable than other construction materials. If used judiciously, reinforced concrete structures may serve for an infinitely long time with their load-bearing capacity unaffected. This is because, in contrast to other materials, the strength of concrete increases with time, and the embedded steel is protected against corrosion. Reinforced concrete owes its good fire resistance to the fact that structures with a proper concrete cover

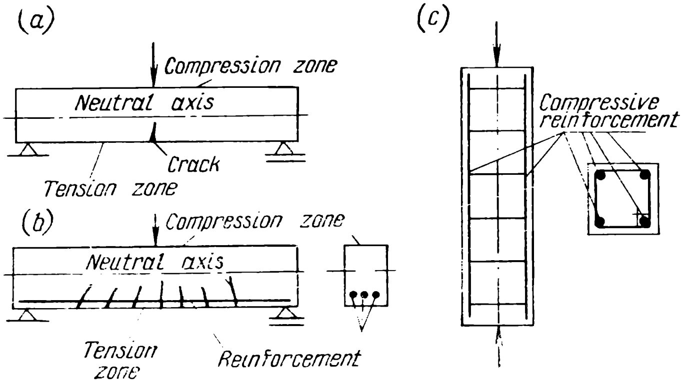


Fig. 1. Behaviour of members under load

subjected to medium fires for several hours suffer damage starting from their surface, so they lose their load-bearing capacity gradually.

Loaded reinforced concrete structures tend to develop cracks in the concrete of the tension zone. In most structures, the crack width under service loads is small, and cracking does not affect their normal behaviour. In practice (especially with high-strength materials), however, it is often necessary to prevent members from cracking, or limit the crack width. For this purpose, the concrete is prestressed before loading; as a rule, this is done by tensioning the steel. In this case, we speak of prestressed reinforced concrete. The relatively large self-weight of reinforced concrete, although useful under certain conditions, is generally undesirable. The weight of structures is reduced by using thin-walled members, members with voids, or members made of porous-aggregate concrete.

2. Fields of Application for Reinforced Concrete

Reinforced concrete structures serve as a basis for modern industrialized surface and underground construction. Reinforced concrete members are used in one- and multi-storey industrial buildings (Fig. 2), thermal power stations (Fig. 3), storehouses, civil buildings, including blocks of flats (Fig. 4), and structures for agricultural

purposes. Reinforced concrete is widely used in large-span thin-walled shells for industrial and civil buildings (Fig. 5), bunkers, tanks, and smoke stacks. It has also proved indispensable in subways, highway and railway bridges and tunnels, hydroelectrical power plants, nuclear power stations and reactors, irrigation works, mine yards, structures for underground workings, etc.

The amount of steel used in linear reinforced concrete structures is one half to one third of that in all-steel structures. Reinforced-concrete floors, pipes, bunkers and the like require one tenth of the amount of steel used in similar sheet-steel structures.

It is very important to combine reinforced concrete with metals and other materials so that the properties of each can be utilized to advantage.

According to the manner of manufacture, we distinguish precast reinforced concrete products, that is, those made at an off-site factory and erected on the site; products and structures cast in-situ; and their combination consisting of in-situ and precast members.

Precast products offer the utmost in the efficiency of industrialized construction. Precast reinforced concrete makes it possible to improve the quality of structures, reduce labour consumption as compared with in-situ structures, cut down (or, sometimes, eliminate altogether) the need for formwork and falsework, and shorten considerably the time of construction. Reinforced concrete structures

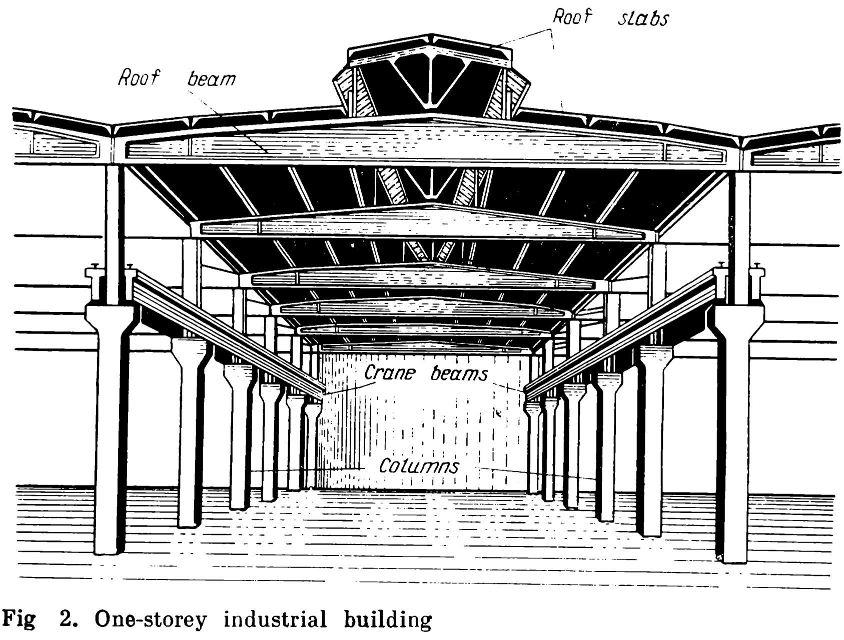


Fig 2. One-storey industrial building

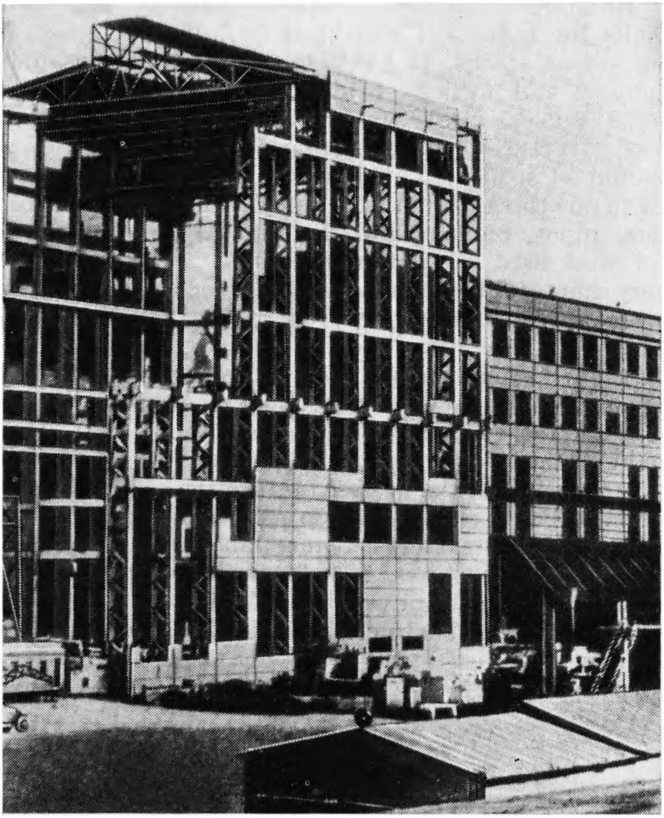


Fig. 3. Thermal power station building

can be erected the whole year round, including the winter season, without any significant rise in the cost of construction; whereas structures cast in-situ are still mostly erected during the warm season of the year, and, if erected in winter, involve additional expenditures (to heat the concrete while it hardens, and so on).

3. Historical Outline of Reinforced Concrete

Reinforced concrete was invented in the latter half of the 19th century when rapidly growing industry, trade and transport necessitated the construction of a large number of factories, bridges, ports and other structures. By that time, the cement industry and iron-and-steel making had already been sufficiently developed to provide the necessary basis for reinforced concrete.

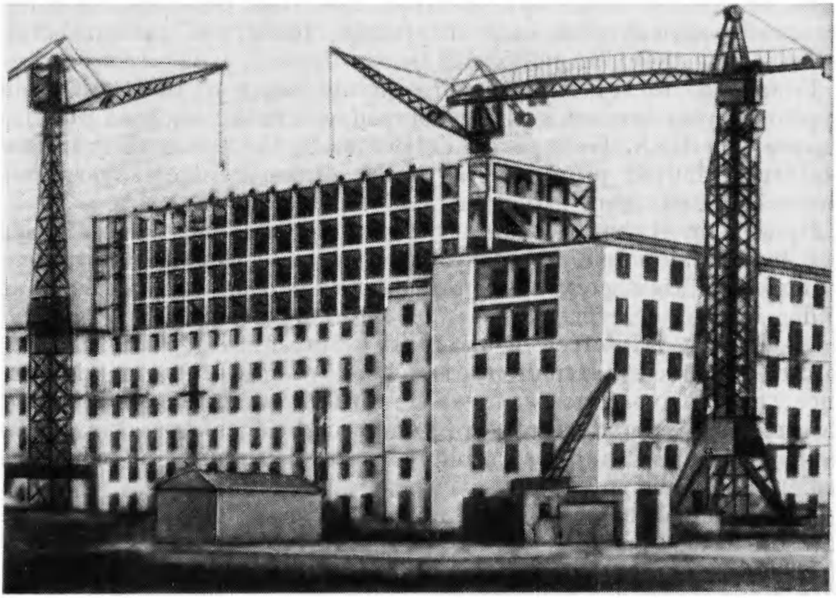


Fig. 4. **Construction** of a block of flats

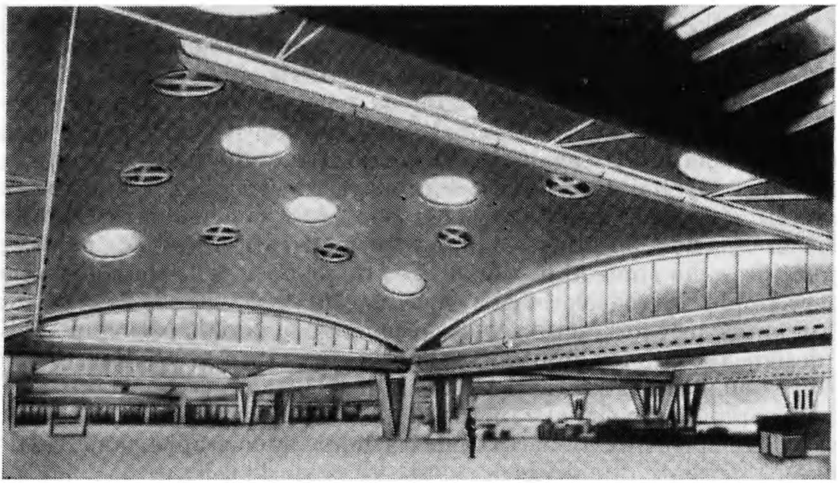


Fig. 5. Reinforced concrete shell

In the period from 1850 to 1885, the first reinforced concrete structures were erected in France (1850, 1854, 1867-1880), Britain (1854), and the USA (1855-1877).

From 1885 to 1917, reinforced concrete began to find still wider application in economically developed countries, such as Britain, France, the USA, Germany and Russia, in the floors of industrial buildings, buried pipes, wells, walls, tanks, bridges, overpasses, trestles, fortification structures and so on.

By the end of the 19th century, the work conducted by researchers and engineers in various countries had given birth to the main principles of elastic theory based on the elastic behaviour of materials under load.

In Russia, reinforced concrete structures developed under the influence of foreign experience and home practice. Of great importance were the experiments on various structures (such as slabs, tanks, vaults, funnels, precast corn-bins, and arch bridges), spiral reinforcement of columns, indirect reinforcement of compression members, cementation and reinforced concrete piles, arrangement of floors and ceilings without beams, precast flat floor slabs (solid and cored), and the like.

The development and application of reinforced concrete in the Soviet Union can be divided into two periods.

During the first period (1918-1945), reinforced concrete was mainly used in industrial and hydraulic construction. In the late 20s, large institutions were set up to design industrial structures. At the same time, various research institutes and laboratories came into being to carry out research work on concrete and reinforced concrete.

To satisfy the demands of large-scale construction and to save metal, reinforced concrete was widely used instead of steel structures. These were the years when reinforced concrete was a dominating construction material. It was used in in-situ multispan beam floors, multispan and multistage frameworks, arches and similar structures for industrial buildings, hydroelectrical power plants, elevators, silos, etc. In 1928, the first precast reinforced concrete was used in some industrial buildings and hydroelectrical power plants. At about the same time, thin-walled in-situ roofs were first applied in domes, folds, cylindrical shells, and marquees.

Advances in construction were accompanied by the development of design theory for multispan beams and frames, shells, slabs, plates, and other structural systems.

The early achievements in the field of reinforced concrete were covered in scientific research publications, monographs, and textbooks.

During this period, further steps were taken in preparing, transporting and placing the concrete mix, winter concreting, and standardization of formwork.

With the growing scale of construction, the drawbacks of elastic theory treating reinforced concrete as an elastic material became more evident. To overcome them, the basic principles of plastic (or collapse) design were formulated at the end of 1931. This theory suggested that plastic strains in the steel and concrete of a bending beam at failure reached their ultimate values and that this determined the critical bending moment.

This approach was backed by a number of experiments and wide theoretical work which gave rise to a fundamentally new theory of designing and reinforcing concrete structures. Some time later the new method was extended to cover members in eccentrical compression and reinforced structures containing stiff reinforcement.

Plastic theory served as the basis for new Soviet standards and specifications requiring that reinforced concrete structures should be designed in terms of the breakdown stage.

In the 30s, the idea of an improved prestressed concrete suggested at the turn of the century was put into practice owing to the work done by French and German scientists. In the Soviet Union, the first experiments on prestressed concrete began in 1930.

In the early 40s, Soviet researchers put forward a theory of crack resistance and stiffness of reinforced concrete, which gained general recognition.

The second period in the use of reinforced concrete in the USSR began at the end of World War II (1945) and still continues.

In the meantime, reinforced concrete has become the basic material not only in industrial and hydraulic structures, but also in civil buildings, thermal power stations, highways, railroads, and agricultural structures. The advent of precast reinforced concrete structures has ushered in a new era in construction. Now, structural members are factory-made and erected at the site. This technology has improved the strength of reinforced concrete members, and has gone a long way towards mechanized erection. Progress has been made in the design of statically indeterminate reinforced concrete structures with allowance for inelastic strains on the basis of the equilibrium between the ultimate external and internal forces. Considerable advances have been made in the field of creep in concrete, and design and arrangement of buried structures (such as subways and various tunnels). In the 50s, the theory of design and arrangement of heat-resistant reinforced concrete structures was developed.

During this period, the design layout of structures has undergone a significant change spurred by the shift to structures made entirely of precast members and application of prestressed members, which are manufactured nowadays by almost every plant. Precast skeleton and bearing-wall multi-storey structures have been built and a theory of their design has been advanced.

A major step in the development of reinforced concrete in the

Soviet Union has been the establishment of a complete range of standard structural members for large-scale manufacture and use.

In 1955, the limit-state design of reinforced concrete members underlying the present-day standards and specifications was developed and put into practice in the USSR.

The latest relevant standards embody the results of the experiments on new types of reinforcement; design of reinforced concrete members; layout of structures; specifications for heavy, heat-resistant, lightweight and cellular concretes; and so on.

A considerable contribution to the modern theory and structural design of reinforced concrete in the Soviet Union has been made by higher-educational establishments which have carried out research in the composite action of precast structures in plane and space systems; design of civil multi-storey buildings; economic design of reinforced concrete structures; spatial behaviour of spans in trestle structures; polymerized concretes and structures using polymerized concrete members; limit-state design of domes; creep in concrete; states of stress in some roofs; systems used to strengthen reinforced concrete structures; biaxial bending and eccentrical compression; bond between steel and concrete; etc.

CHAPTER 1

MAIN PHYSICAL AND MECHANICAL PROPERTIES OF CONCRETE, REINFORCING STEEL AND REINFORCED CONCRETE

1.1. CONCRETE

1. Concrete for Reinforced Concrete Structures

As a material for reinforced concrete structures, concrete must have certain predetermined physical and mechanical properties. Among other things, it must be sufficiently strong, have a high bond resistance, and be watertight enough to protect reinforcing bars against corrosion.

Also, according to the purpose of a reinforced concrete structure and exposure conditions, concrete must meet some special requirements. It must be frost resistant (it must stand up well to freezing and thawing cycles) for example, in exterior wall panels, outdoor structures, etc.; resist exposure to high temperatures for a long time; stand well attack by corrosive environments, and so on.

Concretes may be classified according to:

(a) inner structure, into "no-voids" concretes where the space between the aggregate grains is entirely occupied by a hardened binding agent; no-fines concretes using no or little sand; induced-porosity concretes using aggregates and binding agents with artificially produced open pores; and cellular concretes with artificially produced closed pores;

(b) average density (bulk unit weight), into superheavy concretes with an average density of more than 2 500 kg/m^3; heavy concretes with an average density of more than 2 200 to 2 500 kg/m^3; medium-heavy concretes with an average density of more than 1 800 to 2 200 kg/m^3; and lightweight concretes with an average density of more than 500 to 1 800 kg/m^3;

(c) form of aggregate, into nonporous-aggregate (usually, sand-and-broken stone) concretes; porous-aggregate concretes; concretes using special aggregates providing biological shielding, heat resistance, etc.;

(d) aggregate size, into coarse concretes using both coarse and fine aggregates, and fine concretes using fine aggregates;

(e) manner of hardening, into naturally hardened concrete; normal-pressure moist-cured concrete; and high-pressure steam-cured (autoclaved) concrete.

For brevity, concretes used in bearing reinforced concrete structures are referred to in Soviet practice as:

—heavy concretes; these are "no-voids", heavy, nonporous coarse-aggregate concretes using cement as the binding agent and hardened under any conditions;

—porous-aggregate concretes; these are dense, medium-heavy or lightweight, coarse porous-aggregate concretes using cement as the binding agent and hardened under any conditions.

Nonporous aggregates for heavy concretes are broken stone crushed from various rock, such as sandstone, granite and diabase, and natural quartz sand. Porous aggregates are naturally occurring perlite, pumice or shell rock, or manufactured materials, such as ceramsite, slag, etc. Those using porous aggregates are divided into ceramsite concrete, slag concrete, perlite concrete, and so on.

Induced-porosity, cellular, and lightweight-aggregate concretes with an average density of 1 400 kg/m^3 and lower are mostly used for filler walls. Superheavy concretes are used in structures providing biological shielding against radiation. Fine concretes find application mostly for making joints in structures from precast (prefabricated) members.

In order to produce concrete having a predetermined strength and durability, one needs to mix in a correct proportion the necessary ingredients which include various cements, coarse and fine aggregates, admixtures improving the workability or frost resistance of the concrete, and so on.

The strength of concrete depends on the aggregate grading (the aggregate should be graded so that the volume of voids in the mixture would be minimal), aggregate strength and surface condition, cement grade and its amount, amount of water, etc. Uneven and rough aggregate grains provide a better bond in the mix. So, sand-and-broken stone concretes have a greater strength than sand-and-gravel concretes.

In more detail, the proportioning of concrete is discussed in a course on construction materials.

The necessary density of concrete is attained by intelligent aggregate grading, thorough tamping in forms, and use of a sufficient amount of cement (which ranges between 250 and 500 kg/m^3). The strength of the concrete increases with increasing density. In order to reduce the consumption of cement, its strength should exceed the required strength of the concrete.

2. Inner Structure of Concrete. Its Effect on Strength and Stress-Strain Behaviour

The strength and stress-strain behaviour of concrete markedly depend on its inner structure. To get a better insight into the matter, let us see what happens when concrete is produced. When water is added to a mixture of aggregates and cement, the cement combines with the water (hydration is said to take place). Hydration produces what is known as cement gel, a porous mass consisting of cement particles which have not yet reacted and an insignificant amount of complex crystal substances suspended in water. As the mix is agitated, the gel envelopes the aggregate grains, the paste becomes somewhat stiff, and the crystals gradually intertwine into conglomerates growing with time. The stiffening gel turns into a hardened cement combining the coarse and fine aggregates into concrete.

A very important factor affecting the strength and inner structure of concrete is the amount of water added to the mix; it is expressed in terms of the water-cement ratio which is defined as the weight ratio of mixing water to cement per unit volume of the concrete mix. A water-cement ratio of about 0.2 is usually required for all of the cement to be hydrated. However, the concrete thus proportioned would be so stiff that it would be extremely difficult to place and finish. Consequently, additional water must be added to make the mix workable. For example, fluid concrete mixes which can be poured into place have a water-cement ratio of 0.5 to 0.6, and harsh mixes which are vibrated into forms have a water-cement ratio of 0.3 to 0.4.

Later, the excess water that has not participated in the hydration process reacts with the less active cement particles, or partly fills numerous pores and capillaries in the hardened cement and voids between the coarse-aggregate grains and reinforcing steel. It evaporates with time, thereby leaving voids in the concrete. Experiments show that voids occupy about one third of the hardened cement volume; as the water-cement ratio decreases, the volume of voids in the hardened cement falls and the strength of the concrete rises. This is why prefabricated concrete structures are made of harsh concrete mixes having water-cement ratios as low as possible. Concretes made of harsh mixes have a higher strength, require less cement and shorter ageing in forms.

As is seen, the inner structure of concrete is rather nonuniform. It is formed by a space lattice of hardened cement embedding sand and broken stone grains of various size and shape, and containing a vast number of pores and capillaries filled by the free water, vapour and air. Physically, concrete is a capillary and porous material with discontinuities throughout its bulk, in which all the three phases—solid, liquid and gaseous—are present. The hardened

cement is also of nonuniform inner structure and consists of an elastic crystal conglomerate and a viscous gel filling it.

The slow processes taking place in such a material, namely the change in water balance, decrease in volume of the stiffening gel, growth of crystal conglomerates, are responsible for the specific elastic-plastic behaviour of concrete under load and exposure to variations in ambient temperature and humidity.

Experience shows that the theories of strength plausible for other materials are inapplicable to concrete. Relations between the composition, inner structure, strength and stress-strain behaviour of concrete have yet to be studied. Present-day knowledge about the strength and stress-strain behaviour of concrete is based on a great number of experiments carried out in laboratories and under field conditions.

3. Shrinkage and Initial Stress

Concrete decreases in volume when it is allowed to harden under normal atmospheric conditions (this is known as shrinkage) and increases in volume when it hardens in water (this is called swelling). Concretes using expansive or shrinkage-compensating cements do not shrink. Experimental data show that shrinkage increases (1) with increasing amount of cement per unit volume of concrete and also with active and alumina cements; (2) with increasing amount of mixing water, that is, a higher water-cement ratio; and (3) with decreasing aggregate size; fine sand and porous broken stone result in a greater shrinkage.

The amount of shrinkage may be reduced by using aggregates with a higher stress-to-strain ratio, that is, a higher modulus of elasticity. The amount of shrinkage may further be reduced by using unevenly graded aggregates as this reduces the volume of voids. Hydraulic and early-strength admixtures (for example, calcium chloride) usually contribute to shrinkage.

As a rule, the rate of shrinkage is especially high during the initial period of hardening and the first year after manufacture; later, it gradually slows down. The rate of shrinkage and the resultant strain increase with decreasing ambient humidity. Work in sustained compression speeds up, whereas work in sustained tension slows down the shrinkage.

Shrinkage is closely related to the physical and chemical processes associated with the hardening and decrease in volume of the cement gel, the loss of excess water by evaporation into atmosphere and by hydration of the cement particles that have not yet reacted. As the cement gel hardens, reduces in volume and forms crystal conglomerates, shrinkage becomes less pronounced. Capillary phenomena caused in the hardened cement by the excess water also affect the amount

of shrinkage—the surface tension in the menisci exerts pressure on the capillary walls and gives rise to volume strain.

When the concrete hardens, the cement is prevented from shrinkage by the aggregate which acts as an internal restraint and induces initial tensile stresses in it. A similar restraint is produced by the crystal conglomerates formed in the hardening gel. The concrete dries nonuniformly throughout its bulk, so it shrinks also nonuniformly, which fact results in initial (setting) shrinkage strains. The exposed surface layers of the concrete which dry faster are subjected to tension, whereas the wetter internal layers, in restraining the shrinkage of the surface layers, are subjected to compression. Such initial tensile stresses produce shrinkage cracks in the concrete.

Initial shrinkage stresses do not enter into the strength analysis of reinforced concrete structures explicitly. Instead, they are taken care of by coefficients describing various aspects of concrete strength, and also by reinforcement of concrete structures. Initial shrinkage stresses can be reduced by suitably proportioning the concrete mix, by steam and moist curing, and also by providing shrinkage joints in structures.

4. Strength of Concrete

General. Since concrete is a nonuniform material, it is nonuniformly stressed when exposed to external load. In a compressed test specimen, stresses concentrate at the harder particles having a higher modulus of elasticity. They result in forces which appear at the boundaries between the particles and tend to break the bond. In addition, stresses concentrate near pores and voids. As is known from the theory of elasticity, in a compressed test specimen, compressive and tensile stresses concentrate at holes in the material, the tensile stresses acting on planes parallel to the compressive force (Fig. 1.1*a*). Since concrete is abundant in pores and voids, tensile stresses at one hole or pore overlap with those at an adjacent hole. As a result, an axially compressed specimen is subjected to longitudinal compressive and lateral tensile stresses (this is known as a secondary stress field).

Experimental data show that compression test specimens collapse due to the lateral rupture of the concrete. At first, microscopic bond-failure cracks appear throughout the specimen bulk. As the load increases, the bond-failure cracks merge into visible cracks running parallel or at a small angle to the direction of the compressive force (Fig. 1.1*b*). Then, the cracks open up, and this is accompanied by an apparent increase in specimen volume. Finally, the specimen fails completely.

The point at which microscopic cracks begin to form in concrete under load may be determined by ultrasonic tests. The velocity of

ultrasonic waves, v, at right angles to the direction of the compressive stresses falls as the cracks grow. The point on the curve of Fig. I.2 where the velocity of ultrasound begins to decrease defines the compressive stress, R^0_{cr}, at which microscopic cracks begin to appear in the concrete. It is in terms of this stress that we describe the strength and stress-strain behaviour of concrete.

Since the particles and pores making up concrete are spaced non-uniformly and differ in size, there is a certain spread in strength

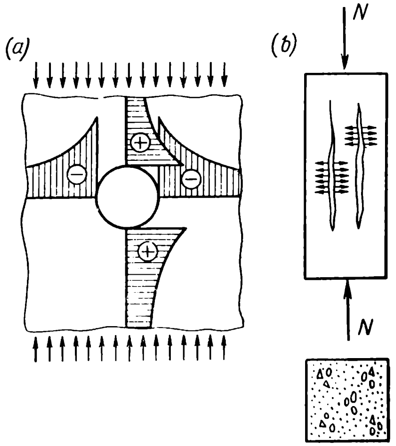


Fig. I.1. Compressed concrete test specimen

(*a*) concentration of stress at micropores and cracks; (*b*) lateral cracking of concrete in axial compression

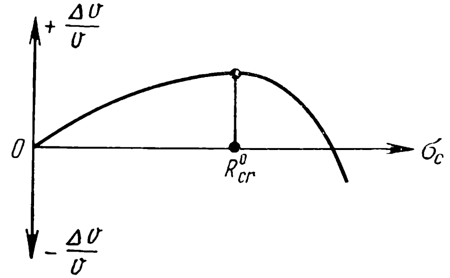


Fig. I.2. To determining the compressive stress in concrete at the limit of microcracking, R^0_{cr}, by ultrasonic pulse test

between different test specimens made of the same concrete mix. The strength of concrete depends on many factors, the main of which are:

(1) type of ingredients and form of mixing;
(2) age and hardening conditions;
(3) shape and size of the test specimen;
(4) type and duration of stress.

The last mentioned point is important because concrete has different ultimate strength in compression, tension or shear.

Design Brands of Concrete. In Soviet practice, concretes are assigned so-called design brands, or brand numbers, depending on the character and purpose of the structure. A design brand, or brand number, refers to some reference quality index specified in the design. Thus, brands can be assigned in terms of axial compressive strength, axial tensile strength, frost resistance, or water resistance. Concretes of the desired brand may be obtained by correct proportioning the concrete mix with subsequent testing for strength. Concrete is

widely used as a construction material owing to its ability to withstand high pressure, so the test of compressive strength is the one most frequently used. Also, compressive strength can be determined most conveniently of all characteristics. This is the reason why the compressive strength of concrete is adopted as its main characteristic.

A relevant Soviet standard requires that the compressive strength of concrete should be determined on test specimens in the form of cubes with a 15-cm edge for a 28-day curing period at a temperature of $20 \pm 2°C$ (the standard provides for some other requirements which we shall leave out in this text). The test result gives the brand of the concrete in terms of ultimate compressive strength in kgf/cm^2. Concrete should be cured long enough for it to reach the required strength by the moment when the structure is subjected to a design load. Structures made of in-situ concrete using normal portland cement are usually aged for 28 days. Precast concrete members may be dispatched to a job before they reach the required strength; in this case, their strength is specified according to transportation and erection conditions, time before loading, etc.

According to compressive strength, the following brands are produced in the Soviet Union:

—heavy concretes, M-100, M-150, M-200, M-250, M-300, M-350, M-400, M-450, M-500, M-600, M-700 and M-800;

—porous-aggregate concretes, M-35, M-50, M-75, M-100, M-150, M-200, M-250, M-300, M-350 and M-400.

The use of heavy-concrete brands M-250, M-350 or M-450 is warranted only if they help to cut down cement consumption and do not affect other technical and economic indices.

If a concrete structure works mostly in tension, one needs to know the tensile strength of the concrete in addition to its compressive strength. According to their tensile strength, Soviet-made concretes are classed as follows:

—heavy concretes, P-10, P-15, P-20, P-25, P-30, P-35 and P-40;

—porous-aggregate concretes, P-10, P-15, P-20, P-25 and P-30.

Here, the numerals give the ultimate tensile strength in kgf/cm^2.

Brand numbers in terms of frost resistance are assigned to concretes used in structures subjected to freezing and thawing cycles in cold climatic regions, exterior structures in regions with frequent freeze-thaw changes, and to porous-aggregate concrete used in filler walls. They are designated from Mрз 25 to Mрз 500. The numerals give the number of freeze-thaw cycles that the concrete can withstand in a water-saturated condition.

Brand numbers in terms of water resistance are assigned to concretes used in structures subjected to water pressure, such as tanks and pressure pipes. These brands are designated by the Russian letter "B", they range from B-2 to B-12. Here, the numerals give

the limit of water pressure at which the flow of water through channels in the concrete is still prevented.

An optimum design brand of concrete is chosen for technical and economic reasons according to the type of reinforced concrete structure, kind of stress, manner of manufacture, exposure conditions, and so on. It is advisable to use concrete brands of at least M-200 for compression members made of heavy or porous-aggregate concrete. Relatively high design concrete brands, such as M-300 and M-400, have proved to be of great utility in structures subjected to considerable compressive forces (columns, arches, and the like). In prestressed reinforced concrete structures, the best results are achieved with brands M-300 through M-500, depending on the type of prestressed reinforcement. The optimum brand for unprestressed members in bending is M-200.

Porous-aggregate cement concretes classed in the same brands in terms of strength, frost and water resistance as heavy concretes are

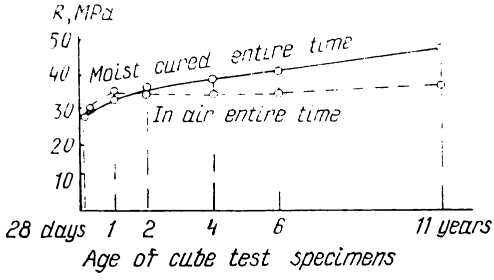


Fig. I.3. Concrete strength as a function of age

used as often in precast or in-situ concrete structures. They are especially effective in many applications where the mass is critical.

Effects of Time and Hardening Conditions on Strength. The strength of concrete increases during a long time, but it grows most quickly during the initial period of hardening. The strength of concrete using portland cement as the binding agent rapidly increases during the first 28 days, whereas that of concretes based on portland-pozzolana and portland-slag cements increases more slowly during the first 90 days. After this period, however, the strength of concrete does not cease rising. Given favourable conditions (positive temperature, moist surroundings), its strength may increase for several years. This is because it takes a long time for the cement gel to harden, and for the crystal conglomerates to grow. Experiments have shown that the strength of test specimens stored in a moist atmosphere for eleven years is twice as great as that at 28 days; the strength of test specimens cured in the air entire time is 1.4 that at 28 days but ceases growing completely after one year in the air (Fig. I.3).

If concrete remains dry, as it often happens in the field, no increase in its strength can be expected one year after manufacture.

The increase in the strength of concrete using portland cement and moist cured at a temperature of about $+15^\circ$C may be determined with the help of the following empirical relation

$$R_t = R \lg t/\lg 28 = 0.7R \lg t \tag{I.1}$$

where R_t is the ultimate compressive strength of a concrete cube at t days, and R is the ultimate compressive strength of the cube at 28 days.

The results obtained with the above formula are sufficiently close to experimental data at $t \geqslant 7$ days.

The strength of concrete rises much faster at an elevated ambient temperature and humidity. In view of this, precast concrete members are cured at a temperature of up to 90°C and a humidity of up to 100%, or autoclaved at a temperature of about 170°C. After one day of such curing, the strength of concrete reaches about 70% of its design value. If concrete is allowed to freeze while hardening, the growth of its strength sharply slows down or stops completely.

Cube Crushing Strength of Concrete. As previously noted, an axially compressed concrete cube fails by lateral rupture (Fig. I.4*a*). That the cracks are inclined is attributable to the friction between the platen and seat of the compression-testing machine and the cube faces. The forces of friction directed inwards restrain the free lateral strain of the cube, thereby causing what we call the grip effect. The restraint decreases with increasing distance from the grips, so, after it has collapsed, the test specimen takes the form of two intact pyramids with their apices within one another. If, before testing, we cap the ends of the specimen with neat cement paste or other material, the lateral strain meets no resistance, the rupture cracks become vertical (parallel to the direction of compression), and the ultimate compressive strength of the concrete is nearly halved (Fig. I.4*b*). According to a relevant Soviet standard, concrete cubes are tested without capping.

Experiments show that the strength of concrete taken from the same batch varies with the size of test specimens. For example, if we designate the ultimate compressive strength of a standard cube with a 15-cm edge as R, this quantity for a 20-cm edge cube will be about $0.93R$, and that for a 10-cm edge cube will be

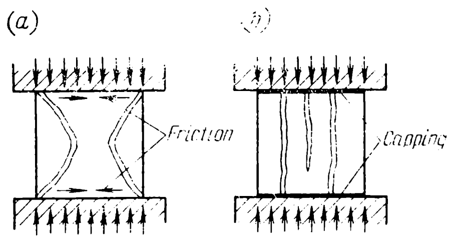


Fig. I.4. Concrete cube failure
(*a*) with friction at the cube faces; (*b*) without friction

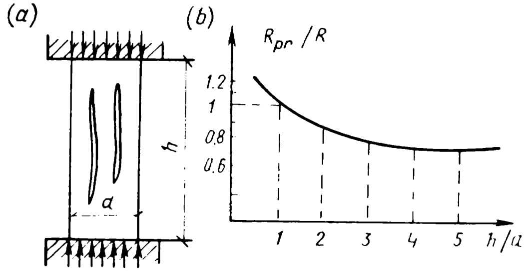


Fig. I.5. Prism crushing strength as a function of the height-least lateral dimension ratio

about $1.1R$. The strength of cube test specimens made of porous-aggregate concrete is less affected by variations in cube size, being $0.97R$ and $1.03R$ for a 20-cm and a 10-cm edge cube, respectively. The variations in strength are explained by the fact that the end grip effect is not the same for different cube sizes. In some countries, for example, in the USA, the test specimens are in the shape of cylinders with a height of 12 inches (30.5 cm) and a diameter of 6 inches (15.2 cm). The ultimate compressive strength obtained with cylindrical specimens ranges between 0.7 and 0.75 that of a 15-cm edge cube.

Prism Crushing Strength of Concrete. Real reinforced concrete structures differ in shape from cubes. Accordingly, the cube crushing strength of concrete cannot be used directly in the strength analysis of members. Instead, the main characteristic for compression members is the prism crushing strength, R_{pr}, defined as the ultimate axial compressive strength of concrete prisms. Experiments with concrete prisms with the least lateral dimension a and height h have shown that the prism crushing strength of concrete is below the cube crushing strength and decreases with increasing h/a ratio. The curve of Fig. I.5 plotted according to averaged experimental results illustrates the relation between R_{pr}/R and h/a.

Referring to the figure, the effect of friction at the prism ends decreases with increasing height and, at $h/a = 4$, R_{pr} becomes almost stable and equal to about $0.75R$. The effect of concrete flexibility becomes appreciable only at $h/a \geqslant 8$.

The strength of the compression zones of members in bending is also expressed in terms of R_{pr}. In this case, the real curved stress diagram for the compression zones in the limit state is replaced by a conventional rectangular stress diagram (Fig. I.6).

Tensile Strength of Concrete. This depends on the strength of the hardened cement and the bonding strength between the cement and aggregate grains. According to experiments, the tensile strength

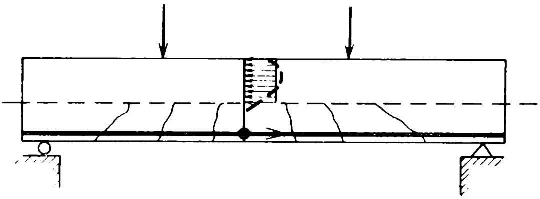


Fig. I.6. State of stress in the compression zone of a bending reinforced concrete beam

of concrete is 1/10 to 1/20 of its compressive strength, and the relative tensile strength decreases with increasing concrete brand number. Here, the spread in strength exceeds that in the case of compression tests. The tensile strength of concrete may be enhanced by raising the cement content, reducing the water-cement ratio, and using broken stone with rough surfaces.

The ultimate axial tensile strength of concrete may be determined by the following empirical formula

$$R_{ten} = 0.5\sqrt[3]{R^2} \tag{I.2}$$

The results obtained with the help of the above relation are not always correct because concrete is nonuniform in inner structure. As a rule, R_{ten} is determined by a rupture test, splitting test and flexure test (Fig. I.7), the last mentioned being most common. Here, a simple beam loaded at third-points is used with a span three times its depth. The moment of rupture of the beam yields the tensile strength of concrete

$$R_{ten} = M_r/\gamma W = 3.5M_r/bh^2 \tag{I.3}$$

where $W = bh^2/6$ is the resisting moment of a rectangular section, and $\gamma = 1.7$ is the factor taking care of the curved nature of the tension area stress diagram due to inelastic deformation.

Shear Strength and Resistance to Spalling of Concrete. Ideally, shear consists in the separation of a member into two parts along a given plane by shearing forces. Grains of the coarse aggregate in

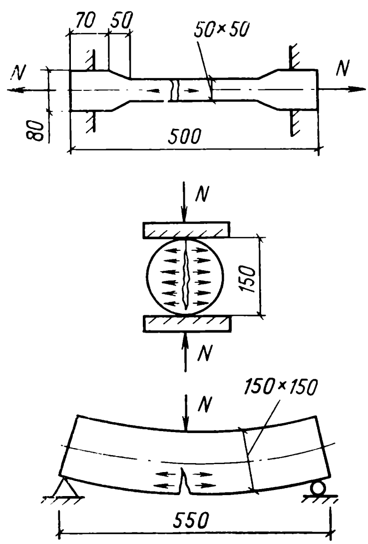


Fig. I.7. Rupture, splitting and flexure tests for determining the tensile strength of concrete

the plane of shear act as keys, so their shear strength markedly affects that of the concrete. In shear, stresses are taken to be uniformly distributed over the entire cross-section of the specimen. The ultimate shear strength of concrete can be found from the following empirical formula

$$R_{sh} = 0.7\sqrt{R_{pr}R_{ten}} \quad \text{or} \quad R_{sh} = 2R_{ten} \tag{I.4}$$

Real reinforced concrete structures are seldom subjected to shearing stresses alone; as a rule, shearing forces act together with longitudinal forces.

Concrete resists spalling when bent until inclined cracks develop. Spalling stresses vary along the depth of a section according to a quadratic parabola. As experimental data show, the ultimate splitting strength of concrete, R_{sp}, is 1.5 to 2 times as great as R_{ten}.

Long-Time Strength of Concrete. Experience shows that concrete subjected to heavy sustained loading fails at stresses below the ultimate axial compressive strength R_{pr}. This happens because such loads develop considerable inelastic strains and structural changes in concrete. According to experimental data, the ultimate long-time axial compressive strength of concrete, designated R_{lt}, may be as low as $0.85R_{pr}$ or even lower. If service conditions are favourable for concrete to gain strength, the ratio of the applied stress to the prism strength of the concrete, σ_c/R_{pr}, gradually decreases, and the negative effect of sustained loading may not be felt.

Fatigue Strength of Concrete. If a concrete structure is subjected to as many as millions repeated compressions, the ultimate compressive strength of the concrete decreases because of microcracking. Experiments show that the fatigue strength of concrete under repeated loading or its fatigue (or endurance) limit, R_f, depends on the number of loading cycles and the ratio of the minimum to the maximum stress in a cycle (or the cycle characteristic), $\rho = \sigma_{min}/\sigma_{max}$. Figure I.8*a* shows the endurance curve of concrete. Here, the number of cycles, n, is laid off as abscissa, and the periodically changing fatigue limit, R_f, is laid off as ordinate. As the number of cycles rises, R_f falls. The value on the horizontal portion of the curve at $n \to \infty$ is called the absolute fatigue limit. Practically, R_f at $n = 2 \times 10^6$ changes almost linearly with the cycle characteristic, ρ, the minimum value being $R_f = 0.5R_{pr}$ (Fig. I.8*b*).

As has been found experimentally, the lowest value of the fatigue limit corresponds to the stress at which microcracking develops in concrete, so $R_f \geqslant R_{cr}^0$. This relation between R_f and R_{cr}^0 makes it possible to find the fatigue limit with a single loading of a test specimen by determining the start of microcracking with the help of ultrasonic testing.

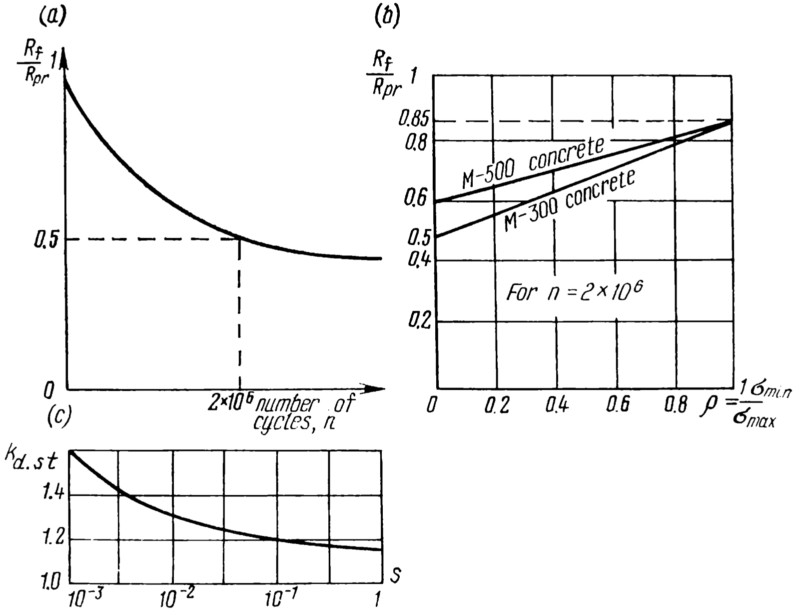


Fig. I.8. (*a*) Fatigue limit of concrete as a function of loading cycles and (*b*) as a function of the cycle characteristic ρ determined at $n = 2 \times 10^6$; (*c*) to determining the dynamic strengthening factor

The fatigue limit is necessary for the fatigue analysis of reinforced concrete structures subjected to dynamic loads, such as crane beams, floors in some industrial buildings, and so on.

Dynamic Strengthening of Concrete. When a concrete structure is subjected to a heavy dynamic load of short duration (impacts or explosion), the ultimate strength of the concrete increases; this may be called dynamic strengthening. Experiments show that the dynamic strengthening factor, $k_{d.st}$, increases with decreasing time during which a test specimen is under a specified dynamic load (or, which is the same, with rising stress growth rate measured in MPa/s). This factor is equal to the ratio of the dynamic ultimate compressive strength, R_d, to the prism strength, R_{pr} (Fig. I.8*c*). For example, if a specimen is exposed to a dynamic load for 0.1 s, $k_{d.st} = 1.2$. This is because concrete subjected to a dynamic load for a short time remains elastic and absorbs energy.

5. Stress-Strain Behaviour of Concrete

Types of Strain. As a rule, two main types of strain develop in concrete. These are volumetric strain produced by shrinkage and temperature variations, which develops in all directions, and strain

produced by applied stresses, usually developing in the same direction as the applied stress. Under the action of longitudinal (axial) stress, concrete deforms laterally. The ratio of lateral to longitudinal deformation (Poisson's ratio) for concrete, designated μ, is equal to 0.2. Concrete is a material having both elastic and plastic properties. When concrete is subjected to a load, no matter how small, elastic deformation in it is accompanied by inelastic or plastic deformation. In view of this, the strain produced by applied stresses is classified according to the nature and duration of loading into three types, namely strain developed by short-duration single loading, strain caused by a sustained load, and strain produced by repeated loading.

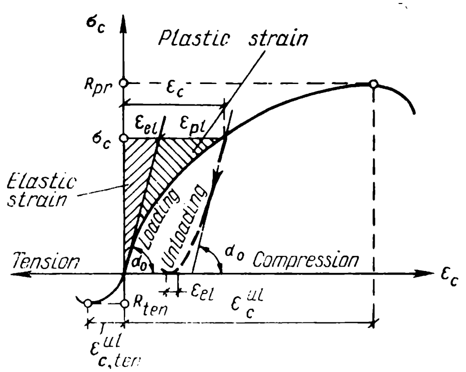


Fig. I.9. General stress-strain curve of concrete

Volumetric Strain. Shrinkage strain in concrete varies over a rather wide range. According to experimental data, ε_{sh} = (approx.) 3×10^{-4} or more for heavy concretes, and ε_{sh} = (approx.) 4.5×10^{-4} for porous-aggregate concretes. Strain caused by swelling ranges between one half and one fifth of that developed by shrinkage.

Temperature strain of concrete depends on the coefficient of thermal expansion of concrete, designated $\alpha_{c,t}$. The thermal coefficient of expansion for heavy concrete and porous-aggregate concrete using quartz sand is $1 \times 10^{-5}/°C$ over the ambient temperature range from —50 to +50°C. The value of this coefficient varies with the character and amount of cement, aggregate, and moisture state of concrete within ±30%. For example, the coefficient of thermal expansion for porous-aggregate concretes using porous sand is equal to $0.7 \times \times 10^{-5}/°C$.

Strain Developed by Short-Duration Single Loading. When concrete is subjected to a short-duration single loading, it undergoes the following strain

$$\varepsilon_c = \varepsilon_{el} + \varepsilon_{pl} \tag{I.5}$$

composed of elastic and plastic strain (Fig. I.9). A small proportion of plastic strain (about 10%) recovers after the load has been removed. In Soviet practice, this share of strain is called the elastic recovery strain. If a test specimen is loaded step by step and the amount of strain is measured twice each time (that is, just as it is loaded and some time after), we shall obtain a stepped stress-strain curve such

as shown in Fig. I.10*a*. The strain measured immediately after loading is elastic in nature and varies linearly with the applied stress, whereas the strain measured some time after loading is nonelastic. it increases with rising stress and is represented by horizontal portions of the stress-strain diagram. With a sufficient number of loading steps, the relation between stresses and strains may be graphically shown as a curve. A similar curve can be obtained if the load is removed step by step. Again, we measure the strain twice at each step (first, just as the load has been reduced and some time

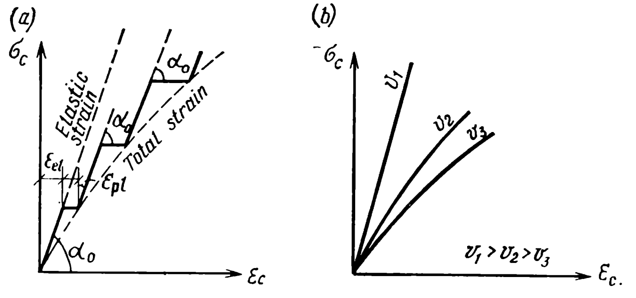


Fig. I.10. (*a*) Stress-strain curve of concrete in compression plotted for several loading steps and (*b*) plotted for different loading rates

after, that is under a new load). With a sufficient number of steps, the stepped line thus produced may be replaced by a smooth curve which in this case is concave (see Fig. I.9).

As is seen, elastic strain occurs in concrete only if it is loaded instantly, whereas nonelastic strain develops with time and depends on the loading rate, v, measured in MPa/s. As the loading rate increases at the same stress, σ_c, nonelastic strain decreases. Stress-strain curves, σ_c-ε_c, plotted for different loading rates, $v_1 > v_2 > v_3$, are shown in Fig. I.10*b*.

When concrete works in tension, the resulting strain likewise consists of elastic and plastic components

$$\varepsilon_{c,ten} = \varepsilon_{el,ten} + \varepsilon_{pl,ten} \qquad (I.6)$$

where $\varepsilon_{el,ten}$ is the elastic strain and $\varepsilon_{pl,ten}$ is the plastic strain.

Strain Caused by a Sustained Load. When a concrete structure is subjected to a sustained load, plastic strain develops in the concrete with time. It has been shown that nonelastic strain is most significant during the first three or four months after loading and may increase out to several years. In the stress-strain curve of Fig. I.11, portion *0-1* represents the strain which occurs immediately when concrete is loaded, the curvature of this portion depends on the rate of loading;

portion *1-2* shows the growth in plastic strain caused by a constant applied stress.

An increase in plastic strain under a sustained stress is called *creep*. Creep deformation may be 3 or 4 times as great as elastic deformation. If creep is allowed to increase freely under a constant sustained load, stresses in concrete remain unchanged. If restraints in concrete, say, reinforcing steel, do not allow creep to develop freely, we speak of restrained creep; in the circumstances, stresses in the concrete are no longer constant.

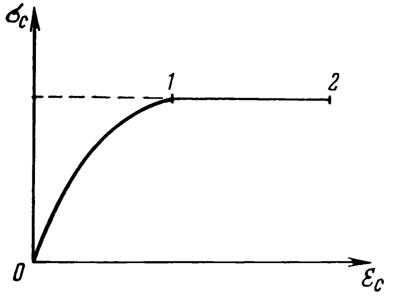


Fig. I.11. Stress-strain curve for a test specimen subjected to a sustained load

If a concrete test specimen is subjected to an initial stress, σ_c^0, developing an initial strain, ε_c^0, and then restrained from further deformation, there is a progressive decrease in stress with time. This is known as *relaxation*. Creep and relaxation of concrete are similar in nature and markedly affect the work of reinforced concrete structures under load.

Tests on concrete prisms show that whatever the loading rate producing the stress, σ_{c1}, the final creep strain corresponding to this

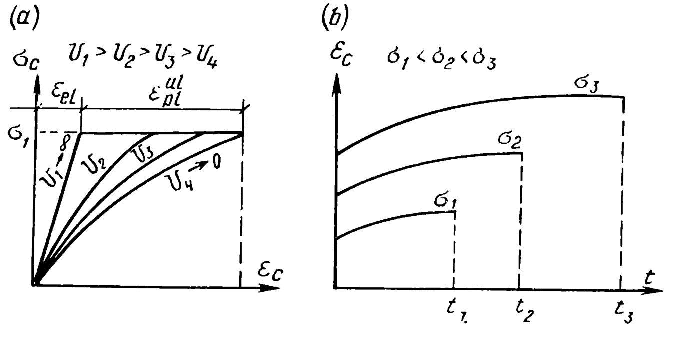


Fig. I.12. (*a*) Creep strain as a function of initial loading rate and (*b*) as a function of time under load, *t*, and stress, σ_c

stress is always the same (Fig. I.12*a*). Creep increases with increasing stress; a plot of strain as a function of time for various stresses, $\sigma_{c1} < \sigma_{c2} < \sigma_{c3}$, is shown in Fig. I.12*b*. Concrete loaded at an early age shows a greater amount of creep than the aged one. Creep in concrete cured in a dry atmosphere is more pronounced than that in moist-cured concrete. The amount of creep also increases with

increasing water-cement ratio and amount of cement per unit volume of the concrete mix. Concretes with stronger aggregate grains and a higher compressive strength show a lower creep. Also, porous-aggregate concretes display a somewhat greater creep than heavy concretes.

The manner in which creep develops in concrete is explained by its inner structure, long crystallization period and decrease in gel volume with the hardening of the cement. When concrete is subjected to a load, the stress is redistributed from the yielding viscous gel to the crystal conglomerates and aggregate grains. At the same time, a proportion of creep is contributed to by capillary phenomena related to the flow of excess water in micropores and capillaries under the applied load. With time, the redistribution of stresses slows down and the deformation ceases completely.

In Soviet practice, creep is customarily divided into linear creep when the stress-strain behaviour of concrete is approximately described by a straight line, and nonlinear creep. When the applied stress exceeds the limit, R^0_{cr}, at which microcracking occurs in concrete, strain begins to develop rapidly and nonlinear creep takes place. This division is arbitrary because in some tests the stress-strain diagram may be nonlinear already at relatively small stresses. It is worth mentioning that nonlinear creep is important in designing prestressed members in bending, and also eccentrically compressed and some other concrete members.

Creep and shrinkage occur simultaneously. So, the total deformation in concrete is the sum of elastic strain, ε_{el}, creep, ε_{cr}, and shrinkage, ε_{sh}. Shrinkage, however, is volumetric in nature, whereas creep develops mainly in the direction of the applied stress.

Strain Caused by Repeated Loading. Repeated load-unload cycles of a concrete prism result in the gradual accumulation of nonelastic strains. After a sufficient number of such cycles, the rate at which the nonelastic strain corresponding to a given stress accumulates gradually slows down. Creep reaches its maximum and the concrete behaves elastically. Referring to Fig. 1.13, nonelastic strains accumulate with each cycle, and the stress-strain curve gradually becomes a straight line characteristic of elastic behaviour. This happens only if the applied stresses do not exceed the fatigue limit, that is, $\sigma_c \leqslant R_f$. Otherwise, nonelastic strains begin to grow without bound, and the test specimen collapses. In this case, the stress-strain curve becomes concave and the angle that it makes with the stress axis progressively decreases.

If concrete is subjected to vibration (200 to 600 load-unload cycles per minute), creep develops at a high rate. This is known as vibrational or dynamic creep.

Strain Capacity. In design calculations concerned with deformations near the ultimate, it is important to know the maximum strains

that can be developed in concrete. They are expressed in terms of the ultimate compressive strain, ε_c^{ul}, and the ultimate tensile strain, $\varepsilon_{c,ten}^{ul}$, which depend on the strength, brand and composition of the concrete and duration of loading. They decrease with increasing concrete brand number, but increase with increasing duration of loading. The ultimate compressive strength obtained in axial compression of prisms ranges between 0.8×10^{-3} to 3×10^{-3}; on the average it is taken to be equal to 2×10^{-3}. The ultimate compressive strain of the compression zone in members in bending is larger than

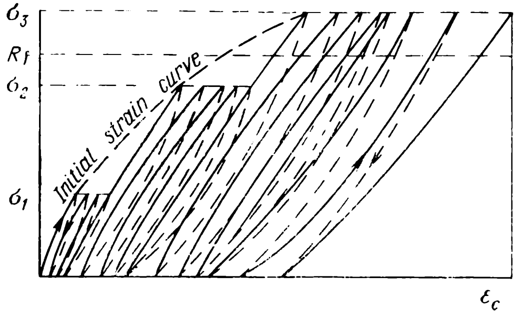


Fig. I.13. Stress-strain curve for a test specimen under repeated loading

that of prisms running into 2.7×10^{-3} to 4.5×10^{-3}. It is lower in members with a cross-section narrowing downwards and in T-sections, and increases with decreasing relative depth of the compression zone.

The ultimate tensile strain of concrete is 1/10 to 1/20 of its ultimate compressive strain; on the average it is taken to be 1.5×10^{-4}. Porous-aggregate concretes have a somewhat higher ultimate tensile strain. The ultimate tensile strain markedly affects the ability of concrete to resist cracking in tension zones.

6. Modulus of Elasticity and Specific Creep

The initial tangent modulus of elasticity of concrete in compression, E_c, gives a measure of only elastic strain produced by momentary working loads. Geometrically, it is defined as the slope of the linear part of the stress-strain curve corresponding to elastic strain (Fig. I.14)

$$E_c = \text{tg}\,\alpha_0 \tag{I.7}$$

The tangent modulus of elasticity of concrete in compression, designated E_c', describes the total strain in concrete (including creep) and is a variable quantity. On the diagram, it is defined as the slope

of the curve at any given point

$$E'_c = d\sigma_c/d\varepsilon_c = \operatorname{tg} \alpha \qquad (1.8)$$

Strain in concrete might be determined analytically in terms of the variable modulus of elasticity by integrating the following function

$$\varepsilon_c = \int (1/E'_c)\, d\sigma_c$$

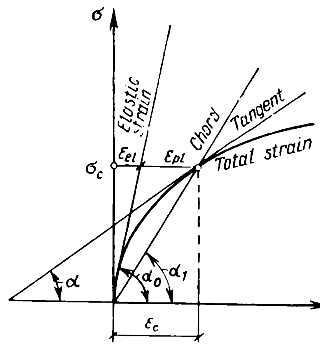


Fig. 1.14. To determining the modulus of elasticity of concrete

This way, however, is rather difficult because variations in E'_c cannot be described analytically. So, in design calculations of reinforced concrete structures use is made of an average quantity known as the secant modulus of elasticity. This is defined as the slope of the chord drawn through a point on the stress-strain curve corresponding to a given stress

$$E'_c = \operatorname{tg} \alpha_1 \qquad (1.9)$$

Since α varies according to the applied stress, the secant modulus of elasticity is also a variable quantity which is smaller than the initial tangent modulus of elasticity.

The initial tangent modulus can be related to the secant modulus by expressing the same stress in concrete, σ_c, in terms of the elastic strain, ε_{el}, and the total strain, ε_c:

$$\sigma_c = \varepsilon_{el} E_c = \varepsilon_c E'_c$$

Hence,

$$E'_c = \nu E_c \qquad (1.10)$$

where $\nu = \varepsilon_{el}/\varepsilon_c$ is the ratio of elastic to total strain in concrete. According to experimental data, ν may vary from 1 (for elastic behaviour) to about 0.15. As the stress-to-strength ratio, σ_c/R_{pr}, and the duration of loading, t, increase, ν decreases. The value of ν as a function of t may be found experimentally or from averaged stress-strain curves (Fig. 1.15).

Experience shows that E'_c for compression zones in concrete members in bending may be 15 to 20% above that in axial compression.

The secant modulus of elasticity of concrete in tension is defined as

$$E'_{c,ten} = \nu_{ten} E_c \qquad (1.11)$$

where $\nu_{ten} = \varepsilon_{el,ten}/\varepsilon_{c,ten}$ is the elastic-to-total strain ratio in tension. If the tensile stress applied is close to the ultimate axial

tensile strength, $\sigma_{c,ten} \rightarrow R_{ten}$, the average experimental value of ν_{ten} is 0.5.

The ultimate tensile strain of concrete is related to its ultimate tensile strength as follows

$$\varepsilon_{c,ten}^{ul} = R_{ten}/E'_{c,ten} = 2R_{ten}/E_c \tag{I.12}$$

The initial tangent modulus of concrete in compression and tension may be found by testing concrete prisms at $\sigma_c/R_{pr} \leqslant 0.2$. There

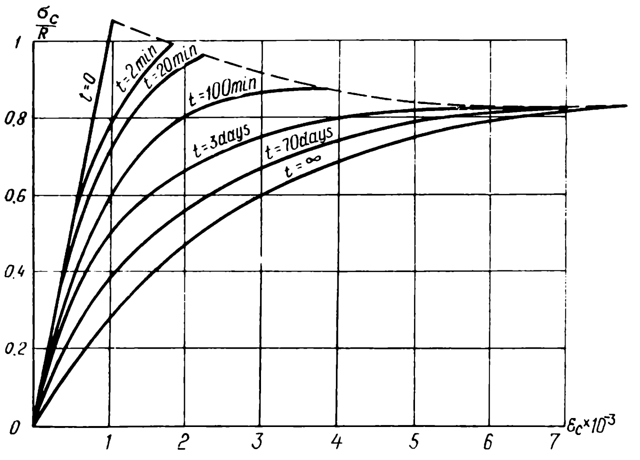


Fig. I.15. Stress-strain curves plotted for different durations of loading

exist various empirical formulas relating the initial tangent modulus to the concrete brand number. For example, for normally hardened heavy concrete, they can be related as

$$E_c = 550\,000\ R/(270 + R) \tag{I.13}$$

Thermal curing of concrete reduces E_c by 10%, and autoclaving by 25%. Porous-aggregate concretes being less resistant to strain have an initial tangent modulus of elasticity which is two thirds or one half of that of heavy concrete. Various empirical formulas for concrete are based on the relation between its initial tangent modulus, average density and cube crushing strength. For example, the ratio of the initial tangent modulus of porous-aggregate concrete to that of heavy concrete is defined by the following empirical relation

$$k = (\gamma_{por}/\gamma)^{3/2} \tag{I.14}$$

where γ_{por} and γ are the average densities of porous-aggregate and heavy concrete for the same concrete brand, respectively.

The values of the initial modulus of elasticity in compression and tension for different concrete types and brands are given in Appendix IV.

The modulus of rigidity of concrete is defined as

$$G_c = E_c/2\,(1 + \mu) \tag{I.15}$$

At a Poisson's ratio of 0.2 it is taken to be $0.4E_c$.

The specific creep of concrete in compression, designated C_c, is used to determine creep strain in concrete from the applied stress

$$\varepsilon_{cr} = C_c \sigma_c \tag{I.16}$$

Recalling that

$$\varepsilon_{cr} = \varepsilon_c - \varepsilon_{el} = \sigma_c/\nu E_c - \sigma_c/E_c = (1 - \nu)\,\sigma_c/\nu E_c$$

we may write for the creep characteristic

$$\varphi = (1 - \nu)/\nu \tag{I.17}$$

and for the specific creep

$$C_c = \varphi/E_c \tag{I.18}$$

The specific creep depends on the concrete brand and the stress-strength ratio, and varies with time.

To describe linear creep in concrete analytically, various mathematical models have been used and various theories of creep have been developed, of which the so-called hereditary ageing theory has gained recognition. However, the equations derived on the basis of this theory are not handy for practical calculations, especially where long-term processes, complex states of stress (such as eccentrical compression and bending of prestressed concrete members), and a high stress-strength ratio are involved. So, in practice, resort is often made to computer-aided analysis and discrete models with a great number of rod members working linearly in axial compression or axial tension at any time. In these models, each loading step has the stress-strain relation of its own taken from average experimental diagrams.

7. Physical and Mechanical Properties of Some Concrete Types

Dense Lime Concrete. This is a high-pressure steam-cured type of concrete using lime instead of cement as the binder. It belongs to heavy concretes using quartz sand as the aggregate. It has a good bond resistance with reinforcing steel and provides a durable anti-corrosive protection. Dense lime concrete comes in various brands up to M-600. Its initial tangent modulus of elasticity is two thirds

or one half of that of cement concrete having the same compressive strength. In addition, it has lower creep. Dense lime concrete is used in precast structural members. It has a limited applicability under adverse exposure conditions (such as atmospheric precipitation, large dynamic load, and so on).

Cellular Concrete. As its name implies, this type of concrete (mainly autoclaved) contains a multiplicity of artificially made cells. It is produced by mixing cement or lime with water and foam (foam concrete, foam-ash concrete, etc.) or by adding aluminium powder to form gas bubbles (gas concrete). The aggregate is fine (crushed) quartz sand. Cellular concrete is less impervious than ordinary concrete, so the reinforcing steel needs a coat of cement-water mixture or cement-asphalt putty for protection against corrosion. Its average density is relatively small, being from 600 to 1 200 kg/m^3. In the USSR, reinforced concrete structures use cellular concrete of M-50, M-75, M-100 and M-150 brands. Its initial tangent modulus of elasticity is one half or one third of that of conventional concrete having the same compressive strength. Cellular concrete has a considerable shrinkage ranging from 4×10^{-4} to 6×10^{-4}. Shrinkage in dry-cured cellular concrete is so high that it may lead to extensive cracking.

Cellular concrete finds application mostly in precast members for filler walls in industrial and civil buildings.

Heat-Resistant Concrete. This is used in structures intended to work at elevated temperature (above 200°C). According to the temperature conditions, its binding agents may be alumina cement, portland cement with admixtures, water glass (an aqueous solution of sodium silicate with the addition of crushed quartz sand and siliceous-fluoric sodium). The aggregates increasing the heat resistance of concrete are chromite, grog, broken brick, slag, basalt, diabase, etc. When heat-resistant concrete is allowed to cool after heating, the bonding strength between the concrete and deformed reinforcing bars remains unchanged. In the USSR, heat-resistant concrete comes in brands up to M-400. Its modulus of elasticity decreases with increasing temperature. Heat-resistant concrete is used in members for tunnel ovens, thermal power plants, blast-furnace foundations, and so on.

No-Fines Concrete. This type of concrete is used in localities lacking natural sand but having materials for coarse aggregates. No-fines concrete contains a great number of large pores, so its density and coefficient of thermal conductivity are low. It is only used in precast and in-situ concrete walls requiring concrete with the brand number below M-100.

Acid-Resisting Concrete. As its name implies, this type of concrete stands well attack by acids dissolved in water or acid fumes. Depending on the concentration of acid, use is made of portland-

pozzolana cement, portland-slag cement and water glass. It is used for underground structures, as a coating material in the chemical industry and nonferrous metallurgy, etc.

Polymerized-Cement Concrete. Sometimes, cement is polymerized by adding various soluble resins, divinyl-styrene latex, polyvinyl-acetate emulsion, etc. These admixtures ranging between 10 and 20% of the cement content of the mix raise the tensile strength several times, and also improve the bond between the concrete and reinforcing steel, and corrosion resistance. However, creep in polymerized-cement concrete is considerably higher than it is in conventional concrete. Also, strength gain of polymerized-cement concrete slows down under conditions of elevated humidity, so it cannot be steam-cured after forming (dry-curing is only permitted). Polymerized-cement concrete is used for waterproofing, surface repair, and so on. At present its application in bearing structures is limited by considerable creep and the relatively high cost of polymerization materials.

Polymerized Concrete. In this type of concrete, cement is completely replaced by polymeric binding agents. Here, the variations in properties are still more pronounced than they are in polymerized-cement concrete. At present, polymerized concrete is not used in bearing structures because its properties have not yet been studied properly and its cost is relatively high.

I.2. REINFORCING STEEL

1. Purpose and Types

Reinforcing steel is primarily the tensile component of reinforced concrete. Also, it is used to reinforce compression areas in members. The necessary amount of reinforcement is determined by appropriate design calculations.

In Soviet practice, reinforcing steel is classed into load-bearing reinforcement whose steel ratio is found by calculations, and erection reinforcement which is used for constructional and other reasons (such as the ease of assembly and erection). Erection reinforcement serves to hold the load-bearing reinforcing bars in place and distribute stresses uniformly between the bars of load-bearing reinforcement in the course of erection. In addition, erection reinforcement may carry some loads, such as shrinkage and thermal variation loads, which generally are not taken care of in design.

Load-bearing and erection reinforcement is combined into welded- or tied-wire fabric or bar mats which are placed in reinforced concrete members according to their behaviour under load (Fig. I.16).

Reinforcing steel may come in the form of *hot-rolled bars* and *cold-drawn wires*. The term bar includes reinforcing bars of any

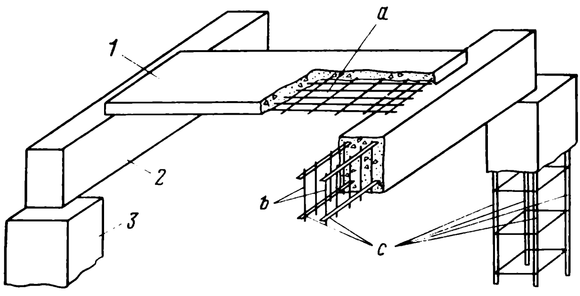


Fig. I.16. Reinforced concrete members and their reinforcement
(a) wire fabric; (b) bar mats; (c) reinforcing cage; 1 — slab; 2 — beam; 3 — column

diameter irrespective of how they are dispatched to the job—in bars ($d \geqslant 12$ mm, up to 13 m long) or hanks ($d \leqslant 10$ mm, up to 500 kg in weight).

After manufacture, hot-rolled reinforcing steel may be strengthened by heat treatment or cold drawing.

The principal forms that standard concrete reinforcement takes are *plain bars* and *deformed bars*. Standard reinforcing bars are rolled with protruding lugs or deformations which, as well as deformations on cold-drawn wire, serve to increase the bond between the reinforcement and the concrete (Fig. I.17).

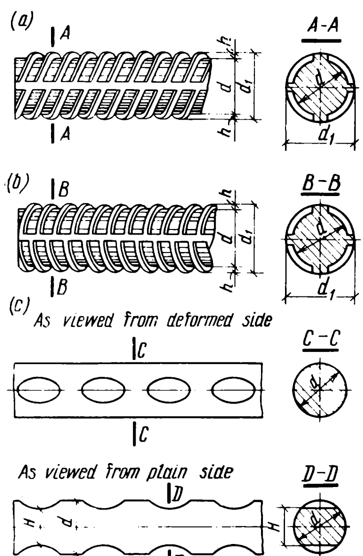


Fig. I.17. Soviet-made deformed reinforcing bars
(a) class A-II reinforcing bars; (b) class A-III and A-IV reinforcing bars; (c) high-strength wire

In a structural capacity, steel reinforcing bars and wire are utilized in concrete either as *prestressed steel* or as *unprestressed steel* that is not purposely prestressed before service loadings are applied.

Sometimes, use is made of rolled I-beams, channels or angles which, before the concrete has hardened, work as a metal structure carrying the dead load including the weight of the falsework and fresh concrete. Such reinforcement may prove efficient in in-situ large-span floors and ceilings, heavily loaded columns in the bottom floors of high-rise buildings, etc.

2. Mechanical Properties of Reinforcing Steel

Strength and Strain Characteristics. These are determined from stress-strain diagrams, σ_s-ε_s, which are plotted on the basis of tension tests (Fig. I.18). Hot-rolled reinforcing steel with a definite yield (soft steel) has a considerable elongation (up to 25%) after rupture (Fig. I.18*a*). The first unit stress at which strain increases without a considerable increase in load is called the *yield point*, σ_y, and the stress (or load) at or just before rupture is known as the ultimate tensile stress, σ_{ul}.

To increase its strength and reduce its elongation at rupture, hot-rolled reinforcing steel is doped with carbon or some other alloying elements, such as manganese, silicon, chromium, and so on. The carbon content, however, should not exceed 0.3 to 0.5%, because otherwise it would reduce the ductility and weldability of the steel. Manganese raises the strength of the steel without considerably reducing its ductility. Silicon contributes to the steel strength but reduces its weldability. The percentage of the alloying elements is low, usually ranging between 0.6 and 2%.

Heat treatment and hardening by drawing raise the strength of hot-rolled reinforcing steel several times. Heat treatment includes hardening (heating to 800 or 900°C and rapid cooling) followed by partial tempering (heating to 300 or 400°C and slow cooling). Thermally hardened steel moves into the plastic region gradually and exhibits no yield region on its stress-strain diagram (Fig. I.18*b*).

The behaviour of reinforcing steel having no definite yield point is characterized in terms of the proof yield strength designated $\sigma_{0.2}$,

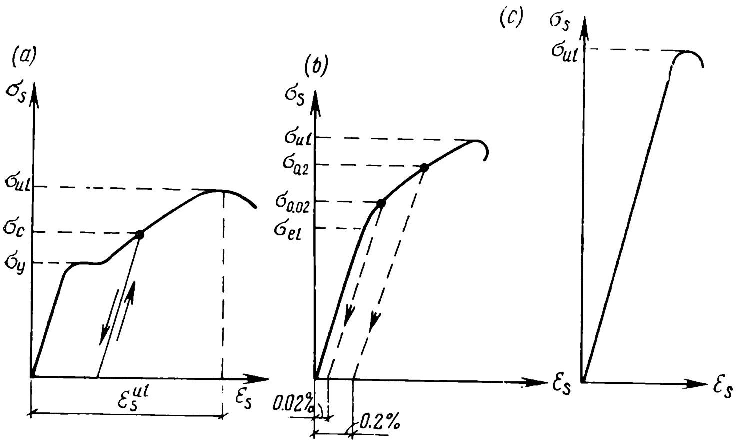


Fig. I.18. Stress-strain curves for reinforcing steel in tension
(*a*) steel with definite yield (soft steel); (*b*) steel without definite yield; (c) hard steel

that is, the stress at which the permanent set is 0.2%, and the proof elastic limit, $\sigma_{0.02}$, at which permanent set is 0.02%.

In hardening by drawing, hot-rolled reinforcing steel is cold-drawn until the stress exceeds the yield point, $\sigma_c > \sigma_y$. Cold drawing (or cold working) changes the crystal lattice so that the reinforcing steel hardens. As a result of a second drawing pass, σ_c becomes a new artificially raised yield point because all of the plastic strain has already been taken up (see Fig. I.18*a*). After several drawing passes through a succession of dies with decreasing diameters, the wire acquires the properties of hard steel, which means that its stress-strain diagram is linear almost till the break. As a result, the ultimate strength of the wire markedly increases, and its elongation at rupture decreases to 4 or 6% (Fig. I.18*c*).

Ductility. This property is of great importance for the behaviour of reinforced concrete structures under load, mechanized installation of reinforcement, prestressing, etc. Although reinforcing steel is inherently very ductile, an excessive reduction in its ductility may lead to brittle rupture under load, brittle fracture in prestressed steel at points where it is sharply bent or near anchorage, and so on. The ductility of reinforcing steel is expressed in terms of the percentage of elongation determined by a tensile test on standard bars having a length equal to five their diametres (or 100 mm), or by bending cold-state standard bars around a mandrel 3 to 5 bar diametres thick.

The total percentage of elongation, designated δ, is found by dividing the increase in the bar length measured across the fracture by the original bar length. The uniform percentage of elongation, δ_{un}, is determined by dividing the increase in the bar length without the neck region by the original bar length. The minimum allowable percentage elongation and requirements placed upon the cold-bending test are set up by relevant standards and specifications.

Weldability. The ability of reinforcing steel to be welded into reliable joints having no cracks or other imperfections in and near the weld is expressed in terms of weldability. This quality is important for shop fabrication of wire fabric and bar mats, splicing reinforcing bars, anchors, various embedded attachment fittings, and so on. Hot-rolled low-carbon and low-alloy reinforcing steels have good weldability. Reinforcing steel hardened by heat treatment or drawing may not be welded because thermally hardened steel would be tempered and cold-drawn wire would be annealed, so their hardening would be wasted.

Cold Brittleness. Deformed reinforcing bars made of semi-killed open-hearth and converter steel fracture at a temperature below —30°C; high-strength wire and thermally hardened bars fracture at a lower temperature.

Creep and Relaxation. Creep of reinforcing steel increases with increasing applied stress and temperature. Relaxation, or reduction in stress, occurs in reinforcing bars at a constant length, that is, when no deformation takes place. Relaxation depends on the mechanical properties and chemical composition of reinforcing steel, manner of its manufacture and exposure conditions. Wire hardened by cold-drawing, thermally hardened reinforcement, and highly-alloyed reinforcing bars show considerable relaxation. Relaxation of hot-rolled low-alloyed reinforcing steels is insignificant. Experimental data show that relaxation is most pronounced during the first several hours after manufacture, but it may continue for a long time. Relaxation of reinforcement markedly affects the behaviour of prestressed structures because it partly reduces the amount of prestressing.

Fatigue Failure. This occurs when a structure is subjected to repeated loading; it is brittle in nature. The fatigue limit of reinforcement depends on the number of loading cycles, the ratio of σ_{min} to σ_{max}, bonding strength, and cracking in the tension area. As the number of loading cycles increases, the fatigue limit of reinforcing steel decreases. Thermally hardened reinforcing steels have a reduced fatigue limit.

Dynamic Hardening. When a structure is subjected to high-rate short-time loads, the reinforcing steel hardens. With a high strain rate, reinforcing steels are elastic even at stresses exceeding the yield point. Plastic deformation sets in with a delay. Due to this delay the dynamic yield point, σ_{dyn}, exceeds the static yield point, σ_y. Dynamic hardening has a smaller effect on the proof yield strength of alloy and thermally hardened steels (having no definite yield point), and practically no effect on the ultimate tensile strength, σ_{ul}, of all types of reinforcing steel including high-strength wire.

High-Temperature Heating. When reinforcing steel is heated to a high temperature, its inner structure changes and strength decreases. For example, the yield point of Soviet-made hot-rolled class A-III reinforcing steel heated to 400°C decreases by 30%, those of class A-II and A-I reinforcing steel fall by 40%, and the modulus of elasticity of all grades decreases by 15%. At a temperature above 350°C, there is significant creep in the reinforcing steel of loaded structures. When heated to a high temperature, wire hardened by cold drawing is tempered and loses its hardness. So, the ultimate strength of high-strength wire decreases quicker than that of hot-rolled reinforcing bars. After heating and subsequent cooling, hot-rolled reinforcing steel recovers its strength completely, and high-strength wire only partly.

3. Classification of Reinforcing Steel

According to their main mechanical characteristics, Soviet-made hot-rolled reinforcing bars are divided into five classes, designated A-I, A-II, A-III, A-IV and A-V (Table I.1). Thermally hardened

TABLE I.1. Classification and Mechanical Characteristics of Reinforcing Steel

Type and class of reinforcing steel	Steel grade	Diameter, mm	Yield point, MPa	Ultimate strength, MPa	Percentage of elongation
Hot-rolled bars:					
plain, class A-I	Ст3, ВСт3	6-40	230	380	25
deformed:					
class A-II	ВСт5	10-40			
	10ГТ	10-32	300	500	19
	18Г2С	40-80			
class A-III	25Г2С	6-40			
	35ГС	6-40	400	600	14
	18Г2С	6-9			
class A-IV	20ХГ2Ц	10-22			
	80С	10-18	600	900	6
class A-V	23Х2Г2Т	10-22	800	1 050	7
Thermally hardened bars:					
class Ат-IV	—	10-25	600	900	8
class Ат-V	—	10-25	800	1 050	7
class Ат-VI	—	10-25	1 000	1 200	6
Ordinary wire:					
plain, class B-I	—	3-5	—	500	—
deformed, class Bp-I	—	3-5	—	550-525	—
High-strength wire:					
plain, class B-II	—	3-8	—	1 900-1 400	4-6
deformed, class Bp-II	—	3-8	—	1 800-1 300	4-6
Seven-wire strands, class K-7	—	4.5-15	—	1 900-1 650	—

bars are divided into three classes: Ат-IV, Ат-V and Ат-VI. The Russian letter "т" in the designation stands for thermal hardening.

Steel grades for each class of reinforcing bars have the same mechanical characteristics, but differ in composition. The grade designation shows how much carbon and alloying elements a given steel carries. For example, in the designation "25Г2С", the first two-digit number indicates the carbon content in points (hundredths of a per cent), the Russian letter "Г" indicates that the steel is alloyed with manganese, the numeral 2 indicates that the manganese content may reach 2%, and the Russian letter "С" stands for silicon. In the designations "20ХГ2Ц" and "23Х2Г2Т", "Х" stands for chro-

mium, "T" for titanium and "Ц" for zirconium (Russian letters throughout).

Class A-I reinforcing steel is plain, bars of all other classes are deformed.

The yield point of class A-I, A-II, and A-III reinforcing steel ranges between 230 and 400 MPa, the proof yield strength of class A-IV and A-V highly-alloyed reinforcing steel is from 600 to 800 MPa, and that of thermally hardened steel ranges between 600 and 1 000 MPa.

The percentage of elongation depends on the class of reinforcing steel. Class A-I has a considerable percentage of elongation ($\delta = 25\%$), classes A-II and A-III have a lower percentage of elongation ($\delta = 14$ to 19%), and classes A-IV, A-V and thermally hardened steel of all classes have a relatively small percentage of elongation ($\delta = 6$ to 8%)

The modulus of elasticity of reinforcing bars, E_s, somewhat decreases with increasing strength. It is equal to 2.1×10^5 MPa for classes A-I and A-II, 2×10^5 MPa for classes A-III and A-IV, and 1.9×10^5 MPa for class A-V and thermally hardened reinforcing steel.

Wire from 3 to 8 mm in diameter is divided into two classes, namely B-I and B-II. The former is ordinary (cold-drawn, low-carbon) wire mainly used in welded-wire fabric, the latter is high-strength (multi-pass drawn, carbon) wire used as prestressed reinforcement in prestressed members. Deformed reinforcing wire has an additional Russian letter "p" in its designation: Bp-I, Bp-II.

The main mechanical characteristic of reinforcing wire is the ultimate tensile strength, σ_{ul}, which increases with decreasing wire diameter. The ultimate tensile strength of ordinary wire is equal to 500 MPa, that of high-strength reinforcing wire ranges between 1 300 and 1 900 MPa. The percentage of elongation of reinforcing wire is relatively small, ranging between 4 and 6%. Rupture of high-strength wire is brittle in nature. The modulus of elasticity of reinforcing wire is equal to 2×10^5 MPa for classes B-I, B-II and Bp-II; 1.7×10^5 MPa for class Bp-I, and 1.8×10^5 for class K-7 strands.

In the Soviet Union, reinforcing steel is classed into standard or gauge sizes according to the nominal diameters. For deformed bars, these are the diameters of plain bars having the same cross-sectional area; for deformed ordinary and high-strength reinforcing wire, these are the wire diameters prior to deformation (see Table I.1 and Appendix VI).

4. Applications of Reinforcing Steel

Nonprestressed structures use class A-III hot-rolled reinforcing bars and ordinary wire available in fabric and mats, which have a relatively high strength. It is also possible to use class A-II reinforc-

ing steel, if not all of the strength of class A-III reinforcement is utilized due to excessive strain or crack width. Class A-I reinforcing steel may be used as erection reinforcement, hooks in tied mats, or transverse reinforcing bars in welded mats.

For prestressed structures, it is recommended to use class Aт-VI, Aт-V and Aт-IV thermally hardened bars and class A-V and A-IV hot-rolled reinforcing steel. It is advisable to reinforce members over 12 m long with reinforcing strands and high-strength wire.

The more recent additions are class Aтп-V reinforcing bars having an increased corrosion resistance, class K-19 19-wire strands, and some other new types of reinforcing steel.

Cold-brittle steels are not used to reinforce structures intended for service at subzero temperatures (out-of-doors and in unheated locations). Below —30°C, use is made of class A-II grade BCт5пс2 and class A-IV grade 80C reinforcing steel; below —40°C, class A-III grade 35ГС reinforcing steel.

One of important factors determining the applicability of reinforcing steel is its weldability. Class A-I, A-II, A-III, A-IV and A-V hot-rolled reinforcing bars and ordinary wire in fabric form are good for resistance welding. Thermally hardened bars of all classes and high-strength wire cannot be welded because they would lose their hardness. Also, arc welding is inapplicable to class A-IV and A-V reinforcing bars.

5. Wire Fabric and Bar Mats

As a rule, reinforcing steel for nonprestressed structures comes in the form of welded-wire fabric and bar mats. Longitudinal and transverse wires in fabric and mats (which are usually at right angles to one another) are joined by resistance spot welding. Fabrication of fabric and mats by welding has industrialized reinforcement work and reduced labour requirements and the cost of reinforcement.

Welded-Wire Fabric. According to an appropriate Soviet standard, welded-wire fabric is made of ordinary reinforcing wire from 3 to 5 mm in diameter and class A-III reinforcing steel from 6 to 9 mm in diameter. Welded-wire fabric comes in rolls and mats (Fig. I.19). The maximum diameter of longitudinal wires in rolls is 7 mm. Both longitudinal and transverse wires may serve as the load-bearing reinforcement. Wires placed at right angles to load-bearing wires work as erection reinforcement serving to distribute stress uniformly among the load-bearing wires. Fabric in which the transverse wire provides the minimum steel area necessary for fabricating and handling is termed "one-way". Where significant reinforcement is provided in both the transverse and longitudinal directions, the fabric is called "two-way". The maximum width, B, of welded-wire fabric in rolls is 3.5 m. The length of this type of fabric is limited by the

weight of rolls, which may range from 100 to 500 kg. The maximum width of fabric in mats is 2.5 m, and the maximum length, L, is 9 m. The dimensions B and L are taken between the centres of edge wires. Type designations of welded-wire fabric contain the main characteristics of the type. The numerator of a designation is the type of fabric stated in terms of $t/t_1/d/d_1$ (see the figure), and the denominator

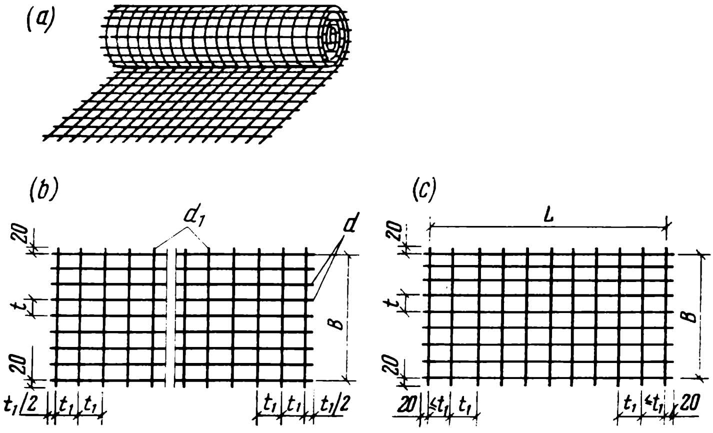


Fig. I.19. Welded-wire fabric
(a) in rolls; (b) unrolled fabric; (c) in mats

gives the width and length of the fabric, BL. For example, a welded-wire fabric mat 2 300 mm wide (B) and 5 900 mm long (L) having longitudinal wires 4 mm in diameter (d) spaced 250 mm apart (t) and transverse wires 8 mm in diameter (d_1) spaced 200 mm apart (t_1) is designated as follows: $\frac{250/200/4/8}{2\ 300.5\ 900}$.

Type designations for welded-wire fabric in rolls give only the width B.

Welded-Bar Mats. These are fabricated from longitudinal load-bearing and erection bars and transverse bars (Fig. I.20). In the plane of a mat, the longitudinal bars may be placed in one row (Fig. I.20*a* through *c*) or in two rows (Fig. I.20*d* and *e*). In addition, they may be located on one side (Fig. I.20*c* and *d*) or either side of the transverse bars (Fig. I.20*a*, *b* and *e*). The longitudinal bars placed on one side of the transverse bars facilitate resistance spot welding and contribute to the bond between the mat and the concrete. Sometimes, use is made of double bar mats in which single bar mats are joined by arc welding (Fig. I.20*f* and *g*), or bar mats in which the longitudinal load-bearing bar is joined by arc welding to an additional load-bearing bar (Fig. I.20*h*).

Concrete structural members may be reinforced by reinforcing cages which may be one-piece or built up from sections. Reinforcing cages (Fig. I.20*i*) may be wholly fabricated by welding in the shop or assembled at the job from bar mats joined by arc-welded transverse bars. In welded-bar mats and reinforcing cages, it is advisable to

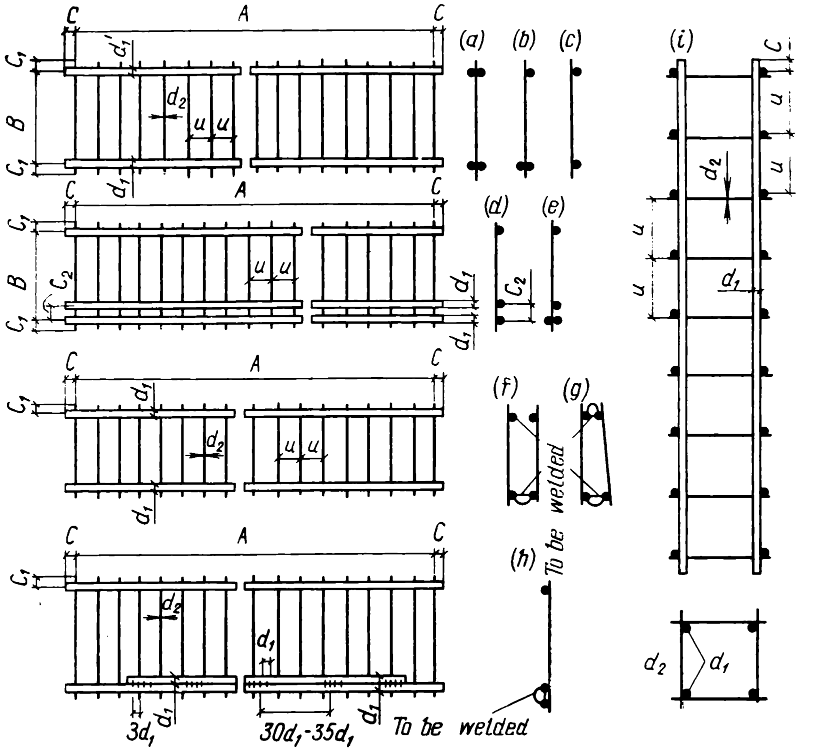


Fig. I.20. (*a*) through (*h*) welded-bar mats; (*i*) reinforcing cage

space transverse bars equal distances apart along the full or some part of mat length so that welding machines need not be readjusted.

The quality of spot welds on welded-wire fabric and bar mats depends on the diameter ratio of the longitudinal and transverse bars, which should be at least 0.3. For column reinforcement and deformed-wire fabric, the ratio may be reduced to 0.25. The minimum bar spacing also depends on bar diameters. The necessary data for the design of welded-wire fabric and bar mats are given in Appendix IX.

6. Prestressed-Concrete Reinforcing

Prestressed-concrete structures are reinforced by wires combined into strands and cables. The time and labour required to place and tension many small prestressing wires can be extensive. Besides,

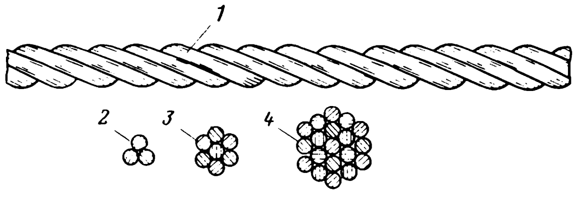


Fig. I.21. Wire strands
1 — side view; *2*, *3* and *4* — cross sections of 3-, 7- and 19-wire strands

the need to maintain the necessary spacing between the wires leads to an excessive cross-sectional area of a prestressed-concrete member.

To reduce the placing and tensioning expense, strands composed of several prestressing wires twisted together can be used (Fig. I.21). They consist of several wires having the same diameter helically

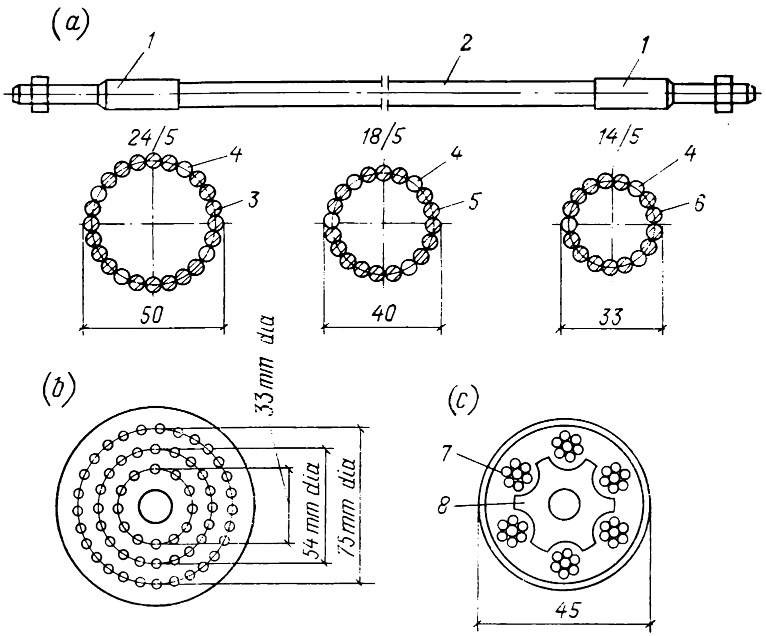


Fig. I.22. Prestressing cables
(*a*) one-layer cables; (*b*) multilayer cables; (*c*) cable using seven-wire strands; *1* — anchorage; *2* — side view; *3*, *5* and *6* — cross sections of 14-, 18- and 24-wire cables; *4* — short wires; *7* — wire strand; *8* — spacer

twisted around a central straight wire so that they cannot untwist. During the manufacture of a strand, the wires are deformed and fit closely to one another. At present, use is most commonly made of class K-7 seven-wire strands made of wires 1.5 to 5 mm in diameter. The diameter of a class K-7 strand is equal to three diameters of its wires. The irregular surface of strands ensures a good bond between the concrete and strands. In addition, very long strands are available, so that they can be used in continuous joint-free structures.

There exist other types of strands; they are made of a large number of wires 1 to 3 mm in diameter. All strands belong to class K-n, where n is the number of wires in a strand. Strands composed of a large number of wires are used as prestressing reinforcement in large structures. Prestressing strands exhibit a somewhat greater elongation under load than prestressing wire. To reduce nonelastic strain, they are tensioned in advance.

Prestressing cables consist of high-strength wires running parallel to one another (Fig. I.22). One-layer cables are made of 14, 18 or 24 wires placed around a circumference with spacings to let grout inside the cable, and wrapped by soft wire. Stronger cables are composed of parallel strands instead of separate wires. In multilayer cables, as many as 100 wires 4 to 5 mm in diameter may be used. Prestressing cables are not supplied by reinforcing steel manufacturers, so they are fabricated directly at the job or erecting shops.

7. Splicing Reinforcing Steel

Types of Welded Joints. The preferred reinforcing-bar splice welds are the butt-welded joints which can be made by various techniques in the shop or directly at the job.

In the shop, class A-I, A-II, A-III, A-IV and A-V reinforcing bars (for example, reinforcing-bar blanks and large diameter splice bars) are joined by resistance welding (Fig. I.23*a*). The diameter ratio of the bars being joined, d_1/d_2, should not be less than 0.85, the minimum bar diameter being $d_1 \geqslant 10$ mm. If a special welding technique is used, the diameter ratio may be reduced to 0.5.

In the field, class A-I, A-II and A-III reinforcing bars (for example, stick-outs in precast members) are arc-welded in a pool of molten metal contained within a permanent (reusable) mould (Fig. I.23*b*). If the bars to be welded are less than 20 mm in diameter, splices are made by double-lap joints. To this end, reinforcing bars are arc-welded to splice bars by four (Fig. I.23*c*) or two longitudinal side fillet welds (Fig. I.23*d*). In the former case, the weld length is $l = 4d$; in the latter, use is made of longer splice bars, and the weld length is $l = 8d$. The throat of the weld, h, should be equal to one fourth of the bar diameter, but it must be not less than 4 mm; the

width of the weld, b, should be one half of the bar diameter but not less than 10 mm (Fig. I.23e).

T-joints of reinforcing bars 10 to 16 mm in diameter and plates $0.75d$ thick (made of steel sheets or strips) are made by submerged arc welding (Fig. 1.23f). Lap splices of reinforcing bars 8 to 40 mm in diameter with plates or flat rolled elements may be made by longitudinal-fillet arc welding (Fig. I.23g).

Nonwelded Lap Splicing. Class A-I, A-II and A-III reinforcing bars may be lap-spliced where not all of the reinforcement strength

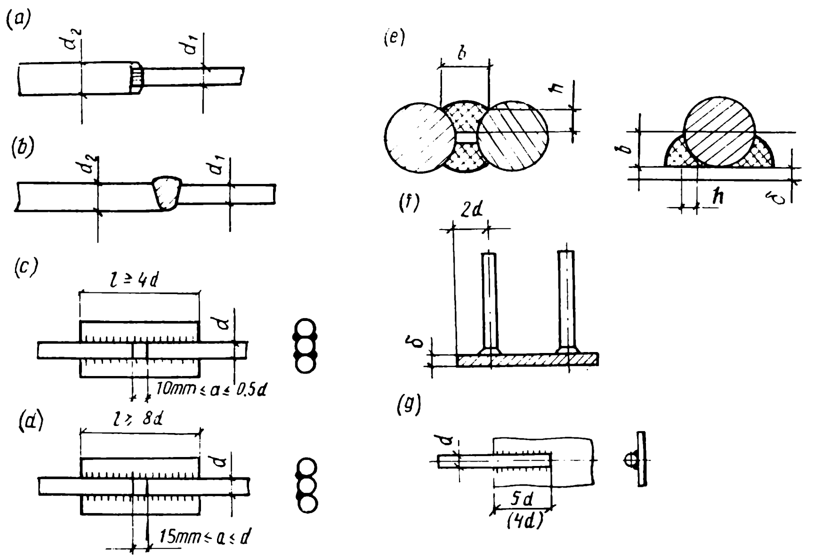


Fig. I.23. Welded joints

(a) butt resistance welding; (b) arc welding in a pool of molten metal contained within a reusable mould; (c) arc welding with splice bars, four side fillet welds; (d) arc welding with splice bars, two side fillet welds; (e) legs of a weld; (f) T-joint of reinforcing bars and a plate; (g) lap splice of a reinforcing bar and a plate

is used. The lap of a splice should be from 20 to 50 bar diameters. This, however, is the least desirable type of splicing because it requires additional steel and is imperfect in construction.

Wire fabric may be lap-spliced in the load-bearing direction (Fig. I.24). The load-bearing bars of the fabric pieces to be joined may be located in one or different planes. In the tension zone, each piece should contain at least two transverse bars welded to all the longitudinal bars of the fabric. If the load-bearing bars are deformed, one or both fabric pieces may have no welded transverse bars within the lap. The necessary lap of a splice, l_l, is determined according to the necessary anchorage by Eq. (I.20).

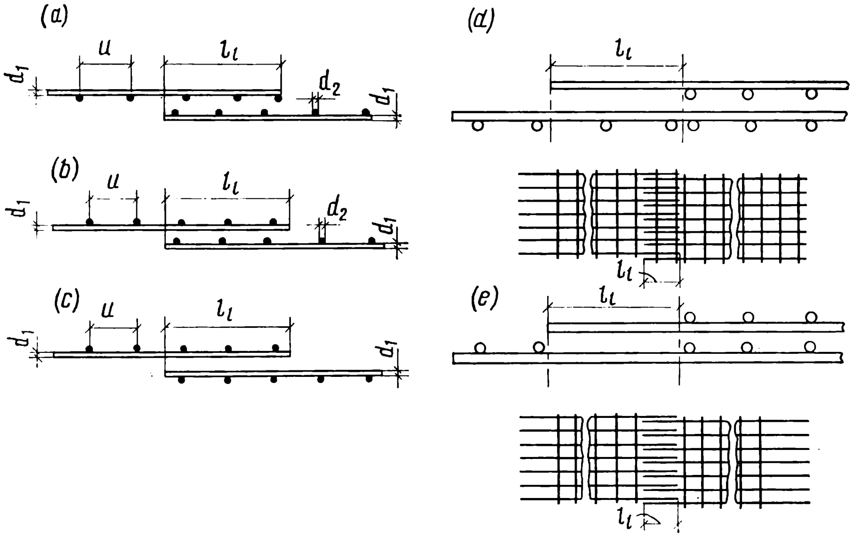


Fig. I.24. Splicing of welded-wire fabric in the load-bearing direction
(*a*) plain wires, transverse wires in the same plane; (*b*), (*c*) plain wires, transverse wires in different planes; (*d*) deformed wires, no transverse wires within the lap in one piece to be spliced; (*e*) deformed wires, no transverse wires within the lap in both pieces to be spliced

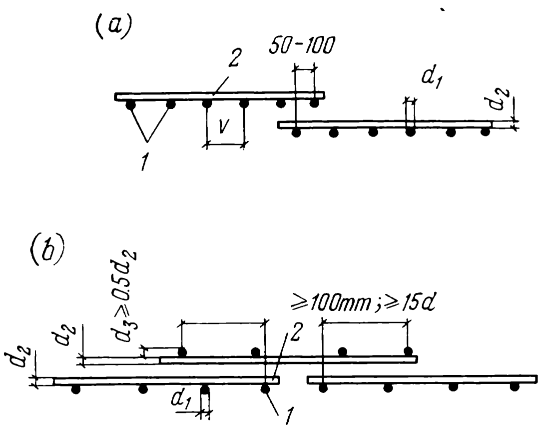


Fig. I.25. Splicing of welded-wire fabric in the direction of the erection bars
(*a*) lap splicing; (*b*) with additional splice fabric; *1* — load-bearing wires; *2*— erection wires

Lap splices on bar mats may be made when the longitudinal bars are located on one side of the transverse bars. Like welded-wire fabric, bar mats are spliced in the load-bearing direction. Here, additional stirrups or transverse bars with a spacing of not more than 5 longitudinal-bar diameters are placed within the lap.

The joints in welded-wire fabric and bar mats in a structure must be staggered.

In the non-load-bearing direction (for erection wires), welded-wire fabric may also be lap-spliced (Fig. I.25). The lap of a splice is taken to be 50 mm for erection wires less than 4 mm in diameter, and 100 mm for erection wires more than 4 mm in diameter. When the load-bearing bars are 16 mm in diameter or more, additional splicing fabric is used, with an overhang of 15 diameters, but not less than 100 mm on either side.

8. Nonmetallic Reinforcement

Investigations to find ways for reducing steel consumption have led to nonmetallic reinforcement. It is produced from thin glass fibres bonded into reinforcing bars by synthetic resins. Glass-fibre reinforcing bars provide a good bond with the concrete; they have a high tensile strength (up to 1 800 MPa) and a low modulus of elasticity (45 000 MPa). Owing to this, glass-fibre reinforcement may be efficient in prestressed-concrete structures. The major drawbacks of glass-fibre reinforcement are low resistance to alkali reactions and a considerable decrease in strength with time.

I.3. REINFORCED CONCRETE

1. Off-Site Fabrication

Reinforced concrete members should be designed so that their overall dimensions, cross-sections and reinforcement would be optimal for manufacture at precasting plants and erection on the site. Members which can be mass-produced by high-efficiency equipment without labour-consuming manual operations are always preferable. The design of a member and the manufacture procedures are closely related. There exist several technologies for precast concrete manufacture.

Line Production. With this type of manufacture, concrete members are made in forms installed on cars moving by rail from one work station to another. As a car moves on, workmen successively carry out all the necessary operations, namely erect the reinforcement, tension the prestressing steel, install void formers into hollow-

core slabs, place and compact the concrete mix, remove the void formers, and cure the concrete in a moist atmosphere and at elevated temperature in order to accelerate its hardening. All cars are advanced at a predetermined speed. This high-efficiency type of manufacture is utilized by major plants in the quantity production of members having a relatively small mass.

Stage-by-Stage Production. With this type of manufacture, all operations are carried out by appropriate plant shops, and a form with a member is transported from station to station by cranes. Here, no speed is predetermined.

Casting-Bed Technology. A distinction of this technology is that members remain at the same place during their manufacture and heat curing, and the units carrying out the necessary operations move along the stationary forms. Casting beds are equipped with travelling cranes, concrete placers, and vibrators to compact the concrete mix. Structural members are fabricated in plain or shaped forms (matrices or cassettes). This technology is used to manufacture large and prestressed members for industrial buildings (such as trusses, roof beams, crane beams, columns, and so on).

Panels for floors, ceilings and walls of civil buildings are widely made in cassettes. Members are made in stationary beds consisting of a faggot of vertical metal cassettes, so that several panels may be produced at a time. The assembly and disassembly of cassettes are mechanized. Reinforcing cages are installed inside each compartment of a cassette. Concreting is by a thin-consistency mix pneumatically applied through pipes. Owing to their vertical position, the surface of such members is even and smooth.

In the vibration-rolling process, thin floor, ceiling and wall panels are manufactured on a continuously moving belt whose smooth or corrugated surface serves as a mould. After a reinforcing cage has been installed, the concrete mix is placed on the belt, which is then vibrated and compacted from above by rollers. The members being rolled are covered from above and heated from below. During their motion with the belt (for several hours), they attain the necessary strength; after cooling on racks, finished products are transported to a storehouse. The operations are carried out at a rate keyed to the speed of the forming belt.

The whole kit of precast members required for the construction of a building cannot be fabricated by any one of the above processes. So, precasting plants and yards use several production processes simultaneously. The advent of new and more promising designs may in some cases require certain improvements in the production techniques or even new processes, which, in turn, may necessitate a re-design of the product.

2. Prestressed Concrete. Methods of Prestressing

Prestressed members are those in which considerable compressive stresses are artificially induced by tensioning the reinforcing steel before they are loaded. Initial compressive stresses are applied to those concrete areas which, when loaded, work in tension. Prestressing

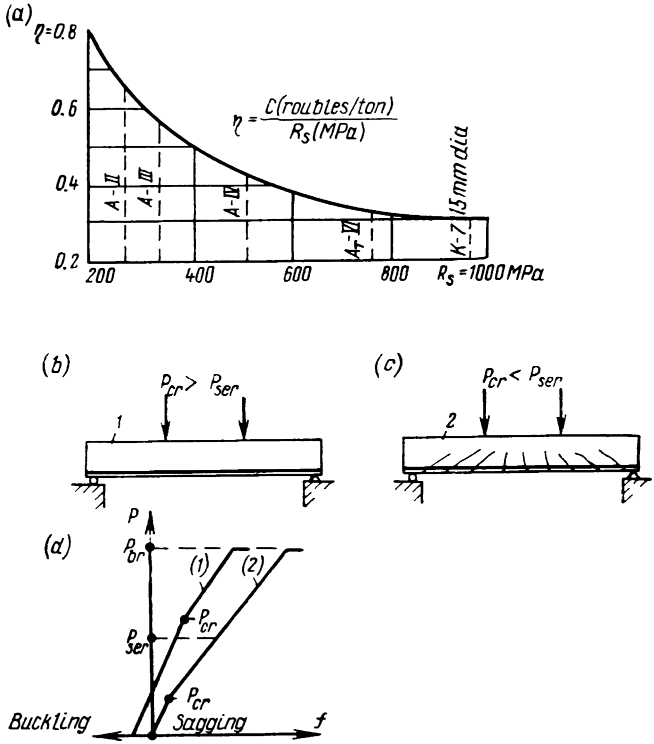


Fig. I.26. To the analysis of prestressed members

(*a*) relative cost of reinforcing steel; (*b*) prestressed beam (*1*); (*c*) nonprestressed beam (*2*); (*d*) load-sag (*P*-*f*) curve

imparts greater crack resistance to the member and enables the designer to take advantage of high-strength steel to cut down steel consumption and the cost of a structure.

The ratio of the cost of reinforcing steel, C (roubles/tonne), to its strength, R_s, designated η, decreases with increasing strength (Fig. I.26*a*). Therefore, it pays to use high-strength steel instead of hot-rolled reinforcing bars. In nonprestressed structures, however, high-strength steel may not be used because the high tensile stresses

in it and the respective tensile strain in the concrete result in wide cracks which bereave the structure of the qualities necessary in service.

The major advantages of prestressed concrete are its cost reduced by the use of high-strength steel and high crack resistance which, in turn, contributes to its stiffness, dynamic load strength, corrosion resistance, and service life.

In a loaded prestressed beam (Fig. I.26*b*), the concrete begins to work in tension only after all of the compressive prestressing has been counteracted. Here, the load, P_{cr}, which causes cracking or limited opening of cracks exceeds the service load, P_{ser}. As the load increases to the ultimate breaking value, P_{br}, the stresses in the reinforcement and concrete reach their ultimate values. In a similar nonprestressed beam (Fig. I.26*c*), P_{cr} is below P_{ser}, but P_{br} is about the same for either beam because the ultimate stresses in the reinforcement and concrete are equal.

As is seen, loaded prestressed structures work with no or narrow cracks ($P_{ser} < P_{cr} < P_{br}$), whereas nonprestressed structures develop cracks ($P_{cr} < P_{ser} < P_{br}$) and sag under load (Fig. I.26*d*). This is the main difference between prestressed and nonprestressed members, affecting their analysis, design and manufacture.

There are two basic methods of prestressing concrete, pretensioning and posttensioning. In the former, the tendons (single wires or wire strands) are erected in a form where they are anchored to an abutment at one end and tensioned at the other by jacks or other devices until the required stress is attained (Fig. I.27*a*). After the concrete has attained the necessary cube crushing strength before the compression, $R_0 \geqslant 0.8\overline{R}$, the wires or strands are released from the abutments. Being bonded to the concrete, the wires or wire strands are unable to return to their original length and transfer the prestressing force to the concrete by bond resistance together with radial compression (Fig. I.27*b*). In the so-called continuous reinforcement, the form is placed on a bed equipped with studs which carry sleeves. A special machine winds the wires on the sleeves at a specified stress, after which the ends of the wires are clamped by a die clip (Fig. I.27*c*). After the concrete has attained the necessary strength, the member with embedded sleeves is removed from the bed studs, and the released wires compress the concrete.

Bar tendons may be pretensioned by the electrothermal process. Bars with upset ends are heated by an electric current to 300-350°C, placed in a bed and anchored to abutments. As they cool, the bars tend to restore their initial length, and are thus pretensioned.

In posttensioning, the tendons (steel wires, strands, or bars) are tensioned against and anchored to the concrete after it has developed adequate strength, $R_0 \geqslant 0.8\overline{R}$ (Fig. I.27*d*). Here, the tendons are inserted in ducts, tensioned, anchored, and grouted (Fig. I.27*e*).

The stress in the tendons is checked after all of the compressive force has been transferred to the concrete. The ducts or raceways in the concrete, exceeding the tendon diameter by 5 to 15 mm, are produced by installing, before the concrete is poured in the forms, withdrawable steel spings, rubber hose or permanent corrugated steel tubing. After the concrete has been compressed, the steel is bonded to it by injecting cement paste or grout under pressure into the raceways. This is done through pipes placed during the manufacture of the

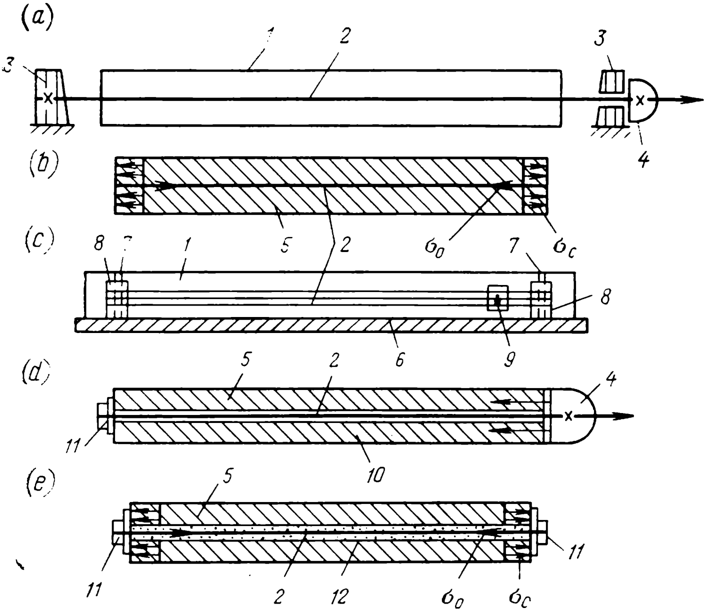


Fig. I.27. Methods of prestressing

(a) pretensioning (schematic diagram); (b) finished member; (c) pretensioning in continuous reinforcement; (d) posttensioning (schematic diagram); (e) finished member; *1* — mould; *2* — reinforcement; *3* — abutment; *4* — jack; *5* — hardened concrete; *6* — casting bed; *7* — studs in the casting bed; *8* — sleeves; *9* — clip; *10* — raceway; *11* — anchorage; *12* — grouted raceway

member. If prestressing steel is located outside the concrete (for example, in pipes, tanks, and so on), it is wound by special wrappers, and the compressive stress is simultaneously transferred to the concrete. Here, after the steel has been tensioned, the protective cover is applied by shotcreting (concreting under pressure).

Pretensioning can be mechanized more easily, so it is mostly used in the off-site manufacture of concrete members. Posttensioning is mainly used in the production of large-size structures and for their joining at the job.

3. Bond Between Reinforcing Steel and Concrete

Owing to the bond between the materials, the reinforcing steel does not slip in the concrete under load. Bonding strength is tested by the "pull-out" and "press-in" tests (Fig. I.28*a*). Experiments show that the reliance for the bond strength is on (1) the bearing of lugs and the strength of concrete between lugs (Fig. I.28*b*); (2) friction between the steel and the concrete resulting from shrinkage; and (3) adhesion between the cement paste and the steel surface, attributed

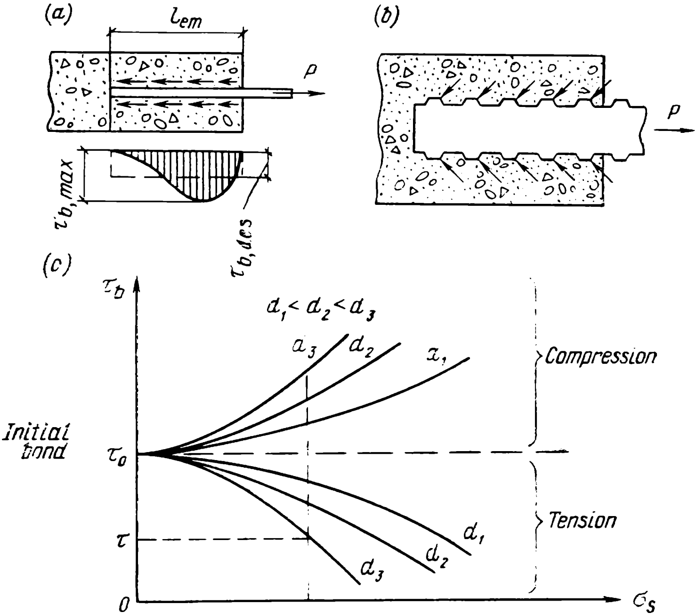


Fig. I.28. Bond between reinforcing steel and concrete

to the adhesive properties of the cement gel. The first factor accounts for about 75% of the total resistance to slip. With plain reinforcing steel, the resistance to slip is one half or one third of that with deformed bars. According to experimental data, the bearing stress is nonuniformly distributed along the embedded length of a bar, and the maximum bearing stress, $\tau_{b,\,max}$, does not depend on the embedded length, l_{em}. The average bearing stress is defined as the ratio of the force in a bar, N, to the surface of the embedded part of the bar

$$\tau_{b,\,av} = N/l_{em}u \tag{I.19}$$

where u is the bar circumference. For plain bars and medium concrete brands, the average bearing stress ranges between 25 and 40 kg/cm^2.

The bond strength increases with increasing concrete brand number, decreasing water-cement ratio, and age. When the embedded length of a bar is insufficient, additional short bars or washers are welded on (class A-I plain bars are hooked at the ends). When a bar is pressed in concrete, the bond strength is greater than in pulling out because the concrete resists the lateral expansion of the compressed bar. As the bar diameter and the stress, σ_s, increase, the bond strength in compression rises, and that in tension falls (Fig. I.28*c*). So, for better bond strength the diameter of bars in tension should not be less than a certain minimum value.

4. Anchorage

The anchorage of reinforcing steel in concrete structures is carried out by embedding the steel in the concrete past the section in question for a length sufficient to transfer stresses from the steel to the concrete (due to the bond between the steel and the concrete), and also by anchoring devices.

Anchorage of Nonprestressed Steel. Class A-I plain bars have hooks at their ends, with a diameter of 2.5d in nonporous-aggregate

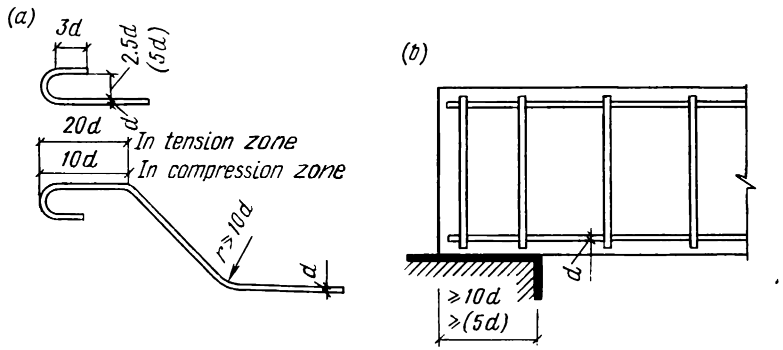


Fig. I.29. Anchorage of nonprestressed reinforcing steel
(*a*) plain bars; (*b*) deformed bars at a free support

concrete, and 5d in porous-aggregate concrete (Fig. I.29*a*). In welded wire fabric and mats made of plain bars, the anchors are transverse bars, so they are used without hooks at the bar ends. Deformed bars form a much better bond with concrete, so no bent bar anchorage is used in this case, either.

Nonprestressed deformed bars are embedded past the section normal to the longitudinal axis of a member, within which the reinforcing steel is assumed to develop the full design strength for the entire anchorage length

$$l_{an} = (m_{an}R_s/R_{pr} + \Delta\lambda_{an})\,d \qquad \text{(I.20)}$$

but not less than $l_{an} = \lambda_{an} d$, where m_{an}, $\Delta\lambda_{an}$, λ_{an}, and also the minimum safe value of l_{an} are taken from Table I.2; R_s is the design strength of the reinforcing steel (see Chapter II); R_{pr} is the design axial compressive strength of the concrete (see Chapter II); and d is the bar diameter.

TABLE I.2. To Determining the Anchorage Length of Nonprestressed Deformed Bars

State of stress and anchorage conditions	m_{an}	$\Delta\lambda_{an}$	λ_{an}	l_{an}, mm, min
Tensile steel in tension concrete	0.7	11	20	250
Tensile or compressive steel in compression concrete	0.5	8	12	200

If reinforcing bars are embedded past the section normal to the longitudinal axis of a member, within which not all of their design strength is used, R_s should be multiplied by the ratio of the cross-sectional area of the steel required when the total design strength is utilized, to the cross-sectional area of the actual reinforcing steel.

At the outer free supports of bent members, the tension longitudinal bars are carried past the internal face of each support for not less than $10d$; if there are no inclined cracks in the tension area, the bars are carried past the internal face for not less than $5d$ (Fig. I.29*b*).

Anchorage of Prestressed Steel. In pretensioning deformed bars or wire strands in structures made of sufficiently strong concrete, no special anchoring devices are needed. In posttensioning wire cables or pretensioning plain high-strength wire having poor bond resistance, anchors are used always. The anchorage length of prestressed steel used without anchoring devices is taken to be equal to the length of the area where the stress is transferred from the reinforcing steel to the concrete, which is defined as

$$l_{tr} = (m_{tr}\sigma_{tr}/R_0 + \Delta\lambda_{tr})\, d \tag{I.21}$$

where m_{tr} and $\Delta\lambda_{tr}$ are taken from Table I.3; R_0 is the transfer strength of the concrete (the cube crushing strength of the concrete attained by the moment of the stress transfer); σ_0 is the preliminary stress in the steel with allowance for losses; and σ_{tr} is taken to be equal to R_s or σ_0, whichever is the greater.

For members made of porous-aggregate concrete, the value calculated by Eq. (I.21) should be multiplied by 1.2. For all types of deformed bars, l_{tr} should be at least $15d$. When the stress is instantaneously transferred from deformed bars up to 18 mm in diameter

TABLE I.3. To Determining the Transmission Length for Stressed Steel without Anchorage

Type and class of reinforcing steel	m_{tr}	$\Delta\lambda_{tr}$
Deformed bars of any class and diameter	0.3	10
Class Bp-II high-strength wire:		
5 mm in diameter	1.8	40
4 mm in diameter	1.8	50
3 mm in diameter	1.8	60
Class K-7 wire strands:		
15 mm in diameter	1.25	25
12 mm in diameter	1.4	25
9 mm in diameter	1.6	30
7.5, 6 and 4.5 mm in diameter	1.8	40

(released from abutments by cutting), l_{tr} is multiplied by 1.25. With structural members used at a design temperature below —40°C, the values of $\Delta\lambda_{tr}$ are doubled.

The stress in the steel is considered to be linearly varying from zero at the edge of a member to its maximum value at the section lying within l_{tr} of the member edge (Fig. I.30).

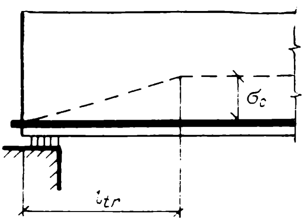


Fig. I.30. Linear variation in prestress within the transmission zone

In order to prevent the concrete from spalling when transferring the stress, the member ends are reinforced by embedded items with anchorage bars, stirrups, mesh, and so on.

Wire strands and deformed bars are gripped, tensioned and fastened to buttresses by grip tensioning units (Fig. I.31*a*); in addition, use is also made of welded short bars or washers (Fig. I.31*b*), threaded attachments which do not weaken the cross section of the tendons (Fig. I.31*c*), button or rivet heads (Fig. I.31*d*), and upset bar ends with bushes (Fig. I.31*e*).

In posttensioning, anchorage should ensure good transfer of the prestress from the tendons to the concrete. At the member ends where anchors are placed, the concrete is reinforced by additional stirrups, welded-wire mesh, and spirals. For uniform transfer of the prestress, anchors are placed on steel bearing plates.

Prestressing cables are anchored by factory-made socket-type anchorages which consist of a threaded stud inserted into a cable, and a mild steel fitting put on the cable (Fig. I.32*a*). When the anchor is drawn through an upsetting ring, the metal of the fitting yields and compresses the cable wires (Fig. I.32*b*). After the cable has been

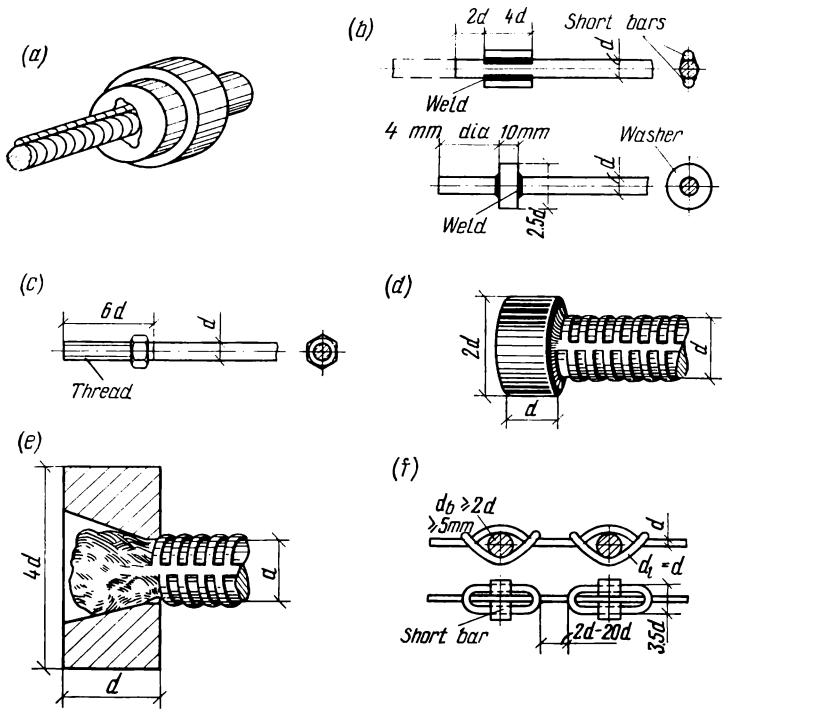


Fig. I.31. Anchorage of prestressed reinforcing steel

(*a*) grip tensioning unit; (*b*) short bars and washer welded to reinforcing bars; (*c*) nut at the end of a threaded bar; (*d*) button or rivet head; (*e*) upset bar end with a bush; (*f*) loops and short bars for the anchorage of plain high-strength wire

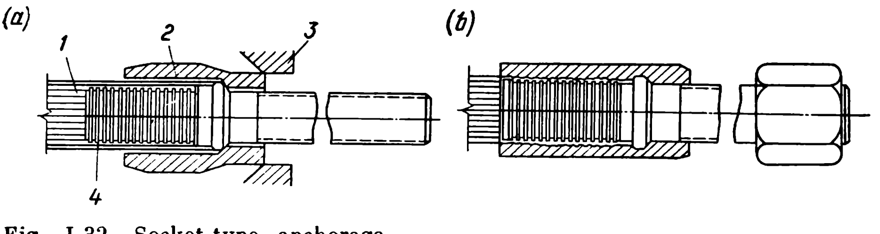


Fig. I.32. Socket-type anchorage

(*a*) before pressing; (*b*) after pressing; *1* — cable; *2* — socket; *3* — upsetting ring; *4* — threaded stud

posttensioned by a jack, it is anchored by turning the nut on the stud as far as it will go.

Figure I.33 shows the Freyssinet system using a wedging principle. A double-acting hydraulic jack is placed in position and gripped to the wires by wedges. The main piston is pumped to the required pressure and, while maintaining that pressure, the inner piston is pumped to drive home the plug.

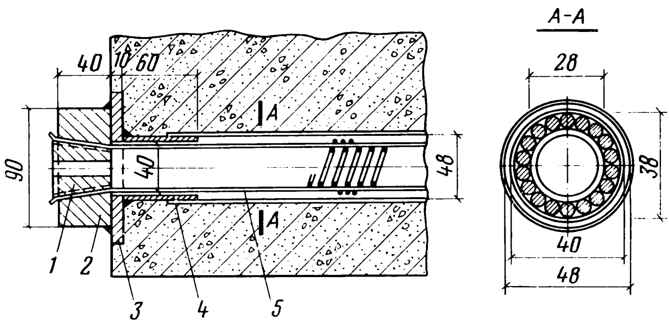


Fig. I.33. Freyssinet anchorage

1 — cone plug; *2* — female anchor cone; *3* — steel bearing plate; *4* — sleeve; *5* — reinforcing cable

Multilayer cables are anchored by basket-type anchorages (Fig. I.34). The cable is posttensioned by a jack to the required stress. Then, the gap between the anchorage and the member end is filled up by notched shims which maintain the tension.

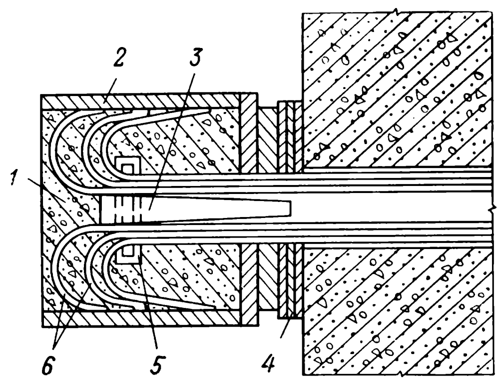


Fig. I.34. Basket-type anchorage for posttensioned multilayer cables

1 — concrete pressed into the basket to hold the cable wires; *2* — steel basket with welded bottom; *3* — steel stud; *4* — steel shims; *5* — ring; *6* — hooks at the wire ends

5. Shrinkage of Reinforced Concrete

In reinforced concrete structures, the bond between the reinforcing steel and the concrete restrains free shrinkage of the concrete. Experimental data show that in some cases shrinkage and swelling of reinforced concrete are one half of those in plain concrete (Fig. I.35). The restrained shrinkage deformation leads to initial internally balanced stresses in reinforced concrete, owing to which the concrete is subjected to tensile stress and the reinforcing steel to compressive stress. The difference between the free shrinkage in the plain concrete area, $\varepsilon_{sh,c}$, and the restrained shrinkage in the reinforced area, $\varepsilon_{sh,s}$ (Fig. I.36)

$$\varepsilon_{c,ten} = \varepsilon_{sh,c} - \varepsilon_{sh,s} \tag{I.22}$$

is responsible for the average tensile stress in the concrete

$$\sigma_{c,\,ten} = \varepsilon_{c,\,ten} E'_{c,\,ten} \tag{I.23}$$

This stress is maximal where the concrete is in contact with the reinforcing steel. The strain, $\varepsilon_{sh,s}$, is elastic for the reinforcing

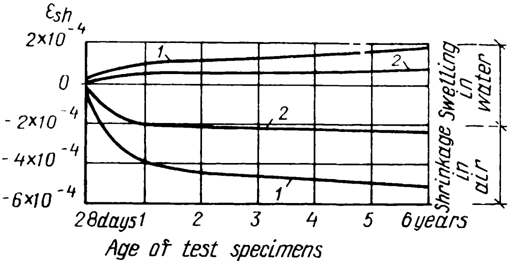


Fig. I.35. Shrinkage and swelling
(1) plain concrete; (2) reinforced concrete

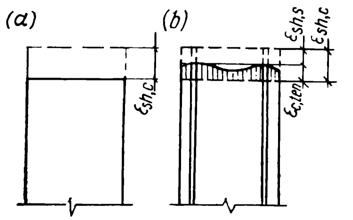


Fig. I.36. Shrinkage strain
(a) plain concrete test specimen; (b) reinforced concrete test specimen

steel, so the steel is subjected to compressive stress

$$\sigma_s = \varepsilon_{sh,s} E_s \tag{I.24}$$

The equilibrium between the internal stresses in a member with symmetrical compressive and tensile reinforcement is described by the following equation

$$\sigma_s F_s = \sigma_{c,\,ten} F \tag{I.25}$$

where F_s is the steel area, and F is the cross-sectional area of the member.

Hence,

$$\sigma_s = \sigma_{c,\,ten} F/F_s = \sigma_{c,\,ten}/\mu \tag{I.26}$$

where $\mu = F_s/F$ is the reinforcement ratio.

Substituting into Eq. (I.22) the strains expressed in terms of stresses according to Eqs. (I.23), (I.24) and (I.26)

$$\sigma_{c,\,ten}/E'_{c,\,ten} = \varepsilon_{sh,c} - \sigma_{c,\,ten}/\mu E_s$$

gives the tensile stress in the concrete

$$\sigma_{c,\,ten} = \varepsilon_{sh,c} E_s/(1/\mu + n/\nu_{ten}) \tag{I.27}$$

where $n = E_s/E_c$ is the ratio of the modulus of elasticity of the reinforcing steel to the modulus of elasticity of the concrete.

As reinforced concrete shrinks, the tensile stress in the concrete is a function of the amount of free shrinkage, $\varepsilon_{sh,c}$, the reinforcement ratio, μ, and the concrete brand. An increased amount of steel results in a higher tensile stress, $\sigma_{c,\,ten}$; should the stresses reach

the ultimate tensile strength, R_{ten}, shrinkage cracking takes place. In members without compressive reinforcement, subjected to restrained shrinkage, the forces in the steel are eccentrically applied to the member section, so the tensile stress in the concrete increases

$$\sigma_{c,\,ten} = 2.25\varepsilon_{sh,\,c}E_s/(1/\mu + 2.25n/\nu_{ten}) \tag{I.28}$$

Initial tensile stresses caused in the concrete by shrinkage contribute to earlier cracking in the areas tensioned by loading. Cracking, however, reduces the effect of shrinkage. During breakdown, shrinkage does not affect the bearing capacity of statically determinate reinforced concrete members.

In statically indeterminate reinforced concrete structures (such as arches, frames, and so on), redundant restraints prevent the reinforced concrete from shrinkage, so shrinkage results in additional internal stresses. The effect of shrinkage is equivalent to a certain decrease in temperature. The average value of $\varepsilon_{sh,\,s}$ for heavy concrete may be as high as about 1.5×10^{-4}, which, at a coefficient of linear thermal expansion of 1×10^{-5} per degree, is equivalent to a fall of about 15°C. For porous-aggregate concrete, $\varepsilon_{sh,\,s}$ = (approx.) 2×10^{-4}.

In order to reduce additional stresses caused by shrinkage, large-size industrial and civil buildings are divided by shrinkage joints into blocks.

6. Creep of Reinforced Concrete

This is a result of creep in concrete. As with shrinkage, reinforcing steel restrains free creep deformations. In a loaded reinforced concrete member, restrained creep leads to the redistribution of stress between the reinforcing steel and the concrete. The stress redistribution is most pronounced during the first several months after manufacture, then it gradually decays during a long time (more than a year). Owing to the bond between the materials, the longitudinal strains in the concrete and the steel of an axially compressed prism (Fig. I.37*a*) are equal

$$\varepsilon_s = \varepsilon_c = \sigma_c/E_c' \tag{I.29}$$

Hence, the compressive stress in the longitudinal reinforcing bars is

$$\sigma_s = \varepsilon_s E_s = \sigma_c n/\nu \tag{I.30}$$

The transverse reinforcing bars or stirrups mainly serve to prevent the longitudinal compressed bars from buckling.

The equilibrium between the external load and internal forces in the concrete and longitudinal reinforcing bars is described by an equation of the form

$$N = \sigma_c F + \sigma_s F_s = \sigma_c F\,(1 + \mu n/\nu) \tag{I.31}$$

Hence, the compressive stress in the concrete is

$$\sigma_c = N/F\,(1 + \mu n/\nu) \tag{I.32}$$

The coefficient of elastic deformation for the concrete

$$\nu = \varepsilon_{el}/[\varepsilon_{el} + \varepsilon_{pl}\,(t,\ \sigma)]$$

decreases with time, t, and also depends on the stress-strength ratio, σ_c/R_{pr}. Thus, according to Eq. (I.32), with the external load, N, held constant the stress in the concrete decreases with time due to

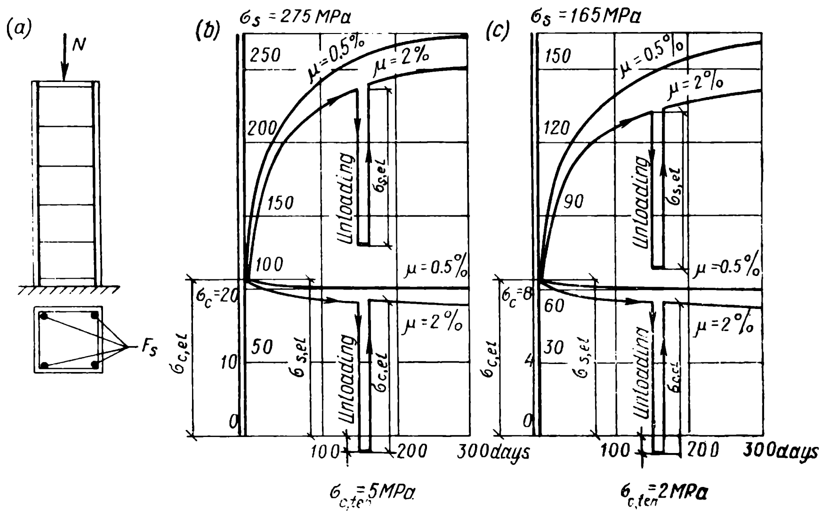


Fig. I.37. Redistribution of stresses in the steel and concrete of a compressed prism due to creep in the concrete

(*a*) reinforced concrete prism; (*b*) design brand M-500; (*c*) design brand M-200

the decrease in ν, whereas the stress in the steel increases. Figure I.37*b* and *c* shows the stress in the concrete and reinforcing steel of a loaded prism as a function of time. With $\mu = 0.5\%$, the stress in the reinforcing steel 150 days after manufacture is 2.5 times its initial value. As the percentage of reinforcement increases to 2%, the stress in the reinforcing steel grows not so rapidly. In the case of instantaneous unloading, the concrete and steel tend to strain elastically; the permanent set in the concrete, however, prevents the steel from elastic strain recovery, and, after unloading, the steel remains in compression and the concrete in tension. If the tensile stress in the concrete after unloading exceeds the ultimate tensile strength, $\sigma_{c,ten} > R$, cracking occurs in the concrete. When the member is reloaded, the cracks close.

Relaxation in the concrete of a reinforced concerete prism takes place also when the stress in the reinforcing steel is held constant (Fig. I.38*a*). If we create an initial compressive strain ε_c^0 in a reinforced concrete prism, impart an initial compressive stress σ_c^0 to the concrete and σ_s^0 to the steel, and apply restraints to maintain the prism length constant and prevent the prism from further deformation, the stress in the concrete at any time after the application of the restraints will be

$$\sigma_c(t) = \varepsilon_c E_c' = \varepsilon_c \nu E_c < \sigma_c^0$$

The stress in the concrete becomes less pronounced with time because ν decreases with time.

As this takes place, the restraint reaction defined as

$$N(t) = \sigma_c(t) F + \sigma_s F_s$$

decreases with time at constant stresses in the reinforcing steel (Fig. I.38*b*).

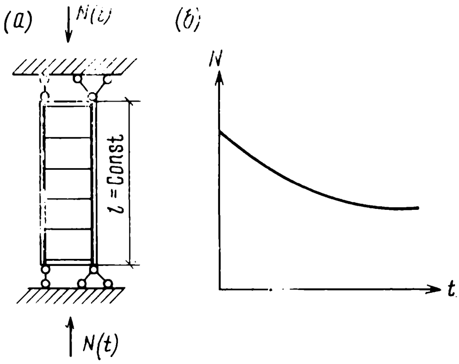


Fig. I.38. Stress relaxation in concrete with the stress in the steel of a prism held constant

(*a*) reinforced concrete prism with restraints; (*b*) restraint reaction N as a function of time t

Creep has a positive effect on the behaviour of short compression reinforced concrete members because it leads to the full use of the concrete and steel strength. In flexible compression members, creep raises initial eccentricities which may reduce the bearing capacity of the members. In members working in bending, creep contributes to sagging. In prestressed structures, creep reduces the prestress.

Shrinkage and creep in reinforced concrete take place simultaneously, and are inseparable in their effect on the structure behaviour.

7. Concrete Cover

The protective concrete cover in reinforced concrete structures is produced by placing the reinforcing steel within a certain distance of the structure surface. The cover is necessary for the composite action of the steel and the concrete at all stages of manufacture, erection and service. It protects the steel against exposure to external factors, high temperature, attacks by corrosive environments, etc. Experience shows that the concrete cover should be chosen according to the steel type and diameter, cross-sectional area of

the member, type and design brand of concrete, exposure conditions, and so on.

For nonprestressed and pretensioned longitudinal steel, the concrete cover should be not less than the bar or strand diameter. For slabs and walls up to 100 mm thick, the minimum cover is 10 mm; for slabs and walls more than 100 mm thick and also beams with a depth of less than 250 mm, it is 15 mm; for beams with a depth of 250 mm or more, it is 20 mm; and for precast foundation, it is 30 mm.

At the ends of longitudinal prestressed steel where the stress is transferred from the steel to the concrete, the cover should be at least two diameters for class A-IV and Aᴛ-IV bars and stranded wire, and not less than three bar diameters for class A-V, Aᴛ-V and Aᴛ-VI reinforcing steel. Also, the concrete cover in this area should be not less than 40 mm for reinforcing bars of all classes and not less than 20 mm for wire strands. If use is made of steel supports, the concrete cover at the ends of a structure may be as thick as it is in the span.

For longitudinal posttensioned steel placed in ducts, the concrete cover (the distance from the structure surface to the nearest duct surface) should not be less than 20 mm, nor less than half the duct diameter; for cables with a diameter of 32 mm or more, it should not be less than the cable diameter.

The distance from the ends of longitudinal nonprestressed steel to the end surface of a member should be at least 10 mm; for long precast members (such as panels over 12 m long, girders over 9 m long, and columns over 18 m high), the minimum distance is 15 mm.

The minimum concrete cover for transverse cage bars and stirrups in members less than 250 mm deep is 10 mm; for members with a depth of 250 mm or more, it is 15 mm.

8. Average Density of Reinforced Concrete

The average density of heavy reinforced concrete vibrated into place is 2 500 kg/m^3, and that of heavy reinforced concrete placed without vibration is 2 400 kg/m^3. If the percentage of reinforcement exceeds 3%, the density of reinforced concrete is defined as the combined mass of the concrete and steel per cubic metre of the structure volume. The average density of lightweight-aggregate reinforced concrete is also defined as the combined mass of the concrete and steel per cubic metre of the structure volume.

9. Fine-Mesh Wire-Fabric Reinforced Concrete

This type of reinforced concrete is fabricated from sand concrete reinforced by thin-wire (0.5 to 1 mm in diameter) fine-mesh (up to 10 × 10 mm) wire fabric. The layers of wire fabric are spaced 3 to

5 mm apart, so the material is sufficiently uniform in its properties. This type of concrete is available in M-300 and M-400 brands. It is used for thin-walled (10 to 30 mm) structures (such as shells and corrugated vaults). The reinforcement ratio is established by calculation, so that $\mu = f_f/\delta = 0.004$ to 0.025, where f_f is the cross-sectional area of wire fabric per unit length (in cm^2/cm) and δ is the thickness of the member (in cm).

Owing to a considerable increase in the contact area between the steel and the concrete, the ultimate extensibility of concrete in such structures rises. The main distinction of this type of concrete is narrow crack opening, which makes it possible to use all of the wire fabric strength without prestressing. Tension areas may also be reinforced by both wire fabric and prestressed steel.

Members made of fine-mesh wire-fabric reinforced concrete may only be used at normal ambient humidity and in the absence of corrosive environments because their corrosion resistance is poor. Their resistance to fire is also lower than that of normal reinforced concrete members. It is not recommended to use such members under conditions of regular repeated impact loads.

10. Reinforced Polymer Concrete

This is polymerized-cement concrete or polymer concrete containing steel or nonmetallic reinforcement. The bond between the reinforcement and the polymerized-cement or polymer concrete is good. There is no corrosion of steel reinforcement in polymer concrete. This type of concrete has a high resistance to corrosion, so its use is advantageous in structures exposed to corrosive environments and a high hydrostatic pressure. Information on the properties of polymer concretes is still meagre and the cost of reinforced polymer concrete structures is high, so they have not yet found wide application.

11. Effect of Temperature on Reinforced Concrete

Exposure of reinforced concrete to an elevated temperature gives rise to mutually balanced internal stresses caused by some difference in the coefficient of thermal expansion between the hardened cement, aggregate grains and reinforcing steel. At a temperature below 50°C, the internal stresses are low and do not practically affect the concrete strength. If a structure is repeatedly and regularly exposed to temperatures from 60 to 200°C, it is necessary to take into account some decrease in the mechanical strength of the concrete (about 30%). If a member is heated to and held at 500 or 600°C for a long time and then cooled, the concrete breaks down.

Concrete breaks down after exposure to high temperatures mainly because of considerable internal tensile stresses produced by the

difference in thermal deformation between the hardened cement and aggregate grains, and also by the increase in the volume of free lime which is liberated by the dehydration of the cement minerals and slacked by the atmospheric moisture.

Structures intended for long exposure to elevated temperatures are fabricated from special heat-resistant concrete. At temperatures up to 500°C the bond between deformed reinforcing bars and concrete decreases by 30%. The bond between plain reinforcing bars and concrete begins to fall significantly already at 250°C.

In statically indeterminate structures, seasonal temperature variations are responsible for additional stresses which may be considerable in extended structures. In order to reduce the additional stresses caused by temperature variations, large buildings are divided into separate blocks by expansion joints which are usually combined with shrinkage joints.

12. Corrosion of Reinforced Concrete and Anti-Corrosion Measures

The corrosion resistance of reinforced concrete members depends on the tightness of concrete and the corrosiveness of the environment. The corrosion of an insufficiently impervious concrete may be caused by infiltrating water which dissolves calcium hydroxide in the hardened cement. This type of corrosion manifests itself as white flakes on the concrete surface. This effect is most pronounced in the case of soft water. Another type of concrete corrosion is caused by a gaseous or liquid corrosive environment such as acidic gases acting jointly with an elevated ambient humidity, solutions of acids, sulphates, and so on. When an acid reacts with the calcium hydroxide of the hardened cement, the concrete breaks down. The products of this reaction crystallize and gradually fill the pores and channels in the concrete. The growing crystals break the pore and channel walls, thereby breaking the concrete itself. Most harmful for concrete are salts of some acids, especially sulphates; they produce calcium sulphate and aluminium sulphate in the cement. The calcium sulphate and aluminate dissolve, flow out of the concrete, and leave white stains on the concrete surface. Ground waters containing calcium sulphate, magnesial and ammoniacal salts are also very corrosive. Systematic exposure to seawater also has a negative effect on concrete, because seawater contains magnesite sulphate, magnesia chloride and some other harmful salts.

Corrosion of reinforcing steel (rusting) takes place due to the chemical and electrolytical action of the environment; as a rule, it occurs simultaneously with concrete corrosion, although, it may take place independently. The product of steel corrosion occupies a volume several times as great as the volume of the reinforcing steel, so it

exerts a considerable radial pressure on the surrounding concrete layer. As this takes place, cracking occurs along the reinforcing bars, and the concrete spalls so that the reinforcement is bared at places.

Depending on the corrosiveness of the environment, reinforced concrete structures may be protected against corrosion by reducing the filtrating capacity of concrete with special admixtures, increasing concrete tightness, using a deeper concrete cover, giving concrete a coat of varnish or putty, using roll insulation, replacing portland cement by alumina cement, and using special acid-resistant concrete.

CHAPTER II

EXPERIMENTAL BASIS FOR THE STRENGTH THEORY OF REINFORCED CONCRETE. METHODS OF REINFORCED CONCRETE STRUCTURAL DESIGN

II.1. EXPERIMENTAL DATA ABOUT THE BEHAVIOUR OF REINFORCED CONCRETE MEMBERS UNDER LOAD

1. The Importance of Experiments

Experimental studies into the composite action of concrete and reinforcing steel differing in physical and mechanical properties have been carried out since the invention of reinforced concrete. It has been experimentally shown that nonlinear strain in concrete and cracks in tension zones have a significant effect on the stress-strain behaviour of reinforced concrete members. Indeed, the assumption that stress-strain relations are linear, and the associated strength equations for elastic materials are often invalid for reinforced concrete.

The strength theory of reinforced concrete is based on experimental data and laws of mechanics, and proceeds from the real stress-strain state of members at various stages of external loading. As more experimental data are accumulated, the methods for the design of reinforced concrete structures are improved.

2. Three Stages in the Stress-Strain State

Experiments on various reinforced concrete members, including those in bending, eccentrical tension, and eccentrical compression with negative and positive stress diagrams, have shown that as the external load is gradually raised, three characteristic stages can be traced in the stress-strain behaviour of the member. Stage I continues until cracking begins in the tension zone of the concrete; during this stage, the stress in the concrete is below the ultimate tensile strength, and both the concrete and the steel are subjected to the tensile stress. Stage II begins after cracking has occurred in the tension zone; during this stage, the tensile stress at the cracks is taken by the steel and the concrete areas above the cracks, and both the concrete and the steel are subjected to the tensile stress between

the cracks. Stage III is the breakdown stage taking place during a relatively short period of time; during this stage, the stress in the tension reinforcing bars reaches its physical or proof yield point, that in high-strength wire reaches the ultimate tensile strength, and that in the compression zone of the concrete reaches the ultimate compressive strength. Depending on the amount of reinforcement, the tension zone may break down ahead of the compression zone, or vice versa.

Let us discuss the three stages for a reinforced concrete member in pure bending (Fig. II.1).

Stage I. When the load is small, the stress in the concrete and and steel is low, the strain is mostly elastic in nature, relation between the stress and the strain is linear, and the diagrams of normal

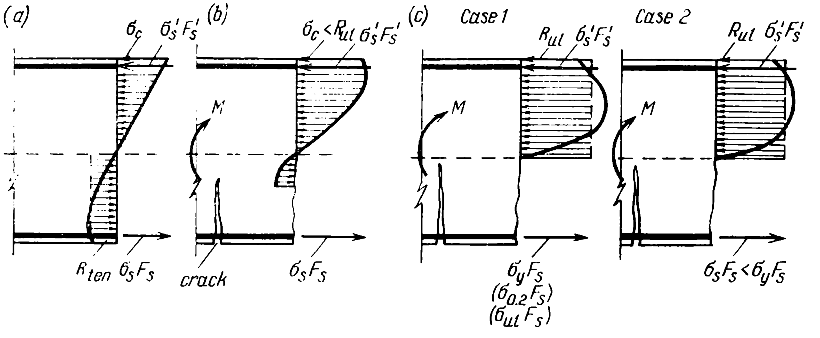


Fig. II.1. Stress-strain stages at normal sections of a nonprestressed member in bending

(*a*) Stage I; (*b*) Stage II; (*c*) Stage III

stresses in the tension and compression concrete zones are triangular. As the load increases, nonelastic strain develops in the tension zone, the stress diagram becomes curved, and the stress approaches the ultimate tensile strength. This is the end of Stage I. Any further increase in the load results in cracks in the tension zone, which signals the beginning of Stage II.

Stage II. As already noted, at the cracks appearing in the tension zone the tensile stress is carried by the steel and the tension concrete areas above the cracks. Between the cracks, the steel retains its bond to the concrete, and the tensile stress in the concrete increases and that in the steel decreases on moving away from the crack edges. As the load increases, nonelastic strain develops in the compression concrete zone, the normal stress diagram becomes curved, and the stress peak moves from the edge towards the centre of the section. Stage II ends when appreciable plastic strain develops in the reinforcing steel.

Stage III. As the load increases further, the stress in the reinforcing bars reaches the physical or proof yield point; under the action of the increasing sag and decreasing depth of the compression zone, the stress in the compression zone of the concrete reaches the ultimate compressive strength. As this takes place, the reinforced concrete member collapses; the steel in the tension zone is the first and the concrete in the compression zone is the last to break down. This failure is plastic in nature; we shall call it Case 1. If in the tension zone the member is reinforced by a high-strength wire having a low

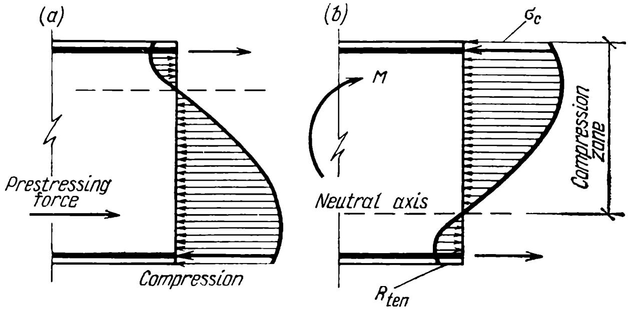


Fig. II.2. Stress distribution at normal sections of a prestressed member in bending
(a) during the transfer of the prestress; (b) after the external load has been applied, Stage I

percentage elongation at rupture (about 4%), the wire breaks down simultaneously with the concrete of the compression zone. The failure is brittle in nature, and it also falls under Case 1.

If a member is overreinforced, the failure is caused by the breakdown of the concrete in the compression zone. Stage II changes to Stage III suddenly. The failure of overreinforced members is always brittle in nature, with the strength of tensile reinforcing steel only used in part; we shall call it Case 2.

In Stage III, the nonprestressed steel in the compression zone is subjected to a compressive stress whose value depends on the ultimate strain of the concrete, $\sigma'_s = \varepsilon_c^{ul} E_s =$ (approx.) 400 MPa, or the yield point $\sigma'_s = \sigma_y$, which is less than 400 MPa for hot-rolled steel.

A particular stage of the stress-strain state is not the same for different sections along a reinforced concrete member. For example, Stage I takes place in the areas with low bending moments, Stage II in the areas with higher bending moments, and Stage III in the areas of a maximum bending moment. Similarly, different stress-strain stages may occur during manufacture, prestressing, transportation, erection, and exposure to various service loads.

When the prestress is transferred from the steel to the concrete, a rather high stress may develop in a prestressed member. Owing to nonelastic strain, the compressive stress diagram becomes curved. As the member is gradually loaded, the compressive prestress is balanced out and the appearing tensile stress approaches the ultimate tensile strength of the concrete (Fig. II.2). As the strain in the concrete, ε_c, gradually increases and E'_c decreases from the axis to the outer fibre of the section, the maximum stress, $\sigma_c = \varepsilon_c E'_c$, moves closer to the centre of the section. The difference between ordinary and prestressed reinforced concrete members is most pronounced in Stage I of the stress-strain state. In prestressed members, the external load causing cracking is several times that in ordinary members, the stress in the compression zone and the depth of this zone also considerably increase. The interval between Stage I and Stage III decreases. After cracking has occurred in Stages II and III, the stress-strain states of prestressed and nonprestressed members are similar.

3. Crack Development in Tension Zone

Cracks in reinforced concrete members may be caused by hardening and shrinkage conditions, compressive prestress during manufacture, and overstressing of the materials in service, that is, overloading, settlement of supports, temperature variations, and the like. Overstressing is likely to cause cracking in tension zones rather than in compression zones. Invisible cracks appear even in the tension zones of perfectly designed reinforced concrete structures. This is a result of low extensibility of concrete which is unable to follow the considerable steel elongation at high working stresses. In prestressed members, cracking occurs at a relatively high load. Experience shows that as long as the width of these cracks lies within certain limits, they are not dangerous and do not affect the integrity of reinforced concrete.

Reinforcing steel in the tension zone of a member somewhat reduces the negative effect of the nonuniform inner structure and discontinuity of concrete, but with the usual amount of reinforcement, the tensile strength of reinforced concrete only slightly exceeds that of plain concrete. Cracks in compression zones usually are an indication that the section is inadequate to carry the compressive stress. Such cracks constitute a threat to the strength of the structure.

Cracking in tension zones may be divided into three stages, namely (1) incipient cracking when cracks may still be invisible, (2) visible cracking, and (3) crack opening until a maximum width is attained. In members with a normal amount of reinforcement, the first two stages merge together, so we may consider only two stages, namely visible cracking and crack opening.

II.2. ELASTIC DESIGN

This method of design of members in bending was the first to develop. As the point of departure, it uses Stage II of the stress-strain state and the following assumptions: (1) the concrete does not resist tensile stresses; all of the tensile stress is applied to the reinforcing steel; (2) the concrete in compression zones is elastic, the stress-strain relation is linear and obeys Hooke's law; and (3) right sections which were plane before bending are plane after bending (that is, Bernoulli's assumption holds).

As a consequence of these assumptions, the stress diagram in the compression zone is taken to be triangular and the ratio of the elastic moduli of steel and concrete, $n = E_s/E_c$, is constant (Fig. II.3).

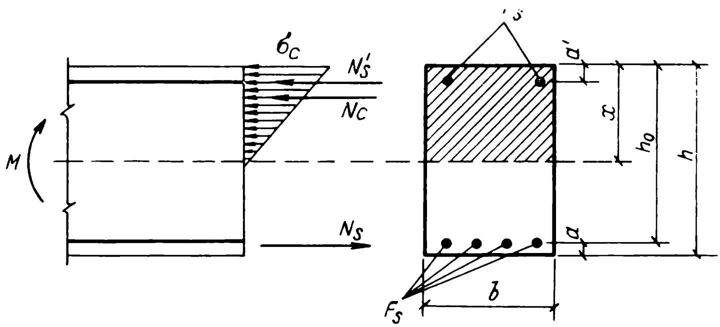


Fig. II.3. To the elastic design of a rectangular beam

Here, we consider a uniform section in which the steel area, F_s, is replaced by its transformed "concrete area" equal to nF_s. Since the strains of the concrete and the steel are equal

$$\varepsilon_s = \sigma_s/E_s = \varepsilon_c = \sigma_c/E_c$$

we may write

$$\sigma_s = n\sigma_c \tag{II.1}$$

The stress in the outer fibre is determined as for the transformed uniform section

$$\sigma_c = Mx/I_{tr} \tag{II.2}$$

the stresses in the tensile and compressive steel can be determined by the following formulas

$$\sigma_s = nM\,(h_0 - x)/I_{tr} \tag{II.3}$$

$$\sigma_s' = nM\,(x - a')/I_{tr} \tag{II.4}$$

where $h_0 = h - a$ is the effective depth of the section; h is the overall depth of the section; a is the distance from the tensile face to the centroid of the tensile steel; a' is the distance from the compressive face to the centroid of the compressive steel; and x is the depth of concrete in compression.

The value of x is found from the condition that the static moment of the transformed section about the neutral axis is zero

$$S_{tr} = bx^2/2 + nF'_s (x - a') - nF_s (h_0 - x) = 0 \tag{II.5}$$

The moment of inertia of the transformed section is

$$I_{tr} = bx^3/3 + nF_s (h_0 - x)^2 + nF'_s (x - a')^2 \tag{II.6}$$

In elastic theory, the permissible stress for concrete is defined as a certain fraction of the ultimate compressive strength of concrete, $\sigma_c = 0.45R$ (where R is the concrete brand number equal to the cube crushing strength of concrete). The permissible stress for steel is defined as a fraction of its yield point, $\sigma_s = 0.5\sigma_y$.

The main drawback of elastic design is that concrete is regarded as an elastic material. The actual stress distribution in a concrete section in Stage II cannot be described by a triangular stress diagram. Also, n is not constant, being dependent on the stress in concrete, the duration of this stress and some other factors. Adjustment of n to suit a given concrete brand number has also proved ineffective. Furthermore, it has been determined that actual stresses in reinforcing steel are below those calculated. With elastic theory, it is impossible not only to design structures with a predetermined safety factor, but also to determine real stresses in the materials. In some cases, this leads to an excessive material consumption or the use of reinforcement in the compression zones of the concrete.

The disadvantages of elastic design became especially obvious with the advent of new types of concrete (such as high-strength heavy concretes and lightweight porous-aggregate concretes) and stronger reinforcing steel.

II.3. PLASTIC OR COLLAPSE DESIGN

The drawbacks of elastic theory necessitated the development of a new design method which would better conform to the elastic and plastic properties of concrete. In Soviet practice, new standards and specifications based on plastic or collapse design were put in force in 1938.

As the point of departure, plastic design uses Stage III of the stress-strain state. Again, tensile stresses in the concrete are neglected as an element of strength. Permissible (working) stresses in the

design formulas are replaced by the ultimate compressive strength of concrete and the yield point of steel. With these quantities, the modular ratio n need not be known. In the early days of plastic analysis, the stress diagram in the compression zone was taken to be curved; nowadays, it has been replaced by a rectangular diagram. The maximum safe force which may be applied to a structure in service is determined by dividing the limit or collapse force by the total safety or load factor, k (whence another name: the load factor method). Accordingly, for members in bending

$$M = M_{col}/k \tag{II.7}$$

and for members in compression

$$N = N_{col}/k \tag{II.8}$$

When determining the collapse load for members working in Case 1 (breaking down in the tension zone), Bernoulli's assumption

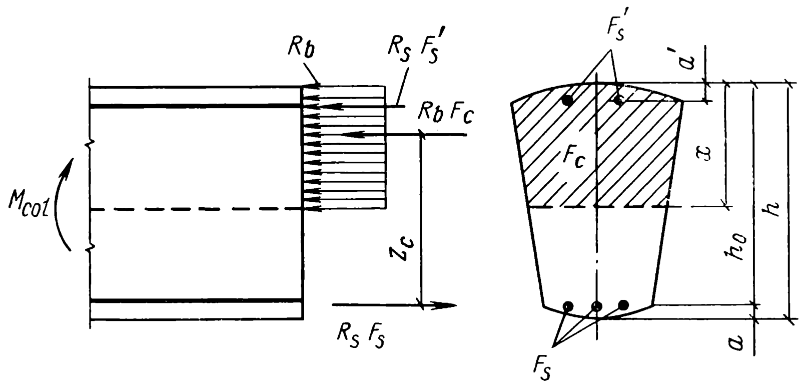


Fig. II.4. To the plastic design of a beam symmetrical in cross section

is replaced by the plastic collapse principle according to which the stress reaches its ultimate value simultaneously in the concrete and the steel. This principle (first formulated by the Soviet scientist A. F. Loleyt) yields the design formulas defining the collapse load for bending and axially loaded members.

For a bending member of any symmetrical cross section (Fig. II.4), the depth of the compression zone is determined from the equilibrium of internal forces in the breakdown stage

$$R_b F_c + R_s F_s' = R_s F_s \tag{II.9}$$

where R_b is the ultimate compressive strength of the concrete in bending equal to $1.25R_{pr}$; R_s is the yield point of the steel; and F_c is the concrete area in compression.

The collapse moment is determined as the moment of internal forces about the centroid of the tensile steel

$$M_{col} = R_b S_c + R_s F'_s (h_0 - a') \tag{II.10}$$

where $S_c = F_c z_c$ is the static moment of the compression zone about the centroid of the tensile steel; and z_c is the distance from the centroid of the tensile steel to the centroid of the compression zone.

The demarkation line between Case 1 and Case 2 is drawn according to experimental data: at $S_c/S_0 \leqslant 0.8$, concretes of M-400 brand and below fall into Case 1. Here, S_0 is the static moment of the entire effective area of concrete about the centroid of the tensile steel. For rectangular and T-beams with compressed flanges, the depth of the concrete in compression should not exceed $0.55h_0$.

The safety (load) factor used in the design equation by this method is the same for the entire member. It is established according to the stress distribution at failure, load combination and the ratio of the live load, T_l, to the dead load, T_d. If live load prevails, a structure is more likely to be overloaded, so the safety factor must be greater. For example, for slabs and beams subjected to the basic combination of loads and at $T_l/T_d \leqslant 2$, $k = 1.8$, at $T_l/T_d > 2$, $k = 2$, and so on. For off-site precast members, the safety factor decreases by 0.2, but its specified value should not be less than 1.5.

In plastic design, the internal forces M, Q and N due to loading are also determined in the breakdown stage, that is, with allowance for the formation of plastic hinges. For some structures (such as slabs, continuous beams and frames), this design method results in a significant economy.

Advantages and Disadvantages of Plastic Design. Plastic design, which takes into consideration the elastic and plastic properties of concrete, describes the actual behaviour of members under load more correctly and has been an important advance in the strength theory of reinforced concrete.

A major advantage of this method over elastic design is that the total safety factor of a structure thus found is closer to its actual value.

In addition, structures designed by the load factor method require less reinforcing steel than those designed by the elastic method. For example, plastic design calculations usually show that members in bending do not require compressive reinforcing steel.

A disadvantage of this method is that the likely departures of actual loads and strength of the structural materials from their design values cannot be taken care of by a common factor of safety.

II.4. LIMIT-STATE DESIGN OF REINFORCED CONCRETE

1. General

Limit-state design has originated from plastic design. The main difference between the two methods is that limit-state design clearly establishes the limit states of structures and sets up a system of design coefficients which guarantee that a structure will not attain such states under the worst load combinations and at the minimum strength of the materials. This method is likewise based on the breakdown stage, but the safety of a structure under load is expressed in terms of several design coefficients rather than a total safety factor. Structures designed by this method are more economical.

2. Two Groups of Limit States

Limit states are those in which structures no longer meet the service requirements, that is, they cease to resist external loads and other factors or are displaced or locally damaged more than it is allowed.

Limit-state design involves two groups of limit states. One refers to the necessary load-bearing capacity (this is the first group of limit states) and the other to fitness for a particular normal service (the second group of limit states).

The design in terms of the first group of limit states is carried out to prevent a structure from:

— brittle, viscous or any other failure (for this purpose we find the strength of a structure with allowance, where necessary, for the sag before failure);

— loss of shape (stability analysis of thin-walled structures) and position (overturning and sliding analysis of retaining walls and eccentrically loaded high foundations; analysis of buried and underground tanks for tendency to float, etc.);

— fatigue failure (endurance analysis of structures subjected to repeated live or pulsating load, such as crane beams, sleepers, frame foundations and floors for unbalanced machines, and so on);

— failure under the combined action of load and unfavourable exposure conditions (periodic or permanent exposure to corrosive environments, freeze-thaw cycles, and the like).

The design in terms of the second group of limit states is done to prevent a structure from:

— excessive or long-term crack opening (if it is not allowed by the service conditions);

— excessive displacement (sagging, rotation, warping, and vibration).

The limit-state design of an entire structure and of its members or parts is carried out for each stage, namely manufacture, transportation, erection and service. For each stage, an appropriate loading scheme should be used, compatible with the type of structure involved.

3. Design Factors

Design factors which include various loads and the mechanical characteristics of concrete and reinforcing steel (such as the ultimate strength and the yield point) are statistical quantities, that is, vary from case to case probabilistically. For example, loads and other factors may exceed, and the mechanical characteristics of the materials may be below the average values as found by probability laws. Limit-stage design takes into account the statistical variability in load and mechanical characteristics of the materials, nonstatistical factors, and various favourable and unfavourable exposure, manufacture and service conditions to which concrete and reinforcing steel are subjected in buildings and other structures.

All loads, mechanical characteristics of materials and design coefficients are subject to relevant standards and specifications.

4. Classification of Loads. Basic and Design Loads

Classification of Loads. According to their duration, loads are divided into dead and live. In turn, live loads may be subdivided into long-time, short-time and special loads.

Dead loads are loads which are related to the self-weight of the load-bearing and filler members of the structure, the mass and pressure of the soil, and the prestress in prestressed concrete structures.

Long-time loads include the weight of stationary equipment on floors (machines, apparatus, motors, tanks, etc.); the pressure of gases, liquids, and particulate materials in tanks; loads in storehouses, refrigerators, archives, libraries and the like; the specified part of the live load in civil buildings; long-time exposure to heat from stationary equipment; the load due to one hoist or travelling crane multiplied by 0.6 for medium-duty cranes and by 0.8 for heavy-duty cranes; and the snow load for some climatic regions reduced by 70 kgf/m^2. The above values of crane, snow and some live loads constitute only a part of the total value; they are used when it is necessary to evaluate their long-time effect on displacement, deformation and cracking. The total values apply to short-time loads.

Short-time loads include the loads constituted by people, spare parts and materials in attendance and maintenance areas, aisles and other locations free from equipment; floor loads in residential and other civil buildings; loads arising during the manufacture,

transportation and erection of structural members; loads presented by hoists and overhead cranes used in the erection and service of buildings and other structures; snow and wind loads; and climatic temperature factors.

Special loads are related to earthquake and explosion loads; loads caused by faulty equipment and sudden changes in temperature; nonuniform strain in foundations accompanied by a radical change in the soil structure (for example, strains caused by the soaking of settling soils or the thawing of permafrost soils); and so on.

Basic Loads. These are established by relevant standards according to a predetermined probability of exceeding average values or are set up according to rated values. For dead loads, the basic values are deduced from the design geometrical and structural variables and average densities. For live erection and service loads, they are deduced from the maximum safe values ensuring normal service; for snow and wind loads, they are deduced from the worst values averaged over a year or some other repetition period.

Design Loads. For the strength and stability analysis of structures, design loads are found by multiplying appropriate basic loads by the respective overload factor, n, which is usually greater than unity, for example, $q = q^b n$. The weight overload factor for plain and reinforced concrete structures is 1.1. For lightweight concrete structures (with an average density of 1 800 kg/m^3 and lower) and various strainers, fillings, heat insulation and similar materials, the weight overload factor is 1.2 when factory-made and 1.3 when cast in-situ. For various live loads it may vary according to their values from 1.2 to 1.4. In the design of structures intended to resist floating, overturning and sliding, and also in some other cases when a reduction in weight might affect the behaviour of a structure, the overload factor is taken equal to 0.9. To determine the loads existing during the erection stage, the design short-time loads must be multiplied by $n = 0.8$. For the purpose of strain and displacement analysis (the second group of limit states) design loads are taken equal to the respective basic values, so $n = 1$.

Load Combinations. Any structure should be designed for different load combinations or, in the case of plastic design, for the respective forces. According to the loads involved, there are basic load combinations consisting of dead, long-time and short-time live loads or the appropriate forces, and special combinations which are composed of dead, long-time, likely short-time and one of special live loads or the respective forces.

In turn, basic load combinations are subdivided into two groups. The first group includes dead, long-time and one short-time live load, the second contains dead, long-time and two (or more) short-time live loads. Here, the values of short-time loads or the respective forces should be multiplied by the combination factor equal to 0.9.

When structures are designed to resist special load combinations, short-time loads or the respective forces should be multiplied by a combination factor of 0.8, except the cases covered by specifications for structures in earthquake regions. The special load proper should be taken into account without any reduction.

Load Reduction. When designing columns, walls and foundations for industrial buildings, the vertical forces induced by short-time loads (such as people, spare parts, repair materials, and so on) acting in areas free from equipment, transportation facilities and materials stored for a long time and occupying the whole floor area may be reduced by multiplying by the coefficient η

$$\eta = 0.5 + 0.6/\sqrt{m} \tag{II.11}$$

where m is the number of floors above the section in question; with $m = 1$, $\eta = 1$; with $m \geqslant 9$, $\eta = 0.7$.

For residential buildings, health farms and the like

$$\eta = 0.3 + 0.6/\sqrt{m} \tag{II.11a}$$

Relevant standards and specifications also permit reduction in short-time loads for beams and girders, in proportion to the floor area being actually loaded.

The design climatic temperature is defined as the average ambient temperature on the hottest day, or as the average temperature over the five coldest days, depending on the climatic region in which the project site is located.

5. Basic and Design Concrete Strength

Basic Concrete Strength. This is established with allowance for the statistical variability of strength and is taken as the least observable ultimate strength of concrete. The minimum confidential probability for the basic strength is set by relevant specifications to be 0.95. For example, in the compressive test of a large number of standard cubes, n_1 cubes may have an ultimate strength of R_1; n_2 cubes, R_2; n_k cubes, R_k. The total number of cubes is $n = n_1 + n_2 + \ldots + n_k$. Plotting $R_1, R_2 \ldots R_k$ as abscissa, and $n_1, n_2 \ldots n_k$ as ordinate, we get a statistical distribution curve (Fig. II.5). The statistically processed test results yield the average or mean ultimate compressive strength

$$\overline{R} = (n_1R_1 + n_2R_2 + \ldots + n_kR_k)/n$$

the deviations from the mean

$$\Delta_1 = R_1 - \overline{R}; \quad \Delta_2 = R_2 - \overline{R}; \ \ldots; \quad \Delta_k = R_k - \overline{R}$$

and the root-mean-square error known as the standard deviation

$$\sigma = \sqrt{(n_1\Delta_1^2 + n_2\Delta_2^2 + \ldots + n_k\Delta_k^2)/(n-1)}$$

The average, or mean, ultimate compressive strength $\bar{R}$ is required to have a certain definite value. It is the cube crushing strength or the ultimate compressive strength corresponding to a particular concrete brand number M.

The least observable ultimate strength R^b lies on the x-axis to the left of $\bar{R}$ and is separated from the latter by the distance $\varkappa\sigma$, where $\varkappa$ is the number of standard values. By definition, the basic strength is

$$R^b = \bar{R} - \varkappa\sigma$$

or

$$R^b = \bar{R}\,(1 - \varkappa v) \quad \text{(II.12)}$$

where $v = \sigma/\bar{R}$ is the coefficient of strength variation (the variability factor).

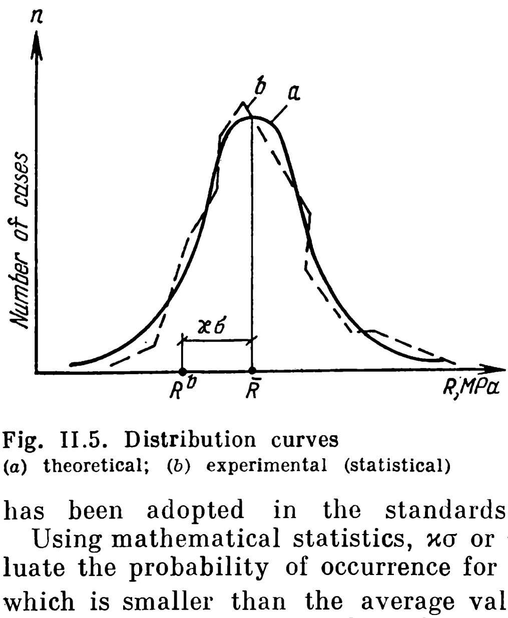


Fig. II.5. Distribution curves
(a) theoretical; (b) experimental (statistical)

The experiments carried out at factories making reinforced concrete structures have shown that the variability factor for heavy and porous-aggregate concrete is 0.135; this value has been adopted in the standards and specifications.

Using mathematical statistics, $\varkappa\sigma$ or $\varkappa v$ makes it possible to evaluate the probability of occurrence for the ultimate strength $< R^b$ which is smaller than the average value $\bar{R}$. If $\varkappa = 1.64$, $< R^b$ is likely to occur in not more than 5% of the test cubes (and R^b is likely to occur in at least 95% of the test specimens). Thus, the probability of occurrence for the basic strength is at least 0.95, as is required.

The following basic strength values are used for heavy and porous-aggregate concrete:

— the basic cube crushing strength found for $\varkappa = 1.64$ and $v = 0.135$ by Eq. (II.12) to be $R^b = 0.778\bar{R}$;

— the basic prism crushing strength as determined by the following empirical formula

$$R^b_{pr} = R^b\,(0.77 - 0.0001\bar{R}) \quad \text{(II.13)}$$

but not less than $0.72R^b$;

— the basic axial tensile strength determined by Eq. (I.2) with a reduction coefficient applied

$$R^b_{ten} = 0.5k\sqrt[3]{(R^b)^2} \tag{II.14}$$

where $k = 0.8$ for concretes of the M-450 brand and lower; or $k = 0.7$ for concretes of the M-500 brand and higher. For a check on the axial tensile strength of concrete, it may alternatively be taken equal to

$$R^b_{ten} = 0.778\bar{R}_{ten}$$

where $\bar{R}_{ten}$ is the ultimate tensile strength of the concrete as given by its respective brand number.

Basic strength values for concrete in round figures are given in Appendix III. (All Appendices are given in Volume 2.)

Design Strength of Concrete. For the first group of limit states, it is found by dividing the basic strength by the respective safety factors for concrete: in compression, $k_{c,com} = 1.3$ and in tension, $k_{c,ten} = 1.5$ (or $k_{c,ten} = 1.3$ as a check on the tensile strength). Thus, the design axial compressive strength is

$$R_{pr} = R^b_{pr}/k_{c,\,com} \tag{II.15}$$

and the design axial tensile strength of concrete is

$$R_{ten} = R^b_{ten}/k_{c,\,ten} \tag{II.16}$$

For design brands M-600, M-700 and M-800 of heavy concrete, the design strength should be multiplied by the coefficient taking care of the mechanical properties of high-strength concrete (reduced creep strain). This coefficient is 0.95, 0.925 and 0.9, respectively.

Design strengths for concrete in round figures are given in Appendix I.

In the design of structural members, the design concrete strengths R_{pr} and R_{ten} are reduced or, sometimes, increased by multiplying them by an appropriate service factor, m_c. These factors take care of the duration and cycling of loading; conditions, nature and stage in the behaviour of a structure; manner of manufacture, cross-sectional area and so on. The values of the various service factors will be found in Appendix II.

For the second group of limit states, the design strength of concrete is determined assuming that the safety factor for concrete is $k_c = 1$; in other words, use is made of the basic values

$$R_{pr\,II} = R^b_{pr}; \qquad R_{ten\,II} = R^b_{ten}$$

In calculations, these values are multiplied by the service factor $m_c = 1$, except when reinforced concrete members are designed to resist cracking under the action of repeated loading. In such cases, use is made of the service factor m_{c2}.

6. Basic and Design Strength of Reinforcing Steel

Basic Strength. This quantity, designated R_s^b, is established with allowance for the statistical variability of strength and is taken as the least observable value of the physical yield point σ_y or that of the proof yield point $\sigma_{0.2}$ for reinforcing bars, and as the least observable ultimate strength σ_{ul} for reinforcing wire. The minimum confidential probability for the basic strength of reinforcing steel required by relevant specifications is 0.95. The basic strengths for various classes of reinforcing bars and wire are given in Appendices V.1 and V.2.

Design Strength. For the first group of limit states, it is found by dividing the design strength by the respective safety factors for reinforcing steel

$$R_s = R_s^b / k_s \tag{II.17}$$

The safety factors for reinforcing steel are given in Table II.1.

The design tensile strength of reinforcing steel is given in Appendices V.1 and V.2.

The design compressive strength of steel, $R_{s,\,com}$, used in the design on the basis of the first group of limit states on the assumption that

TABLE II.1. Safety Factors for Reinforcing Steel

Type of steel	Safety factor
Bars:	
class A-I and A-III	1.1
class A-II	1.15
class A-IV and AT-IV	1.2
class A-V, AT-V and AT-VI	1.25
Wire:	
class Bp-I, B-II, Bp-II and K-7	1.55
class B-I	1.75

the steel remains bonded to the concrete, is taken equal to the respective tensile strength of the steel, R_s, but not more than 450 MPa (according to the compressive strain capacity of the concrete, ε_c^{ul}). For structures made of heavy or porous-aggregate concrete, the design strength is chosen for long-time loading and taken with the

service factor $m_{c1} = 0.85$ applied, so $R_{s,com}$ may be taken equal to 450 MPa for class A-IV and Aт-IV reinforcement and to 500 MPa for class A-V, Aт-V, Aт-VI, B-II, Bp-II and K-7 reinforcement. This is legitimate because long-time loading leads to an increase in the compressive strain capacity. At the same time, the transverse reinforcement should be positioned so as to prevent the longitudinal compressive steel from buckling; the spacing should not exceed 500 mm or twice the width of a given member face. When there is no bond between the concrete and the steel, $R_{s,com} = 0$.

In the design of structural members, the design strength of reinforcing steel can be reduced or increased, as the case may be, applying an appropriate service factor, m_s. This factor takes care of the possibility that the steel will not act at its full strength because of the nonuniform stress distribution in the section, low concrete strength, anchorage condition, bends, changes in the steel properties caused by exposure conditions, and so on.

When structural members are designed to carry a shearing force, this service factor is $m_{s,tr} = 0.8$. It allows for the nonuniform stress distribution in the steel along the inclined section. Besides, with welded transverse reinforcement made of class B-I and Bp-I wire and class A-III bars, the design strength is multiplied by the service factor $m_{s,tr} = 0.9$ which allows for the possibility of brittle breakdown in the welded joints between stirrups and load-bearing steel. For tied reinforcing cages made of class B-I wire, the service factor is $m_{s,tr} = 0.75$ which takes care of its low bond with the concrete. The design strength of transverse steel, $R_{s,tr}$, with the service factors, $m_{s,tr}$, applied, is given in Appendices V.1 and V.2.

In addition, the design strength R_s, $R_{s,com}$ and $R_{s,tr}$ should be multiplied by the following service factors:

— m_{s1} and m_{s2} for repeated load (see Chapter VIII);

— $m_{s3} = l_{tr}/l_{transm}$ or $m_{s3} = l_{tr}/l_{an}$ in the stress transmission zone or in the anchorage zone of nonprestressed steel without anchoring devices, respectively;

— m_{s4}, when high-strength reinforcing steel is working under stresses exceeding the proof yield point $\sigma_{0.2}$.

For design in terms of the second group of limit states, the safety factor is $k_s = 1$, so the design strength of steel is equal to the basic strength $R_{s\,II} = R_s^b$, and in calculations it is introduced with the service factor $m_s = 1$ applied.

7. Requirements for Crack Resistance of Reinforced Concrete Structures

The crack resistance of a reinforced concrete structure is its ability to resist cracking in Stage I and crack opening in Stage II of the stress-strain state.

The requirements for the crack resistance of a reinforced concrete structure or its parts vary according to the exposure conditions and type of reinforcing steel. They are related to normal and inclined cracks and may be divided into three categories:

— Category one, no cracks are allowed;

— Category two, short-time opening of limited-width cracks is allowed, provided that they close tightly after the load has been removed;

— Category three, short- and long-time opening of limited-width cracks is allowed.

"Short-time" refers to the opening of cracks under the action of dead, long- and short-time live loads; "long-time" refers to the opening of cracks under the action of dead and long-time live loads only. The limiting crack width (designated $a_{cr,sh\text{-}t}$ for short-time opening and $a_{cr,l\text{-}t}$ for long-time opening) at which structures continue to behave normally, reinforcing steel remains protected against corrosion and the service life of the structure is unaffected, ranges from 0.05 to 0.4 mm according to the category of crack resistance requirements.

The requirements of the first category apply to prestressed concrete members subjected to the pressure of liquids or gases (such as tanks or pipes), members reinforced by bars or wire used below the groundwater level and with their sections entirely in tension, and members reinforced by wire 3 mm in diameter or smaller with their sections partly in compression. The requirements of the second and third categories hold for other classes of prestressed members, depending on service conditions and type of reinforcement. The requirements of the third category apply to nonprestressed structures reinforced by class A-I, A-II and A-III bars. The categories of crack resistance requirements for various service conditions and reinforcement are given in Table II.2.

These requirements also hold in the calculation of forces arising during transportation and erection.

In crack-resistance analysis, the loads taken into consideration are treated according to the applicable category of crack resistance requirements. For example, with Category one, the design loads are multiplied by the overload factor $n > 1$ (as in strength analysis); with Categories two and three, the loads are multiplied by the overload factor $n = 1$. With Category two, the cracking analysis to make sure whether a check on crack width is necessary or not is carried out, using the design loads with the overload factor $n > 1$ applied; with the third category of requirements, this is done, using the design loads multiplied by the overload factor $n = 1$. The crack resistance calculations are carried out for the joint action of all loads, except special. In the cracking analysis, special loads are taken into account only when cracks may cause failure. In crack-

TABLE II.2. **Categories of Requirements for Crack Resistance of Reinforced Concrete Structures According to Service Conditions and Type of Reinforcement**

<table>
<tr><th rowspan="2">Service conditions</th><th colspan="4">Categories of requirements for crack resistance of structures and limit short-time, $a_{cr,\ sh\text{-}t}$, and long-time, $a_{cr,\ l\text{-}t}$, crack widths</th></tr>
<tr><th>Class A-I, A-II and A-III bars</th><th>Class A-IV, AT-IV, A-V and AT-V bars, class B-I and Bp-I wire</th><th>Class AT-VI bars, class B-II and Bp-II wire and class K-7 strands at wire diameter of 4 mm and more</th><th>Class B-II and Bp-II wire 3 mm in diameter, class K-7 strands at wire diameter of 3 mm and less</th></tr>
<tr><td>Members subjected to pressure of liquids or gases, with sections entirely in tension</td><td>Category three,
$a_{cr,\ sh\text{-}t}=0.2$ mm
$a_{cr,\ l\text{-}t}=0.1$ mm</td><td colspan="3">Category one
—</td></tr>
<tr><td>Same members with sections partly in compression</td><td colspan="2">Category three,
$a_{cr,\ sh\text{-}t}=0.3$ mm
$a_{cr,\ l\text{-}t}=0.2$ mm</td><td>Category two,
$a_{cr,\ sh\text{-}t}=0.1$ mm
—</td><td>Category one,
—
—</td></tr>
<tr><td>Members subjected to pressure of particulate materials</td><td colspan="2">Category three,
$a_{cr,\ sh\text{-}t}=0.3$ mm
$a_{cr,\ l\text{-}t}=0.2$ mm</td><td>Category two,
$a_{cr,\ sh\text{-}t}=0.1$ mm
—</td><td>Category two,
$a_{cr,\ sh\text{-}t}=0.05$ mm
—</td></tr>
<tr><td>Other exterior members</td><td colspan="2">Category three,
$a_{cr,\ sh\text{-}t}=0.4$ mm
$a_{cr,\ l\text{-}t}=0.3$ mm</td><td>Category two,
$a_{cr,\ sh\text{-}t}=0.15$ mm
—</td><td>Category two,
$a_{cr,\ sh\text{-}t}=0.05$ mm
—</td></tr>
<tr><td>Other interior members</td><td colspan="2">Category three,
$a_{cr,\ sh\text{-}t}=0.4$ mm
$a_{cr,\ l\text{-}t}=0.3$ mm</td><td>Category three,
$a_{cr,\ sh\text{-}t}=0.15$ mm
$a_{cr.\ l\text{-}t}=0.1$ mm</td><td>Category two,
$a_{cr,\ sh\text{-}t}=0.15$ mm
—</td></tr>
</table>

closure analysis for the second category of requirements, dead and long-time live loads are multiplied by the overload factor $n = 1$. Various load combinations used in crack-resistance design are given in Table II.3.

TABLE II.3. Loads for Crack Resistance Design

Category of requirements	Loads for			
	cracking analysis	crack-width analysis		crack-closure analysis
		short-time	long-time	
One	All loads (except special) act together, $n > 1$ (as in strength analysis)	—	—	—
Two	All loads (except special) act together, $n > 1$ (analysis is carried out to make sure whether checks on short-time crack width and crack closure are necessary or not)	All loads (except special) act together, $n = 1$	—	Dead and long-time live loads act together, $n = 1$
Three	All loads (except special) act together, $n = 1$ (analysis is carried out to make sure whether crack-width check is necessary or not)	Same	All loads (except special) act together, $n = 1$	—

Within the transmission length l_{transm} at the ends of prestressed members, it is required that the combined action of loads (except special) multiplied by the overload factor $n = 1$ should cause no cracking. This is because the premature cracking of the concrete at the ends of members might force the reinforcement to slip under load and the structure to fail suddenly.

Cracks that might form during manufacture, transportation or erection in a zone which will be in compression when loaded, would reduce the force required to cause cracking in the tension zone and increase crack width and sagging. The effect of such cracks is taken care of in the structural design. In members subjected to repeated loads, and designed for endurance such cracking is not allowed.

8. Main Points of Design

The First Group of Limit States. As already noted, strength analysis is based on Stage III of the stress-strain state. A section is said to be sufficiently strong if the forces induced by the design load do not exceed the design strength of the materials, taken with an appropriate service factor applied. A force due to the design loads, T

(for example, a bending moment or a longitudinal force), is a function of basic loads, q^b, overload factors, n, and some other factors designated C (loading system, dynamic factor, and so on). The force resisted by a section, T_{sec}, is, in turn, a function of the shape and size of the section, S, material strength, R_c^b and R_s^b, safety factors, k_c and k_s, and service factors, m_c and m_s.

The condition for strength is described by the following inequality

$$T(q^b, n, C) \leqslant T_{sec}(S, R_c^b, k_c^{-1}, m_c, R_s^b, k_s^{-1}, m_s) \qquad \text{(II.18)}$$

Since $q^b n = q$, $R_c^b k_c^{-1} = R_c$ and $R_s^b k_s^{-1} = R_s$, Eq. (II.18) may be rewritten thus

$$T(q, C) \leqslant T_{sec}(S, R_c, m_c, R_s, m_s) \qquad \text{(II.19)}$$

Endurance analysis is based on Stage I of the stress-strain state. It consists in finding the prestress and stresses in the concrete and reinforcing steel caused by the external load. The endurance of a structure is said to be adequate if these stresses do not exceed the design strength of the concrete and steel, multiplied by the service factor at repeated loading.

The Second Group of Limit States. Analysis for the formation of normal and inclined cracks is carried out to check the crack resistance of members designed to meet the requirements of the first category, and also to determine whether cracks may occur in members designed to meet the requirements of the second and third categories. It is assumed that no normal cracks will occur in the concrete, if the force, T, (a bending moment or a longitudinal force) induced by the external load does not exceed a force, T_{cr}, which may be resisted by the section, with the tensile stresses in the concrete equal to $R_{ten\ II}$ before the advent of cracking

$$T \leqslant T_{cr} \qquad \text{(II.20)}$$

The load combinations and overload factors necessary to determine the force, T, are given in Table II.3.

It is also assumed that no inclined cracks will occur if the principal tensile stresses in the concrete do not exceed $R_{ten\ II}$ taken with the respective coefficients applied.

Crack-opening analysis consists in determining the width of normal and inclined cracks at the level of tensile reinforcing steel and comparing it with the limiting crack width. It is required that

$$a_{cr} \leqslant a_{cr,lim} \qquad \text{(II.21)}$$

The limiting crack widths are given in Table II.3.

The displacement analysis of a structure consists in finding the sag of a member caused by the external load and adjusted for the duration of loading, and comparing it with the limit of sag. It is

required that

$$f \leqslant f_{lim}$$

The limit of sag is set according to service and structural requirements. The service requirements provide for the normal operation of cranes, machines and other process equipment. The structural requirements arise from the effect of adjacent members limiting the strain, the necessity to maintain specified slopes, and also the external appearance of the structure.

The limit of sag for prestressed members may be increased by the amount of hogging, if this does not run counter to service and structural requirements.

When a structure is to meet certain service or structural requirements, sagging is determined for dead, long- and short-time live

TABLE II.4. Limits of Sag for Reinforced Concrete Members

Member	Limit of sag as a fraction of span	Loads with $n = 1$
Electric crane beams	$l/600$	Dead, long- and short-time live loads
Flat ceiling and roofs with spans:		
$l < 6$ m	$l/200$	Dead and long-time live loads
6 m $< l \leqslant 7.5$ m	3 cm	
$l > 7.5$ m	$l/250$	
Ribbed ceiling and flights with spans:		
$l < 5$ m	$l/200$	The same
5 m $\leqslant l < 10$ m	2.5 cm	
$l \geqslant 10$ m	$l/400$	
Filler wall panels (in plane design) with spans:		
$l < 6$ m	$l/200$	Dead, long- and short-time live loads
6 m $\leqslant l \leqslant 7.5$ m	3 cm	
$l > 7.5$ m	$l/250$	

loads; if sagging is limited by considerations of the external appearance, it is determined for dead and long-time live loads. In either case, the overload factor is taken as unity.

The limits of sag prescribed for reinforced concrete members by appropriate specifications are given in Table II.4. The limits of sag related to the cantilever overhang are doubled.

Reinforced concrete members such as slabs in floors, flights of stairs and landings, not connected to adjacent members should additionally be analyzed for fixity: the additional sag produced by a short-time concentrated load of 100 kg at the most unfavourable disposition of loads should not exceed 0.7 mm.

II.5. PRESTRESS IN STEEL AND CONCRETE

1. The Value of Prestress

The prestress in reinforcing steel and concrete is very important for the subsequent behaviour of members under load. When the prestress in the steel and concrete is low, the prestressing effect will be gone in some time because of the relaxation of stress in the steel, shrinkage and creep of the concrete, and some other service and structural factors. If the stress in the steel is close to its ultimate strength, wire reinforcement may break down in tension and hot-rolled reinforcement may suffer considerable permanent set. Present evidence shows that the prestress in the steel placed in the tension and compression zones, designated σ_0 and σ_0' respectively, should be specified according to the following conditions

— for reinforcing bars

$$\sigma_0 + p \leqslant R_s^b \text{ and } \sigma_0 - p \geqslant 0.3R_s^b \tag{II.22}$$

— for wire reinforcement

$$\sigma_0 + p \leqslant 0.8R_s^b \text{ and } \sigma_0 - p \geqslant 0.2R_s^b \tag{II.23}$$

where $p = 0.05\sigma_0$ MPa for mechanical tensioning; $p = 0.1\,(300 + 3\,600/l)$ MPa for electrothermal tensioning (that is, tensioning by electric heating); l is the length of the bar being tensioned in metres (the length out to out of the bearing plates).

In electrothermal tensioning, the temperature should not exceed 300 or 350°C so as not to impair the steel strength.

In pretensioning, the initial jacking stress in the steel with allowance for anchorage strain losses, σ_3, and friction losses in templates, σ_4, is equal to

$$\sigma_{in,j} = \sigma_0 - \sigma_3 - \sigma_4 \text{ and}$$

$$\sigma_{in,j}' = \sigma_0' - \sigma_3' - \sigma_4'$$

The initial jacking stress in posttensioning (with a part of the prestress transferred to the concrete) is equal to

$$\sigma_{in,j} = \sigma_0 - n\sigma_{pr} \text{ and}$$

$$\sigma_{in,j}' = \sigma_0' - n\sigma_{pr}'$$

where σ_{pr} is the prestress (with regard to the early losses).

Any possible deviation from the specified prestress in manufacture is taken care of by the tension accuracy factor

$$m_{ac} = 1 \pm \Delta m_{ac} \tag{II.24}$$

where Δm_{ac} is the extreme deviation in the prestress in the steel. The plus sign is taken when the prestress has a negative effect on the member behaviour (for example, in the strength analysis of rein-

forcing steel placed in the zone compressed under load, and also in the calculations for the manufacture and erection stages), the minus sign is taken when the effect of the prestress is positive.

In mechanical tensioning, $\Delta m_{ac} = 0.1$; in electrothermal tensioning,

$$\Delta m_{ac} = 0.5\,(p/\sigma_0)\,(1 + 1/\sqrt{n}) \qquad \text{(II.25)}$$

where n is the number of prestressed bars in the section; p and σ_0 are the same as in Eqs. (II.22).

For the prestress loss, crack-width and displacement calculations, Δm_{ac} may be taken as zero.

The transfer or cube strength of concrete by the instant when the prestress is transferred from the steel to the concrete, designated R_0, should be chosen so that the ratio σ_{pr}/R_0 would not be too high, because otherwise there would be considerable creep strain in the concrete and the prestress in the steel would be gone. The recommended value is $R_0 \geqslant 0.8\overline{R}$ but not less than 14 MPa, and not less than 20 MPa for class Aт-VI bars and class K-7 wire strands.

The stress transferred to the concrete during compression, σ_{pr}, is limited for the same reason; it should not exceed the limiting values of Table II.5, given as fractions of the transfer strength, R_0.

TABLE II.5. Limiting Prestress in Concrete (for a Design Ambient Temperature of above −40°C)

State of stress in section	Method of tensioning	Prestress in concrete, fractions of R_0, max	
		axial prestressing	eccentrical prestressing
Prestress decreases under load	Pretensioning	0.65	0.75
	Posttensioning	0.55	0.65
Prestress increases under load	Pretensioning	0.5	0.55
	Posttensioning	0.45	0.5

If σ_{pr} is reduced by external loading (which happens most often), its value for the eccentrical transfer of compression and pretensioning should not exceed 0.75 R_0.

According to the type and class of prestressed steel, its diameter and anchorage, the concrete for prestressed heavy- and porous-ag-

TABLE II.6. Design Concrete Brands for Prestressed Members

Prestressed steel	Design concrete brand
Wire:	
class B-II with anchorage	M-250
class Bp-II without anchorage, max 5 mm in diameter	M-250
the same, 6 mm and more in diameter	M-400
class K-7 strands	M-350
Deformed bars without anchorage, 10 to 18 mm inclusive in diameter:	
classes A-IV and AT-IV	M-200
classes A-V and AT-V	M-250
class AT-VI	M-350
The same, 20 mm and more in diameter:	
classes A-IV and AT-IV	M-250
classes A-V and AT-V	M-350
class AT-VI	M-400

gregate-concrete members ranges in brand number between M-200 and M-400 (see Table II.6). As the diameter and strength of the reinforcing steel increase, the brand number is also increased.

2. Prestress Losses in Reinforcing Steel

The initial prestress in the steel decreases with time. Losses in prestress are customarily divided into the early losses taking place during the manufacture and transfer to concrete, and the late losses occurring after the prestress has been transferred to the concrete.

Early Losses. 1. Losses due to relaxation of stress in the steel in pretensioning and according to the type of tensioning and reinforcing steel:

Mechanical tensioning, MPa:

— high-strength wire and wire strands

$$\sigma_1 = (0.27\sigma_0/R_{s\text{II}} - 0.1)\,\sigma_0$$

— bars

$$\sigma_1 = 0.1\sigma_0 - 20$$

Electrothermal and electrothermomechanical tensioning:

— high-strength wire and wire strands

$$\sigma_1 = 0.05\sigma_0$$

— bars

$$\sigma_1 = 0.03\sigma_0$$

Here, σ_0 is taken without losses.

2. Losses due to the difference in temperature between the steel and devices resisting the prestress during the moist or heat curing of the concrete

$$\sigma_2 = 1.25\Delta t$$

where Δt is the difference in temperature between the steel and abutments resisting the prestress in °C; if no data are available, Δt is taken as 65°C.

3. Losses due to the deformation of anchorages located near tensioning devices, caused by upset shims, crushed button or rivet heads, and displaced bars in grips in mechanical pretensioning

$$\sigma_3 = \lambda E_s/l$$

where $\lambda = 2$ mm with upset shims or crushed button or rivet heads; $\lambda = 1.25 + 0.15d$ with displaced bars in reusable grips; here, d is

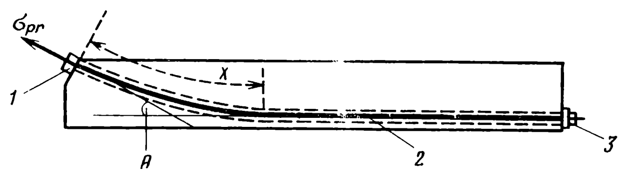


Fig II.6. To determining frictional losses in raceways
1 — jacking end; *2*—tendon in a raceway; *3*—anchorage

the bar diameter in millimetres, and l is the length of the bar being tensioned in millimetres (the distance out to out of the bearing plates). In electrothermal tensioning, $\sigma_3 = 0$.

In posttensioning,

$$\sigma_3 = (\lambda_1 + \lambda_2)E_s/l$$

where λ_1 is the amount of upsetting in shims placed between the anchorages and the concrete, equal to 1 mm; λ_2 is the displacement of basket-type anchorages, Freyssinet anchorages, anchoring nuts and grips, taken as 1 mm; and l is the length of the bar being tensioned (the length of the member).

4. Losses due to steel friction:

(a) against raceway walls or structure surface in posttensioning (Fig. II.6).

$$\sigma_4 = \sigma_0 \,[1 - \exp(-kx - \mu\theta)]$$

where x is the length of the tendon portion; θ is the total angular change of the tendon along the curved portion, in radians; μ is the coefficient of friction between the tendon and surrounding material;

and k is the wobble coefficient. The values of μ and k are given in Table II.7.

TABLE II.7. **Friction and Wobble Coefficients**

Duct	k	cables and trands	deformed bars
With metal sheathing	0.003	0.35	0.4
With concrete surface:			
made by rigid duct-former	0	0.55	0.65
made by flexible duct-former	0.0015	0.55	0.65

(b) against templates in pretensioning

$$\sigma_4 = \sigma_0 [1 - \exp(-0.25\theta)]$$

where θ is the total angular change of the tendon, in radians.

5. Losses due to the deformation of steel forms in tensioning by jacks

$$\sigma_5 = [(t-1)/2t] (\Delta l/l) E_s$$

where Δl is the yield of the abutments along the resultant prestressing force, determined by the mould design; l is the distance out to out of the bearing plates; and t is the number of tendon groups tensioned simultaneously.

If no data about the mould design are available, σ_5 is taken as 30 MPa. In pretensioning by a wrapping machine, σ_5 is halved; in tensioning by electric heating, $\sigma_5 = 0$.

6. Losses due to instantaneous creep and according to the hardening conditions, stress-strength ratio and design concrete brand number. The loss of prestress occurs during the transfer of the prestress from the steel to the concrete (and also during the first 2 or 3 hours after the transfer). When a member is allowed to harden fully in the air, the loss of prestress is

$$\sigma_6 = 50\sigma_{pr}/R_0 \text{ at } \sigma_{pr}/R_0 \leqslant a$$

$$\sigma_6 = 50a + 100b(\sigma_{pr}/R_0 - a) \text{ at } \sigma_{pr}/R_0 > a$$

where $a = 0.6$ and $b = 1.5$ for M-300 concrete, $a = 0.5$ and $b = 3$ for M-200 concrete, and $a = 0.4$ and $b = 3$ for M-150 concrete; σ_{pr} is the prestress in the concrete at the centroid of the prestressed steel F_s and F'_s developed by the prestressing force N_0 taken with allowance for the losses σ_1 through σ_5. In moist curing at atmospheric pressure, the losses are multiplied by 0.85.

Late Losses. 7. Losses due to relaxation in posttensioned high-strength wire and bars, taken equal to the losses in pretensioning, that is, $\sigma_7 = \sigma_1$.

8. Losses caused by shrinkage and shortening of a member, according to the type of concrete, method of tensioning, and hardening conditions. The values of σ_8 will be found in Table II.8.

TABLE. II.8. Loss of Prestress due to Shrinkage (in MPa)

Concrete	Pretensioning		Posttensioning
	in air entire time	heat cured at atmospheric pressure	irrespective of hardening conditions
Heavy concrete:			
M-400 and lower	40	35	30
M-500	50	40	35
M-600 and higher	60	50	40
Porous-aggregate concrete:			
dense fine aggregate	50	45	—
porous fine aggregate, except expanded perlite sand	65	55	—
expanded perlite sand as fine aggregate	90	80	—

9. Losses due to creep of concrete (caused by the respective shortening of a member), according to the type of concrete, hardening conditions, and stress-strength ratio.

For heavy and porous-aggregate concrete (using dense fine aggregate), the loss of prestress is

$$\sigma_9 = 200k\sigma_{pr}/R_0 \text{ at } \sigma_{pr}/R_0 \leqslant 0.6$$

$$\sigma_9 = 400k\,(\sigma_{pr}/R_0 - 0.3) \text{ at } \sigma_{pr}/R_0 > 0.6$$

where σ_{pr} is determined as with the loss due to instantaneous creep (σ_6); $k = 1$ when the concrete is allowed to harden in the air entire time; and $k = 0.85$ when the concrete is cured by heat at atmospheric pressure.

For porous-aggregate concrete using porous fine aggregate (except expanded perlite sand), σ_9 is determined by the above formulas, with the coefficient 1.2 applied; for porous-aggregate concrete using expanded perlite sand as the fine aggregate, σ_9 is also found by the above formulas, but with the coefficient 1.7 applied.

10. Losses caused by crushing of the concrete under the turns of spiral or ring reinforcement (at a pipe and tank diameter of up to 3 m), defined as

$$\sigma_{10} = 30$$

11. Losses due to upset joints between precast blocks, defined as

$$\sigma_{11} = n\lambda E_s/l$$

where λ is the amount of upsetting equal to 0.3 mm for concreted joints and 0.5 mm for dry joints; n is the number of joints along the prestressed steel; and l is the length of the prestressed steel, in millimetres.

The shrinkage and creep losses, σ_8 and σ_9, markedly depend on the time of curing and ambient humidity. If we know the time of loading in advance, the loss is multiplied by the following coefficient

$$\beta = 4t/(100 + 3t) \text{ but not more than } 1$$

where t is the time reckoned from the instant when concreting is completed (for σ_8) or from the day when the prestress is applied to the concrete (for σ_9), in days.

For structures exposed to an ambient humidity of less than 40%, the shrinkage and creep losses increase by 25%. For structures intended for service in dry and hot climatic regions (such as Central Asia), the losses due to shrinkage and creep should be increased by 50%.

With pretensioning, the early losses include relaxation in the steel, thermal losses, anchorage deformation, frictional losses in templates, steel mould deformation, and instantaneous creep

$$\sigma_I = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 + \sigma_6$$

and the late losses include losses due to shrinkage and creep

$$\sigma_{II} = \sigma_8 + \sigma_9$$

With posttensioning, the early losses are composed of anchorage deformation and losses due to friction between the steel and duct walls (or structure concrete surface)

$$\sigma_I = \sigma_3 + \sigma_4$$

and the late losses include relaxation in the steel, shrinkage and creep in the concrete, crushing of the concrete under steel turns, and deformation of joints between blocks (for structures made of precast members)

$$\sigma_{II} = \sigma_7 + \sigma_8 + \sigma_9 + \sigma_{10} + \sigma_{11}$$

The total amount of losses for any method of tensioning is defined as

$$\sigma_{loss} = \sigma_I + \sigma_{II}$$

It may reach 30% of the initial prestress. In design calculations, the total amount of losses should be taken as not less than 100 MPa.

3. Stress in Nonprestressed Steel

In prestressed members, the nonprestressed steel strained together with the concrete is subjected to initial compressive stresses which, during the transfer of the prestress to the concrete, are equal to the losses due to instantaneous creep

$$\sigma_s = \sigma_6$$

and, in service, to the losses due to shrinkage and creep in the concrete

$$\sigma_s = \sigma_6 + \sigma_8 + \sigma_9$$

For the nonprestressed steel placed in the zone which is subjected to tension during the transfer of the prestress, σ_s is taken equal to σ_8.

4. Prestressing Force

The prestressing force is taken as the resultant of the forces in the prestressed and nonprestressed steel

$$N_0 = \sigma_0 F_{pr} + \sigma_0' F_{pr}' - \sigma_s F_s - \sigma_s' F_s' \qquad \text{(II.26)}$$

and the eccentricity of this force relative to the centroid of the transformed section is determined from the equilibrium of the moments

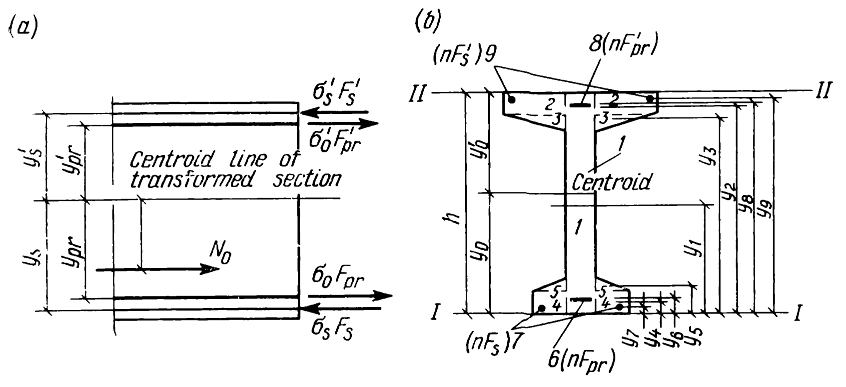


Fig. II.7. Prestressed member

(*a*) distribution of a prestress; (*b*) to determining the geometrical characteristics of the transformed section; *1* through *5* — elementary figures; *6* through *9*—reinforcing steel

of the resultant and the component forces (Fig. II.7*a*)

$$e_{0pr} = (\sigma_0 F_{pr} y_{pr} + \sigma' \boldsymbol{F}' y' - \sigma_0' F_{pr}' y_{pr}' - \sigma_s F_s y_s)/N_0 \qquad \text{(II.27)}$$

where σ_0 and σ_0' are the stresses in the prestressed steel in the tension and compression zones of the loaded member, respectively, determined with allowance for the early losses at the prestress transfer

stage, and the late losses in the service; σ_s and σ'_s are the compressive stresses in the nonprestressed steel in the tension and compression zones of the loaded member, respectively; F_{pr} and F'_{pr} are the cross-sectional areas of the prestressed steel in the tension and compression zones, respectively; F_s and F'_s are the cross-sectional areas of the nonprestressed steel in the tension and compression zones, respectively; and y_{pr}, y'_{pr}, y_s and y'_s are the distances from the centroid of the transformed section to the forces in the steel.

5. Transformed Section

Stresses in the sections of prestressed members in Stage I (before the advent of cracking) are found, using the concept of a transformed section, that is, one in which the steel is replaced by its transformed concrete area. Since the strain in the steel and concrete is the same, the transformed concrete area is n times its true area, where $n = E_s/E_c$. The transformed area is (Fig. II.7b)

$$F_{tr} = F + nF_{pr} + nF_s + nF'_{pr} + nF'_s \tag{II.28}$$

where F is the concrete area minus the duct and slot area.

The static moment of the transformed section about axis I-I on the bottom face of the section is

$$S_{tr} = \Sigma F_i y_i \tag{II.29}$$

where F_i is the area of a part of the section, and y_i is the distance from the centroid of the ith part of the section to axis I-I.

The distance from the centroid of the transformed area to axis I-I is

$$y_0 = S_{tr}/F_{tr} \tag{II.30}$$

The moment of inertia of the transformed section about the centroid of the transformed section is

$$I_{tr} = \Sigma\, [I_i + F_i\,(y_0 - y_i)^2] \tag{II.31}$$

where I_i is the moment of inertia of the ith part of the section about the centroid of this part.

The distances from the top and bottom kern points to the centroid of the transformed area are

$$r_{k,top} = I_{tr}/F_{tr}y_0 \text{ and } r_{k,bot} = I_{tr}/F_{tr}\,(h - y_0) \tag{II.32}$$

6. Prestress in Concrete

When the prestress is transferred to the concrete, it gives rise to nonelastic strain, and the normal stress diagram becomes curved. For simplicity, the prestress in the concrete is determined on the

assumption that the concrete section is elastic and the stress diagram is linear

$$\sigma_{pr} = N_0/F_{tr} \pm N_0 e_{0pr} y/I_{tr} \tag{II.33}$$

According to the purpose of the analysis, the prestress in the concrete is determined at different levels of the section:

(a) when it is necessary to find the jacking stress in posttensioned steel, the prestress in the concrete is found at the level of the forces in the prestressed steel

$$\sigma_{pr} = N_0/F_{tr} \pm N_0 e_{0pr} y_{pr}/I_{tr} \tag{II.34}$$

$$\sigma'_{pr} = N_0/F_{tr} - N_0 e_{0pr} y'_{pr}/I_{tr} \tag{II.35}$$

Here, N_0 is determined with allowance for the early losses at $m_{ac} = 1$;

(b) when checking the ultimate prestress, it is determined at the extreme compression fibre

$$\sigma_{pr} = N_0/F_{tr} + N_0 e_{0pr} y/I_{tr} \tag{II.36}$$

Here, N_0 is determined with allowance for the early losses (less σ_6) at $m_{ac} = 1$;

(c) when calculating the losses due to instantaneous creep and shrinkage, σ_6 and σ_9, the prestress in the concrete is determined at the centroid of the prestressed steel in the tension and compression zones according to formulas (II.34) and (II.35). In this case, N_0 is found with allowance for the early losses (less σ_6) at $m_{ac} = 1$.

7. Gradual Variations in Prestress under Loading

Members in Axial Tension. When a member is manufactured, the steel is pretensioned until the initial jacking stress, $\sigma_{in,j}$, is attained, then the member is concreted, cured by heat and allowed to harden in the mould until it reaches the necessary transfer strength R_0. We shall call this State 1. Here, the early losses σ_{I} (less σ_6) take place (Fig. II.8). When the steel is released from the abutments, the prestress is transferred to the concrete, and instantaneous creep develops in the member, which causes the loss σ_6. This is State 2. In this case, the prestress in the steel with allowance for the elastic compression of the concrete is equal to

$$\sigma_{in,j} - \sigma_{\mathrm{I}} - n\sigma_c$$

where σ_{I} is taken without σ_3 and σ_4 because they are lumped with $\sigma_{in,j}$.

In the course of time, the late losses σ_{II} take place, and the elastic strain in the concrete also decreases in proportion. Now, we speak of State 3. Here, the prestress in the steel with allowance for the

total amount of losses and elastic compression of the concrete is equal to

$$\sigma_{in,j} - \sigma_{loss} - n\sigma_{c1}$$

When the member is subjected to a gradually increasing load, the prestress in the concrete is balanced, and State 4 takes place. In this state, the prestress in the steel, with the losses at zero stress in the concrete, is equal to

$$\sigma_0 = \sigma_{in,j} - \sigma_{loss}$$

As the load increases, the tensile stress in the concrete reaches its tensile strength, R_{ten}. This is State 5 which corresponds to the end of Stage I.

Recalling that the tensile strain capacity of the concrete is $\varepsilon_{c,ten}^{ul} = 2R_{ten}^{b}/E_c$ and taking into consideration that the steel and the concrete resist the strain together, we may write for the stress increment in the tensile steel after the prestress in the concrete has been nullified

$$\sigma_s = \varepsilon_s E_s = \varepsilon_{c,\,ten}^{ul} E_s = 2R_{ten}^{b} E_s/E_c = 2nR_{ten}^{b}$$

The stress in the prestressed steel before cracking is equal to

$$\sigma_0 + 2nR_{ten}^{b}$$

Before the advent of cracking, the stress in the tensile steel is σ_0 above the corresponding stress in nonprestressed members. It is because of this that prestressed members subjected to axial tension have a much greater crack resistance. After cracks have occurred in Stage II, the tensile force is taken by the steel. As the load increases, the cracks open up. A further rise in the load brings about ultimate stresses in the steel, and the member breaks down, that is, Stage III takes place.

In posttensioning, the stress states occur in much the same sequence. The only difference during manufacture and prior to loading is that the initial jacking stress in the steel is determined with allowance for the prestress in the concrete.

Members in Bending. In the case of pretensioning, the tensile and compressive steel is tensioned until the initial jacking stresses, $\sigma_{in,j}$ and $\sigma'_{in,j}$, are attained (Fig. II.9). As a rule, $\sigma_{in,j}$ is taken equal to $\sigma'_{in,j}$. When the member is concreted and cured by heat, most of the early losses take place (State 1). After the member has attained the required strength, the steel is released from the abutments and the prestress is transferred to the concrete; because of instantaneous creep and elastic compression of the concrete, the prestress in the steel decreases (State 2). Since $F_{pr} > F'_{pr}$ and the member is in eccentrical compression, it hogs. In the course of time, the late los-

ses σ_{II} take place (State 3). When the external load is applied to the member, the prestress in the concrete is cancelled (State 4). In this state, the neutral line passes through the tensile steel, and the prestress in the steel is

$$\sigma_0 = \sigma_{in,j} - \sigma_{loss} \tag{II.37}$$

As the load is increased, the stress in the tension zone reaches the tensile strength of the concrete R^b_{ten} (State 5). This is the end of Stage I in bending. In this stage, the stress in the steel is equal to $\sigma_0 + 2nR_{ten}$. As with tension, the stress in the steel before cracking is σ_0 above the corresponding stress in nonprestressed members. Accordingly, prestressed members have a considerably higher crack resistance. As the load is increased further, the member passes into Stage II; the tensile stresses in the steel and the concrete reach the

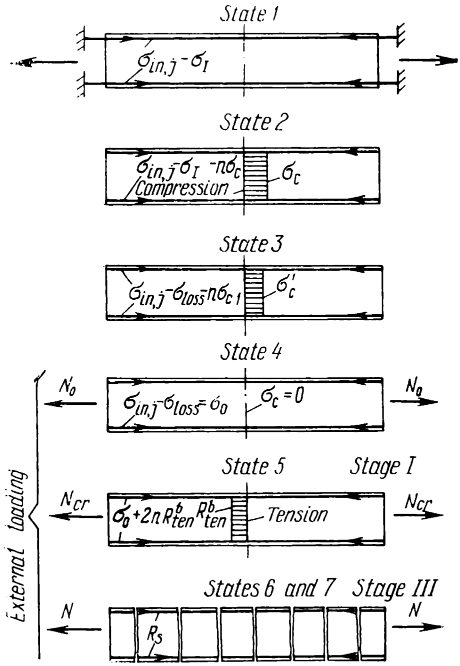


Fig. II.8. Stress variations in a prestressed member subjected to axial tension

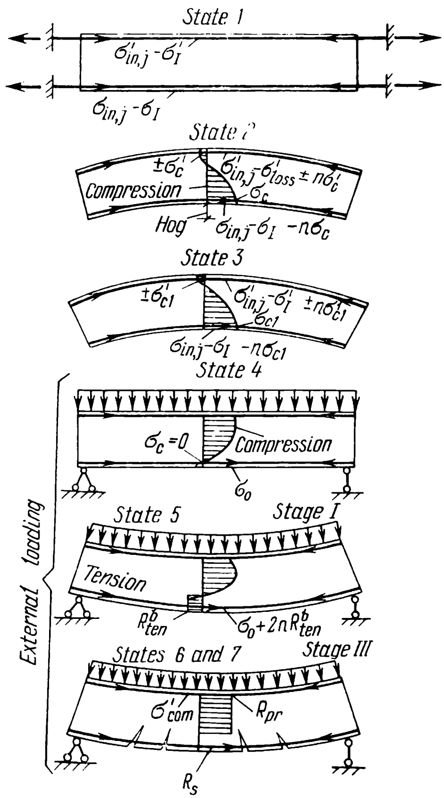


Fig. II.9. Stress variations in a prestressed member under bending

ultimate values, and the member fails (Stage III). The prestressed steel in the compression zone is strained together with the concrete. As this takes place, the prestress in it decreases. When the compressive stress in the concrete reaches its ultimate value, the stress in the prestressed steel in this zone is

$$\sigma_{com} = R_{s,com} - \sigma_0' \qquad (II.38)$$

The value of σ_0' is determined with allowance for losses at $m_{ac} > 1$. At $\sigma_0' < R_{s,com}$, the steel in the compression zone is compressed, and, at $\sigma_0' > R_{s,com}$, it is in tension, in which case it somewhat reduces the bearing capacity of the prestressed member.

II.6. ULTIMATE DEPTH OF COMPRESSION ZONE. LIMITING PERCENTAGE OF REINFORCEMENT

1. Ultimate Depth of Compression Zone

In Stage III, normal sections of members in bending, eccentrical compression and eccentrical tension (that is, those carrying both compressive and tensile stresses) are in the same stress-strain state. Here, the stress distribution in the compression zone is represented by a conventional rectangular diagram, and the design strength of

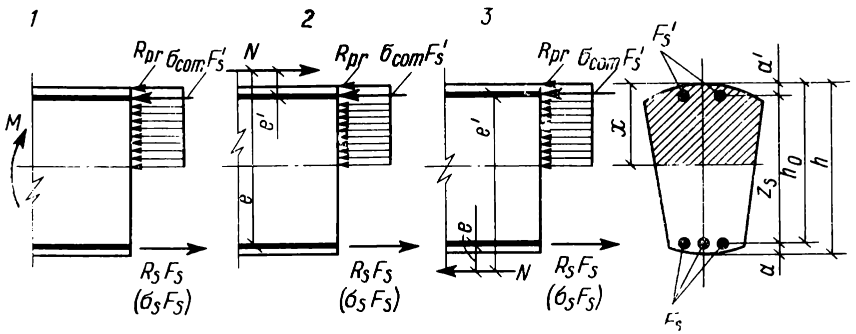


Fig. II.10. To the strength analysis of any symmetrical section

(1) member in bending; (2) member in eccentric compression; (3) member in eccentric tension

the concrete is equal to R_{pr} (Fig. II.10). In any case, a member is strong as long as the moment of the external forces does not exceed that of the internal forces. For the moments about the centroid of the tensile steel we get

$$M \leqslant R_{pr}S_c + \sigma_{com}F_s'z_s \qquad (II.39)$$

where M is the moment of the external forces due to design loads for members in bending, or the moment of the external longitudinal

force for members in eccentrical compression and eccentrical tension, that is, $M = Ne$ (e = distance from N to the centroid of the tensile steel, see Fig. II.10); S_c is the static moment of the compression zone; and z_s is the distance between the centroids of the tensile and compressive steel.

The stress in the prestressed compressive steel is $\sigma_{com} = R_{s,com} - \sigma_0'$, where σ_0' is determined at $m_{ac} > 1$. In nonprestressed members, $\sigma_{com} = R_{s,com}$.

The depth of the compression zone x for members in Case 1 when the stresses in the tensile and compressive steel have not yet reached the ultimate strength is deduced from the equation for the equilibrium of the ultimate forces

$$R_{pr}F_c + \sigma_{com}F_s' - R_sF_s \pm N = 0 \qquad \text{(II.40)}$$

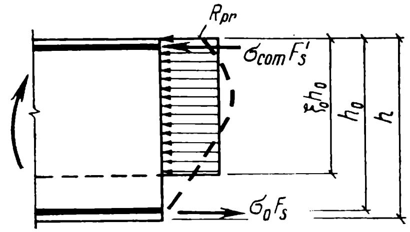


Fig. II.11. To determining the characteristic of the compression zone

where F_c is the area of the concrete in compression, which depends on the depth of the compression zone; for a rectangular section, $F_c = bx$.

In Eq. (II.40), the minus sign corresponds to eccentric compression and the plus sign to eccentric tension; for bending, $N = 0$.

For sections in Case 2 when brittle failure takes place in the concrete and the stress in the steel does not reach its ultimate value, the depth of the compression zone is also determined from Eq. (II.40). In this case, though, the design strength R_s is replaced by the stress σ_s.

Experiments show that σ_s depends on the relative depth of the compression zone $\xi = x/h_0$; it may be determined by the following empirical formula

$$\sigma_s = [R_{s,com}/(1 - \xi_0/1.1)]\,(\xi_0/\xi - 1) + \sigma_0 \qquad \text{(II.41)}$$

Here, $\xi_0 = x_0/h_0$ is the relative depth of the compression zone at the stress in steel $\sigma_s = \sigma_0$ (or $\sigma_s = 0$ in nonprestressed members).

Since at $\sigma_s = \sigma_0$ (or at $\sigma_s = 0$) the actual relative depth of the compression zone is $\xi_{act} = 1$, ξ_0 may be regarded as a correction factor taking care of the difference between the actual and theoretical rectangular stress diagrams. In this case, the force carried by the concrete in compression is $N_c = \xi_0 bh_0R_{pr}$ (Fig. II.11). Experience shows that for heavy concrete

$$\xi_0 = 0.85 - 0.008R_{pr} \text{ (where } R_{pr} \text{ is in MPa)}$$

and for lightweight concretes

$$\xi_0 = 0.8 - 0.008R_{pr}$$

The value of ξ_0 found from the above empirical formulas may be called the characteristic of the strain properties of the concrete in the compression zone.

In Eq. (II.41), the first term on the right side is an increment in the stress in the prestressed steel, $\Delta\sigma_s$, or a stress in the steel of nonprestressed members, σ_s. If ξ is less than ξ_0, σ_s is tensile, if ξ exceeds ξ_0, the stress is compressive (Fig. II.12).

The ultimate relative depth of the compression zone, $\xi_R = x_R/h_0$, at which the tensile stress in the steel reaches the ultimate strength may be determined from Eq. (II.41) if we set σ_s equal to the proof ultimate stress, σ_A. Then

$$\xi_R = \xi_0/[1 + (\sigma_A/R_{s,com})(1 - \xi_0/1.1)] \tag{II.42}$$

The relationship between σ_s and ξ on which Eq. (II.41) is based is derived on the assumption that the steel is subjected to elastic strains only. Accordingly, for reinforcing steel without a definite yield point, we should use the proof stress, σ_A, equal to $\sigma_{0.2}$, assuming that the strain is fully elastic (Fig. II.13). Taking into account that with these stresses the permanent set in the steel is 0.2%, we get

$$\sigma_A = R_s + 0.002E_s - \sigma_0 \tag{II.43}$$

If the value of σ_s calculated by Eq. (II.41) for steel without a definite yield point exceeds the elastic limit, $\sigma_{el} = 0.8R_s$, and ranges between $\sigma_{el} < \sigma_s \leqslant R_s$, it should be refined by the following formula

$$\sigma_s = [0.8 + 0.2(\xi_{el} - \xi)/(\xi_{el} - \xi_R)]\, R_s \tag{II.44}$$

where ξ_{el} is the depth of the compression zone, corresponding to the steel stress σ_{el}; ξ_{el} is found with Eq. (II.42) at $\sigma_A = \sigma_{el}$.

In high-strength wire, the limit-state stress, σ_s, may exceed the proof yield point. According to experimental data, this may happen if the relative depth of the compression zone determined from Eq. (II.40) is smaller than the ultimate depth, that is, $\xi < \xi_R$. The smaller ξ, the greater the difference in stress. This relationship is expressed in terms of the following coefficient

$$\begin{aligned} m_{s4} = \sigma_s/R_s &= 0.95\sigma_{ul}/\sigma_{0.2} \\ &\quad - (0.95\sigma_{ul}/\sigma_{0.2} - 1)\,\xi/\xi_R \end{aligned} \tag{II.45}$$

In strength analysis, the design strength of steel, R_s, is multiplied by the service factor for reinforcing steel

$$m_{s4} = \overline{m}_{s4} - (\overline{m}_{s4} - 1)\,\xi/\xi_R \tag{II.46}$$

where $\overline{m}_{s4}$ is:

for class A-IV and Aт-IV steel 1.2
for class A-V, Aт-V, B-II, Bp-I and K-7 steel 1.15
for class Aт-VI steel 1.1

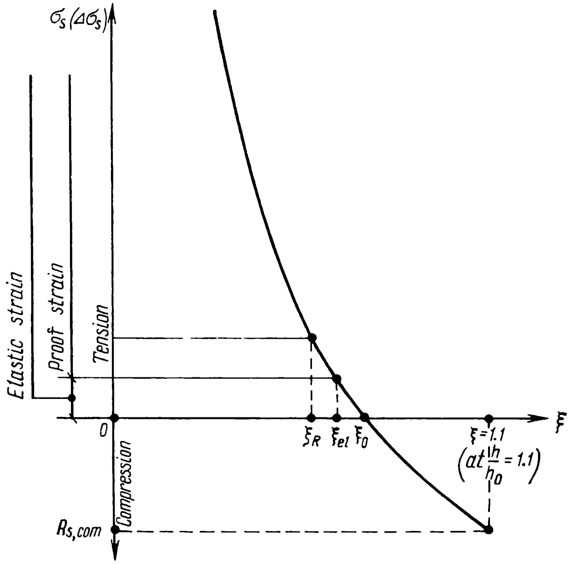


Fig. II.12. Empirical relationship between the ultimate stress in steel and the depth of the compression zone in Stage III

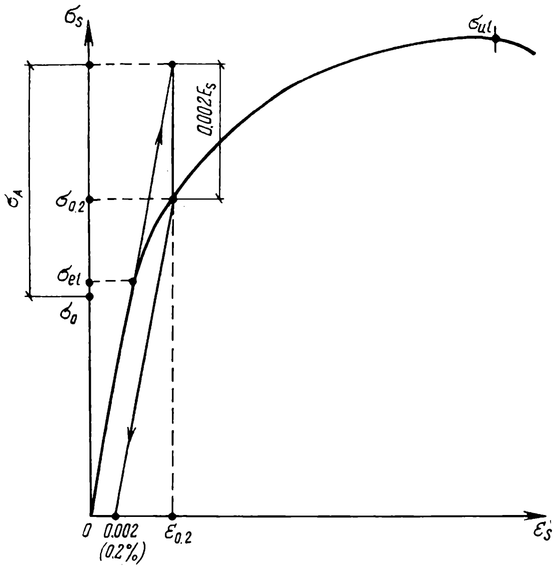


Fig. II.13. To determining the proof stress for a steel having no physical yield point

2. Limiting Percentage of Reinforcement

The limiting percentage of reinforcement for members in bending containing tensile steel only is determined from the equation describing the equilibrium of ultimate forces [Eq. (II.40)] at the ultimate depth of the compression zone. From this we have

$$R_{pr}bx_R - R_sF_s = 0$$

whence

$$\mu = 100\xi_R R_{pr}/R_s \tag{II.47}$$

Recalling Eq. (II.42), we may write the limiting percentage of reinforcement for prestressed members as

$$\mu = 100\xi_0 R_{pr}/[1 + (\sigma_A/R_{s,com})(1 - \xi_0/1.1)]\, R_s \tag{II.48}$$

and for nonprestressed members congested with steel having a definite yield point as

$$\mu = 100\xi_0 R_{pr}/(2 - \xi_0/1.1)\, R_s \tag{II.49}$$

The limiting percentage of reinforcement increases with increasing brand number of the concrete and decreases with increasing class of the steel. The limiting percentage of reinforcement calculated

TABLE II.9. Limiting Percentage of Reinforcement for Members in Bending

Class of steel	Design brand of concrete		
	M-200	M-300	M-400
A-III	$\frac{1.6}{1.37}$	$\frac{2.2}{1.97}$	$\frac{2.7}{2.4}$
A-II	$\frac{2}{1.74}$	$\frac{2.75}{2.45}$	$\frac{3.4}{3}$

Note. The figures in the numerators apply to heavy concrete, those in the denominator to porous-aggregate concretes.

from Eq. (II.49) for some concrete brands and steel classes is given in Table II.9. Members in bending whose percentage of reinforcement exceeds the limiting value are called overreinforced.

The minimum percentage of reinforcement is established from design considerations—to resist various forces (such as shrinkage, thermal stress, and so on) which have not been taken into account

in the analysis. For rectangular sections $b \times h$ in bending and eccentric tension, the minimum percentage of tensile longitudinal reinforcement is $\mu_1 = 0.05\%$; if the longitudinal force is applied within the distance z_s between the compressive and tensile reinforcement, the minimum percentage of each reinforcement for members in eccentric tension is $\mu_1 = 0.05\%$.

In T-sections with their flanges in compression, the minimum percentage of reinforcement refers to the rib cross-sectional area equal to bh.

CHAPTER III

MEMBERS IN BENDING

III.1 CONSTRUCTIONAL FEATURES

The most common reinforced concrete members in bending are slabs and beams. Slabs are defined as flat members whose thickness, h_s, is much smaller than the length, l_s, and width, b_s. Beams are defined as structural members whose length l considerably exceeds their depth, h, and width, b. These members are used in many reinforced concrete structures, most often in precast, in-situ and precast/in-situ beam and slab floors and roofs (Fig. III.1).

There exist single- and multi-span slabs and beams.

Slabs. The thickness of slabs cast in-situ ranges between 50 and 100 mm, whereas precast slabs should be as thin as possible.

Figure III.2*a* shows a one-way single-span slab; Figure III.2*b* shows an in-situ one-way multi-span slab carried by a number of parallel supports. Such slabs deform as beams under various loads, provided the loads do not change in a direction normal to the span.

Slab reinforcement consists of welded-wire fabric (see Sec. I.2). The fabric is placed with its load-bearing wires along the span to resist tensile forces induced by bending according to the bending moment diagrams (Fig. III.2). So, within the span, the fabric is placed in the lower part of the slab; in multi-span slabs, the fabric is additionally placed in the upper part above the intermediate supports. The fabric reinforcement used in multi-span slabs may be continuous (Fig. III.2*b*) and separate (Fig. III.2*c*).

Load-bearing wires are from 3 to 10 mm in diameter, with a spacing of 100 to 200 mm.

The minimum concrete cover for load-bearing reinforcement is 10 mm; in superthick slabs (thicker than 100 mm), it is 15 mm.

Transverse wires (distribution reinforcement) are placed to hold load-bearing wires in the design position, reduce shrinkage and thermal strain in structures, and distribute the stresses due to concen-

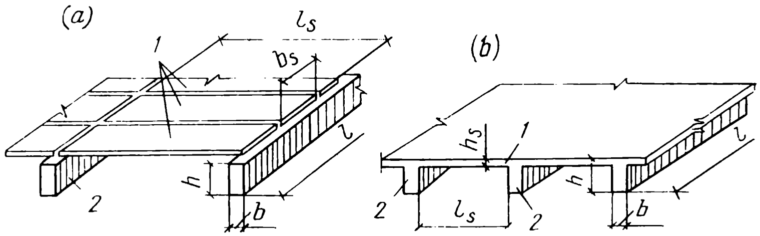


Fig. III.1. Reinforced concrete floors
(*a*) precast floor; (*b*) in-situ floor; *1* —slabs; *2*—beams

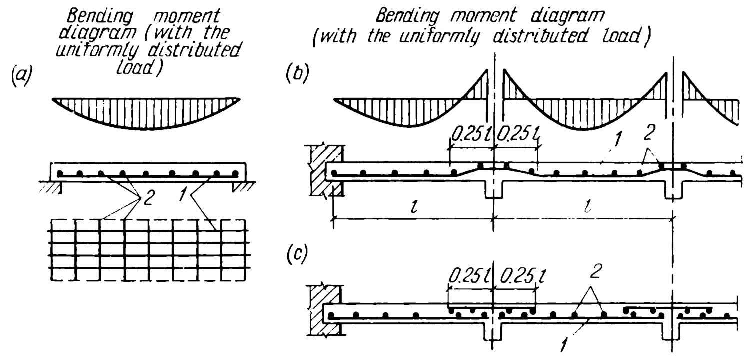


Fig. III.2. Slab reinforcement
(*a*) single-span slab; (*b*) multi-span slab with continuous reinforcement; (*c*) multi-span slab with separate reinforcement; *1* — load-bearing bars; *2* — erection bars

trated loads over a larger area. They are chosen to be smaller in diameter than load-bearing wires, but the total transverse steel area should be not less than 10% of the load-bearing steel area at places where the bending moment is a maximum. Transverse wires are spaced from 250 to 300 mm apart, but not wider than 350 mm.

In some cases (in slabs of a complex configuration or having a large number of voids), where standard welded-wire fabric cannot be used, slabs are reinforced using separate wires held together by manually applied tie wire.

Reinforced Concrete Beams. There exist rectangular beams, T-beams, I-beams and trapezoidal beams (Fig. III.3).

The beam depth, h, varies over a wide range; according to the load and type of structure, it may be from 1/10 to 1/20 of the span. For unification, the beam depth is taken as a multiple of 50 mm if it does not exceed 600 mm, or a multiple of 100 mm if it is greater.

The width of rectangular beams, b, is taken as from $0.3h$ to $0.5h$, namely 100, 120, 150, 200, 220, or 250 mm; the greater sizes being

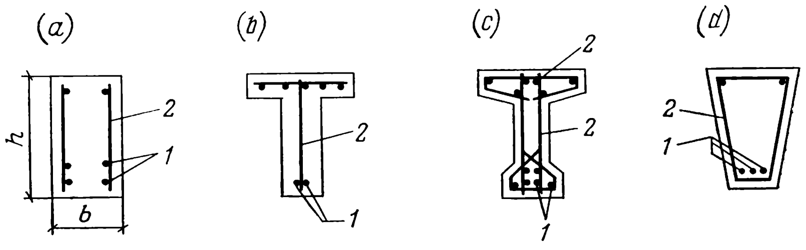


Fig. III.3. Beam cross sections and types of reinforcement
(a) rectangular beam; (b) T-beam; (c) I-beam; (d) trapezoidal beam; 1—longitudinal bars; 2—transverse reinforcement

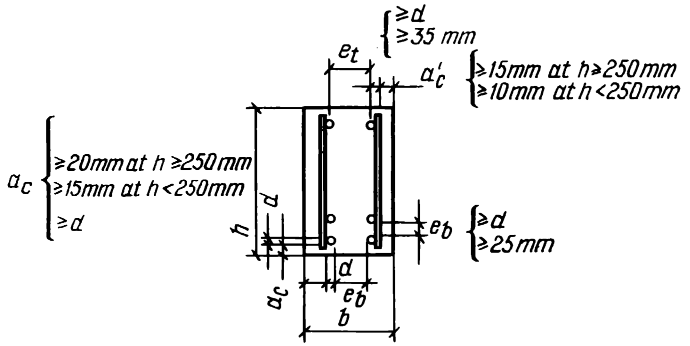


Fig. III.4. Distribution of reinforcement over a beam cross section
a_c—concrete cover for load-bearing reinforcement; a'_c—concrete cover for erection reinforcement; d—largest diameter of load-bearing bars; e_b—clear space between bottom bars (when placing concrete); e_t—clear space between top bars (when placing concrete)

taken as a multiple of 50 mm. To reduce the concrete consumption, the beam width should be taken as small as possible. Over the cross section, the load-bearing reinforcement is positioned in the tension zone in one or two rows with a spacing sufficient for concreting without voids and cavities. The necessary spacings and covers are shown in Fig. III.4. The clear space between longitudinal nonprestressed or pretensioned bars should be not less than the maximum bar diameter; for bottom horizontal bars (so positioned during concreting), it should be at least 25 mm, and for top bars, 30 mm. If there are more than two bottom bar rows, the horizontal separation between the bars in the third and higher rows (as reckoned from the bottom) should be at least 50 mm.

If little space is available, bars may be placed in pairs without separation.

The clear space between deformed reinforcing bars is taken according to their nominal diameters without allowance for lugs.

As in slabs, longitudinal load-bearing reinforcement in beams is placed according to the bending moment diagrams in the tension zones where it is to resist longitudinal tensile forces induced by bending loads.

In members under bending, the calculated area of the longitudinal load-bearing reinforcement, F_s, should be at least 0.05% of the effective area of the section. In rectangular beams, this area is equal to the product of the section width b and the effective depth $h_0 = h - a$ (where h is the overall depth of the section, and a is the distance from the resultant force in the bars to the section face near which the reinforcement is located).

As a rule, longitudinal reinforcement consists of deformed bars 12 to 32 mm in diameter; plain bars are used more rarely.

Beams with a width of 150 mm and more are reinforced with at least two longitudinal bars which extend as far as the beam supports. If a beam is less than 150 mm wide it may be reinforced with one bar (or a bar mat).

In addition to bending moments, reinforced concrete beams are subjected to shearing forces. To carry these forces, beams require transverse (or shear) reinforcement. The area of transverse reinforcement is determined by an appropriate analysis and from design considerations.

Longitudinal and transverse reinforcement is welded into bar mats (see Sec. I.2); if no welding machines are available, they are held together with tie wire. Tied reinforcement is rather labour-consuming, and its use is warranted only when it is impossible to manufacture welded bar mats.

Welded bar mats are combined into reinforcing cages, using horizontal transverse bars spaced 1 to 1.5 m apart.

The reinforcement of a single-span beam using welded bar mats is shown in Fig. III.5*a*. When use is made of tied bar mats (Fig. III.5*b*), stirrups in rectangular beams are of closed design, whereas in T-beams where the rib is inseparable from the solid flange on either side, the stirrups may be open at the top (they are known as U stirrups). Beams wider than 35 cm use multiple stirrups. In tied bar mats, the stirrups should be at least 6 mm in diameter for beams up to 800 mm deep, and not less than 8 mm in diameter for deeper beams.

To meet design and erection requirements, in members without bent bars, the spacing between the transverse bars (or stirrups) in the longitudinal direction should be not more than $h/2$, or 150 mm for beams up to 400 mm in depth, and not more than $h/3$, or 500 mm for beams deeper than 400 mm, whichever is the smaller. In beams subjected to a uniformly distributed load, this requirement holds for 1/4 of the span starting from the support; in beams subjected to concentrated loads, the portion in question extends farther out

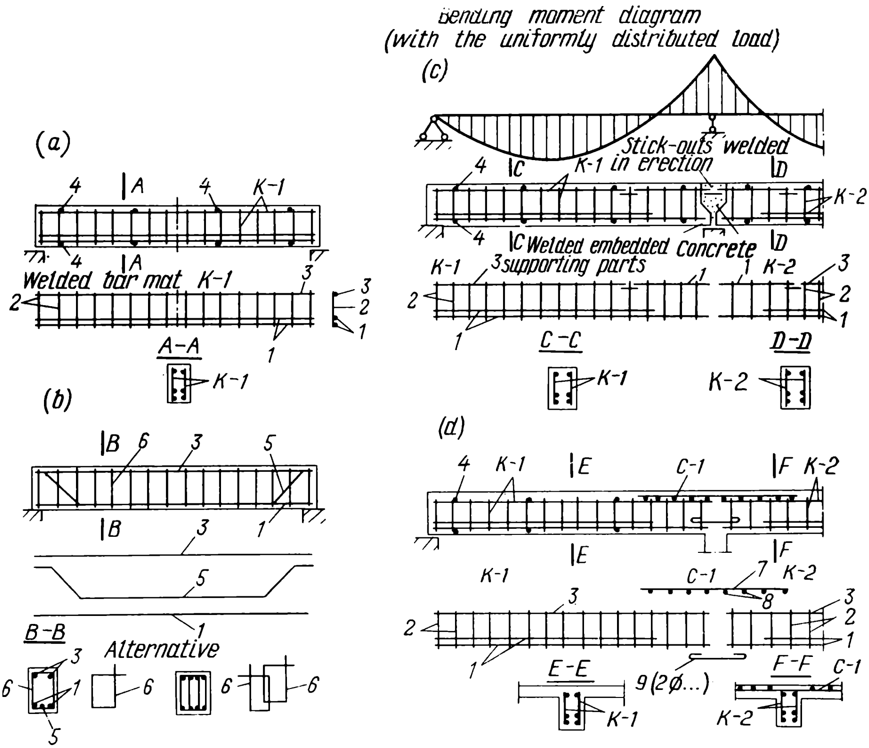


Fig. III.5. Types of beam reinforcement

(*a*) single-span beam with welded bar mats; (*b*) single-span beam with tied reinforcement; (c) precast multi-span rectangular beam; (*d*) in-situ multi-span T-beam; *1*—longitudinal load-bearing bars; *2*—transverse bars in mats; *3*—longitudinal erection bars; *4*—transverse erection bars; *5*—bent load-bearing bars; *6*—stirrups in tied mats; *7*—load-bearing bars in support fabric; *8*—spacer bars in support fabric; *9*—erection bars (two bars with a diameter of not less than 10 mm, not less than one half of the bottom erection bar diameter)

to the nearest load. In the remaining part of the span, the space between transverse bars (or stirrups) may be greater, but not more than $3/4h$ or 500 mm, whichever is the smaller.

Transverse bars (stirrups) in beams (and ribs) more than 150 mm in depth are placed even if they are not required by the analysis; if the depth is less than 150 mm, transverse reinforcement may be dispensed with.

In beams more than 700 mm in depth, the side faces are additionally reinforced by longitudinal bars spaced not more than 400 mm apart in the vertical direction. The cross-sectional area of each bar should be at least 0.1% of the area that they are intended to reinforce (the depth of this area is equal to a half-sum of the distances to the nearest bars, and the width is equal to a half of the member width, but

maximum to 200 mm). These bars together with transverse reinforcement prevent inclined cracks on the side faces from opening.

The top beam faces are reinforced by longitudinal distribution bars 10 to 12 mm in diameter supplied during erection; they are necessary to combine all reinforcing members into a cage that would be stable during concreting, and to anchor the ends of the transverse bars. In precast beams, the distribution bars may serve as load-bearing (or main) bars during transportation and erection.

Instead of, or in addition to, transverse bars, use may be made of inclined bars. They are more effective than the former because they better coincide with the direction of the principal tensile stresses in a beam. Transverse bars, however, are preferable because they are more convenient to install.

Inclined bars are usually placed at 45° to longitudinal bars. In beams deeper than 800 mm, they may be placed at up to 60°, whereas in beams with a smaller depth and beams subjected to concentrated loads, the inclined bars are positioned at 30°.

When beams are reinforced with tied cages (made of class A-I and A-II steel), it is good practice to bend up some longitudinal load-bearing bars to reduce the steel consumption and improve the construction of the cage (Fig. III.5*b*). The minimum bend radius is $10d$. Bends should end in straight portions at least $0.8l_{an}$ long (see Sec. I.3, Subsec. 4), and not less than $20d$ in tension zones or $10d$, whichever is the greater, in compression zones. The straight portions of bends in plain bars should have hooks at their ends.

Precast multi-span beams are fabricated from separate single-span members containing welded reinforcing cages (Fig. III.5*c*). The position and length of load-bearing reinforcement in a cage are determined from the bending moment diagram plotted as for a multi-span beam. At the joints above the intermediate supports, the stick-outs of the top load-bearing bars are field-welded together by means of erection plates in a pool of molten metal contained within a reusable mould; the bottom bars are welded to support pads with the help of special support arrangements embedded into the precast members. After the welding, the joints are concreted.

In-situ-cast multi-span T-beams reinforced by welded bar mats in the spans (see Fig. II.5*d*), have welded fabric placed above the intermediate supports. The load-bearing bars of the fabric are placed along the span; they are intended to carry the forces appearing in the tension zone above the supports.

In prestressed members subjected to bending, the reinforcement is placed according to the diagrams of bending moments and shearing forces induced by the load. The reinforcement using curved prestressed tendons (Fig. III.6*a*) is most efficient because it follows the path of the principal tensile stresses best of all, but it is more complex than the one using straight tendons (Fig. III.6*b*). In addition

to the steel of area F_{pr} in the tension zone, beams reinforced with straight prestressed tendons also use the steel of area F'_{pr} at the opposite face, whose area ranges between $0.15F_{pr}$ and $0.25F_{pr}$. This reinforcement is useful in deep beams where the prestressing force applied outside the kern causes at the top face a tension which may result in cracking during manufacture. Beams with a small depth may do without prestressed steel at the top face. Here, cracking can be cancelled by nonprestressed erection (distribution) steel.

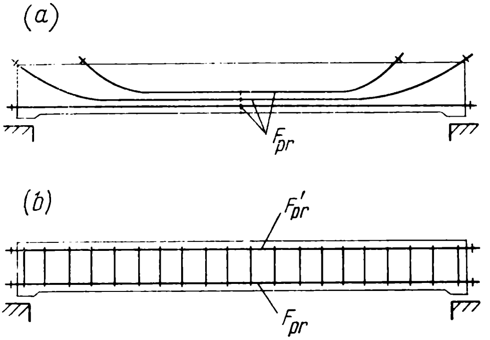


Fig. III.6. Types of prestressed beam reinforcement

(a) curved prestressed tendons; (b) straight prestressed tendons

The best choice for beams in bending are I-beams (see Fig. III.3c) and T-beams with sufficiently wide ribs (see Fig. III.3b). The flanges in compression are proportioned to suit the compressive resultant of the internal couple induced by the bending moment under load. The tension zone should be sufficiently wide to accommodate the necessary reinforcement and resist the prestress in prestressed members.

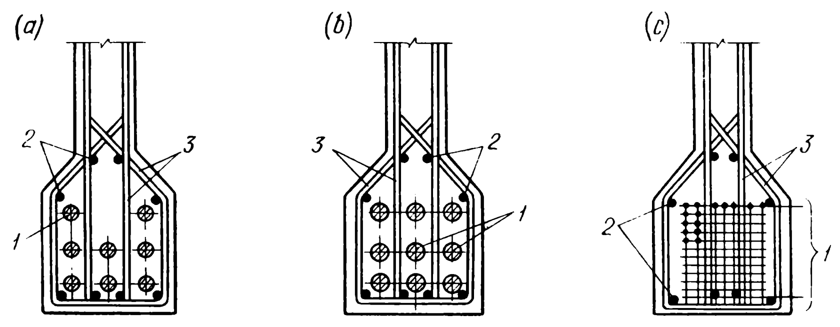


Fig. III.7. Distribution of reinforcement in the tension zone of prestressed members

(a) deformed bars; (b) strands or cables in ducts; (c) high-strength wire; 1—prestressed steel; 2—longitudinal nonprestressed steel; 3—transverse reinforcement

Prestressed steel is placed in tension zones as shown in Fig. III.7 Here, the concrete cover and the spacing between the pretensioned bars and strands are in accordance with Fig. III.4. In posttension-

ing, the concrete cover should not be less than 40 mm, nor less than the duct width. For the side faces, the cover should be not less than half the duct height. Prestressed steel placed in slots or outside the faces of a member should additionally be covered by at least 20 mm of concrete. The clear space between the ducts for posttensioned tendons should be not less than the duct diameter, nor less than 50 mm.

The maximum total angular change of curved posttensioned tendons between two points is taken as 30°, and the minimum radius of curvature is taken (to prevent large losses of prestress) as follows:

—wire 5 mm (and less) in diameter and strands 4.5 to 9 mm in diameter . . . . 4 m
—wire 6 to 8 mm (and less) in diameter and strands 12 to 15 mm in diameter . . . . 6 m
—bars up to 25 mm in diameter . . . . 15 m
—bars 28 to 40 mm in diameter . . . . 20 m

For prestressed members, special emphasis should be placed on the design of the reinforcement of beam ends near supports. Here, considerable prestressing forces are transferred from the steel to the concrete via end anchorages (in posttensioning) or grip length of steel (in the absence of anchorages). Also, the eccentric action of the prestressed steel results in local overstressing at the end of the member, which may cause cracking on its end and top surface. So, the ends of prestressed members should be reinforced additionally.

Prestressed members can locally be reinforced at anchorages and jacks by embedded parts attached to the prestressed tendons, by additional transverse bars, and also by enlarging the section area within the transmission length at the ends of a prestressed member. The concrete cover should be increased to at least $2d$ for bars of class A-IV (Aт-IV) or lower and strands, and to at least $3d$ for bars of class A-V (Aт-V) and higher (where d is the bar or strand diameter). Here, the concrete cover should be at least 40 mm for bars of all classes and 20 mm for strands. For the ends of a prestressed member with steel supporting parts reliably anchored in the concrete, or additionally reinforced by transverse steel or reinforcement-acting fixtures embracing all longitudinal prestressed bars, the cover may be the same as in the rest of the beam.

If at the end of a member the prestressed steel is concentrated at the bottom and top faces, the end should be additionally reinforced by prestressed or nonprestressed transverse steel. The transverse steel should be tensioned prior to the longitudinal reinforcement, with the prestressing force being at least 15% of that in the longitudinal steel in the tension zone at the support. The ends of nonprestressed transverse bars should be welded to embedded parts. The

cross-sectional area of these bars should be chosen to suit the acting force equal to at least 20% of the force in the longitudinal prestressed reinforcement (in the bottom zone at the support) determined by strength analysis.

The concrete at the ends of prestressed members containing steel with and without anchorages is reinforced by additional fabric or stirrups around all of the longitudinal bars (Fig. III.8). The length

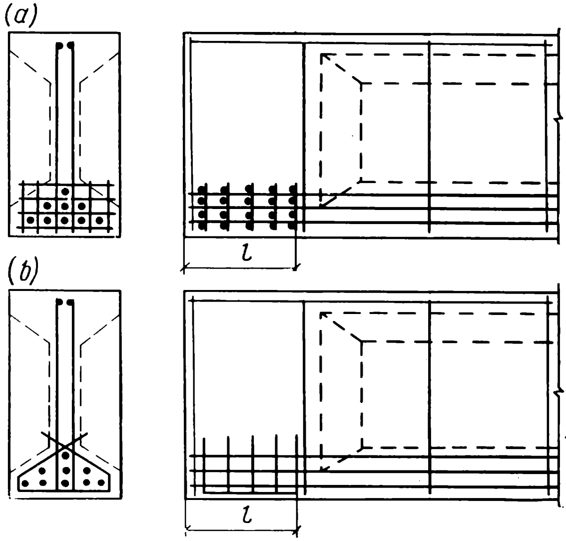


Fig. III.8. Local reinforcement of prestressed beam ends
(a) transverse welded fabric; (b) stirrups or embracing welded fabric

of the zone in question, l, is taken equal to twice the anchorage length, or, in the absence of anchorages, it should be not less than $0.6l_{tr}$ (see Sec. I.3), nor less than 20 cm.

At the ends of prestressed members reinforced without anchorages, cracking is not allowed under any load combinations (except special). During the transfer of the prestress from the steel to the concrete, such cracking is allowed in the prestress-tensioned zone of the section if it contains no prestressed steel without anchors, the transmission length does not exceed $2h_0$ (where h_0 is the effective depth of the beam), and the web is additionally reinforced in the prestress-tensioned zone near the support by nonprestressed longitudinal bars placed over a length equal to at least $2h_0$ starting from the beginning of the transmission zone; the cross-sectional area of this steel should be not less than 0.2% of that of the member at the support.

To ensure the necessary grip length for the longitudinal steel at the free (unfixed) ends of beams and slabs in bending, the bars should extend at least $5d$ beyond the internal face of the support if the calculations by formula (III.62) show that no cracking is likely to occur at the support; otherwise, the bars should extend at least $10d$.

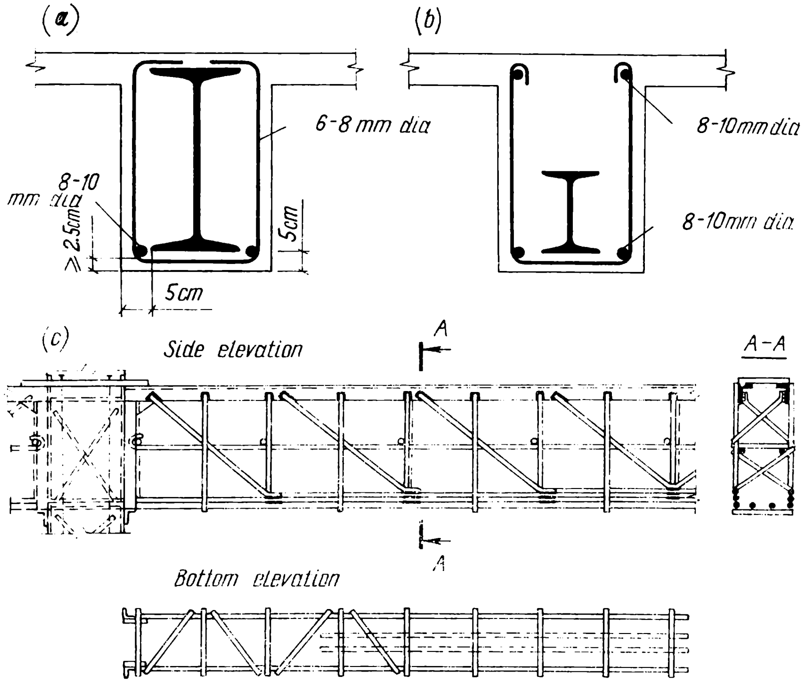


Fig. III.9. Beam reinforcement
(a) and (b) encased steel shapes; (c) welded reinforcing cage

The necessary grip length at a simply supported end is determined either by Eq. (I.20) or from the second line in Table I.2. If the grip length found by the equation is less than $10d$, it may be taken as given in the table, but not less than $5d$. In this case and also when the bar ends are welded to reliably anchored, embedded steel parts, the design strength of the steel at the support is not reduced.

The compressive stress in the concrete at a support (found by dividing the support reaction by the bearing area of the member) should not exceed $0.5R_{pr}$.

Under certain conditions, the load-bearing reinforcement of members in bending consists of steel shapes encased in concrete (known as stiff reinforcement) or welded reinforcing cages.

In a member with stiff reinforcement, steel shapes may occupy the entire depth (Fig. III.9*a*) or only the tension zone (Fig. III.9*b*). In either case, the member is additionally reinforced by welded-wire fabric or stirrups and longitudinal erection (distribution) bars 8 to 10 mm in diameter. Such a reinforcement reduces the crack width and contributes to the bond between the concrete and the stiff reinforcement. In the former case, lateral steel 6 to 8 mm in diameter is placed without calculations, whereas in the latter case, the necessary amount of lateral reinforcement is found by an appropriate design. Here, stirrups and fabric may be supplemented by diagonal bars welded to the top flange of steel beams. The minimum concrete cover for stiff reinforcement is 50 mm.

Load-bearing welded cages are made in the shape of trusses from plain and deformed bars and also small rolled-steel sections (Fig. III.9*c*). Such cages are designed for loads which may be applied during erection, until the concrete hardens. After the concrete has attained the necessary strength, the chords of the trusses act as longitudinal reinforcement, the diagonals as diagonal bars, and the struts as transverse bars.

III.2 NORMAL-SECTION STRENGTH ANALYSIS OF MEMBERS WITH ARBITRARY CROSS SECTION

Let us examine a single-span simply supported reinforced concrete beam loaded by two symmetric concentrated forces. The beam area between the loads is under pure bending, that is, it is subjected only to the bending moment, M, the shearing force being zero (Fig. III.10). At a certain amount of loading, normal cracks (those at right angles to the longitudinal axis of the beam) begin to form in the tension zone of this area. The portions between the support and load are subjected to both the bending moment, M, and the shearing force, Q. It is here that inclined cracks occur.

Accordingly, the strength analysis of members in bending is carried out for normal (*a*-*a*) and inclined (*b*-*b*) sections.

Modes of failure at normal sections and the theory of analysis have been discussed in Sec. II.1.

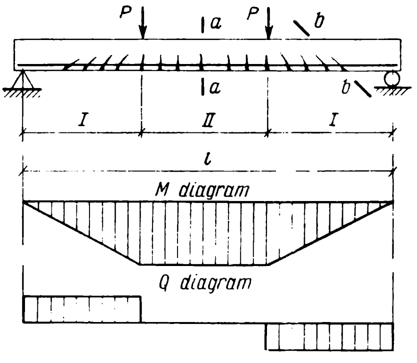


Fig. III.10. Reinforced concrete beam in bending

I—portion subjected to both M and Q; *II*—portion subjected to M only; *a*-*a*—normal section; *b*-*b*—inclined section

In terms of the first group of limit states, the normal-section strength of bending members is determined on the basis of Stage III of the stress-strain state (see Fig. II.1).

The adopted loading system consists of a bending moment, M, computed at design loads, and forces in the steel and concrete, determined at the stresses equal to their design strengths (Fig. III.11). For simplicity, the curved stress distribution diagram in the compression zone is replaced by a rectangular diagram, which fact affects the value of the moment insignificantly. The stress in the concrete is taken to be the same for the entire compression zone, and equal to R_{pr}.

A member may have a cross section of any shape, provided it is symmetric about the plane of flexure. In the general case, the tension zone contains nonprestressed reinforcement of area F_s and of

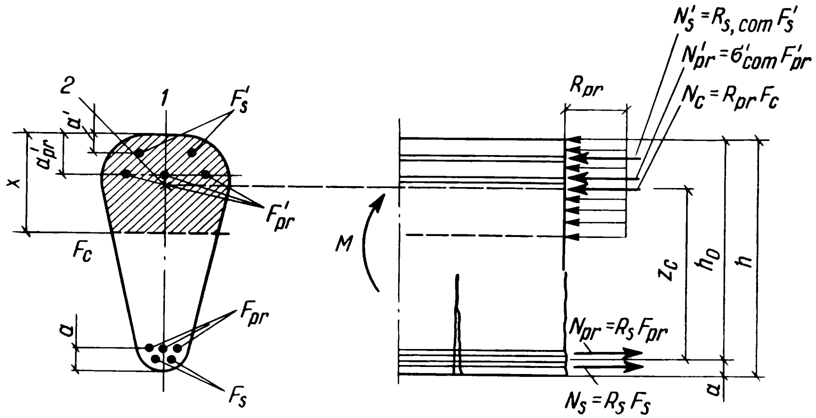


Fig. III.11. Loading system for the normal section strength analysis of members under bending

1—symmetry axis of the member section; *2*—centroid of the concrete in compression

design tensile strength R_s, and prestressed reinforcement of area F_{pr} and the design tensile strength R_s. The compression zone may also be reinforced with nonprestressed steel of area F'_s and of design compressive strength $R_{s,com}$, and prestressed steel of area F'_{pr}, subjected to a certain compressive stress σ'_{com}.

In Fig. III.11, the letter a designates the distance from the resultant of the forces in the tensile steel of area F_s and F_{pr} to the tensile face. The resultants of the normal stresses in the steel and the concrete are

$$\left.\begin{aligned} &N_s = R_s F_s; \quad N_{pr} = m_{s4} R_s F_{pr}; \quad N_c = R_{pr} F_c \\ &N'_s = R_{s.com} F'_s; \quad N'_{pr} = \sigma'_{com} F'_{pr} \end{aligned}\right\} \qquad \text{(III.1)}$$

Here R stands for the design strengths of the materials, and m_{s4} is the additional service factor taking care of the increase in the strength of the tensile prestressed high-strength steel stressed above the proof yield point. This service factor is determined from the empirical formula (II.46).

$$m_{s4} = \overline{m}_{s4} - (\overline{m}_{s4} - 1)\ \xi/\xi_R \tag{III.2}$$

where $\overline{m}_{s4}$ is the maximum value of m_{s4}, which is equal to 1.2 for class A-IV and Aт-IV steel; 1.15 for class A-V, Aт-V, B-II, Bp-II and K-7 steel; and 1.1 for class Aт-VI steel.

If, however, there are welded joints in class A-IV and A-V steel placed in the area where the bending moments exceed 0.9 of the maximum design moment, m_{s4} is adopted equal to not more than 1.1.

In Eq. (III.2), the relative depth of the compression zone, $\xi = x/h_0$, is determined from formula (III.3) at $m_{s4} = 1$, and the ultimate relative depth of the compression zone, ξ_R, (defined as one at which the limit state in the member is reached the same instant as the tensile steel reaches the stress equal to its design strength) is determined from formula (III.7).

From the equilibrium of the internal forces

$$R_sF_s + m_{s4}R_sF_{pr} - R_{pr}F_c - R_{s,com}F'_s - \sigma'_{com}F'_{pr} = 0 \tag{III.3}$$

we may find the cross-sectional area, F_c, of the compression zone in the concrete and, finally, its depth, x.

The member is considered to be sufficiently strong if the external design bending moment does not exceed the design bearing capacity of the member, expressed in terms of the opposing moment of the internal forces. In the case of moments about the axis normal to the plane of flexure and passing through the point where the resultant of the forces in the tensile steel of area F_s and F_{pr} is applied, the condition for strength is expressed by the following inequality

$$M \leqslant R_{pr}F_cz_c + R_{s,com}F'_s(h_0 - a') + \sigma'_{com}F'_{pr}(h_0 - a'_{pr}) \tag{III.4}$$

When using Eqs. (III.3) and (III.4), the stress σ'_{com} in the steel of area F'_{pr} is determined from the following expression

$$\sigma'_{com} = 400 - m_{ac}\sigma_0 \tag{III.5}$$

where 400 MPa is the compressive stress determined to suit the composite strain in the steel and the concrete at the ultimate strain capacity of the concrete, that is, $\varepsilon_c^{ul}E_s =$ (approx.) $0.0020 \times 200\,000 = 400$ MPa; m_{ac} is the tension accuracy factor for F'_{pr}, taken equal to 1.1; and σ_0 is the tensile prestress in F'_{pr}, taken with allowance for the appropriate losses.

At $m_{ac}\sigma_0 < 400$ MPa, the steel F'_{pr} is in compression, which is shown in Fig. III.11; otherwise, it is in tension.

If a member does not contain one type or another of tensile or compressive steel, the respective terms in the above expressions should be dropped.

Members most effective in bending are those in which

$$x \leqslant \xi_R h_0 \tag{III.6}$$

The ultimate relative depth of the compression zone for rectangular, T- and I-beams is given by formula (II.42) at $R_{s,com} = 400$ MPa as

$$\xi_R = \xi_0/[1 + \sigma_A (1 - \xi_0/1.1)/400] \tag{III.7}$$

where ξ_0 is the characteristic of the compression zone in the concrete. For heavy and porous-aggregate concrete it is found from the following expression

$$\xi_0 = a - 0.008R_{pr} \tag{III.7a}$$

where R_{pr} is in MPa, $a = 0.85$ for heavy concrete, and $a = 0.8$ for porous-aggregate concrete.

In Eq. (III.7), the proof stress in the steel is taken according to formula (II.43) at $0.002E_s = 400$ MPa. It is

$$\sigma_A = R_s + 400 - \sigma_0 \tag{III.8}$$

for steel without a definite yield point (class A-IV and higher), class B-II and Bp-II wire, and strands, and

$$\sigma_A = R_s$$

for steel with a definite yield point (classes A-I, A-II and A-III) and wire (classes B-I and Bp-I) which are not generally used for prestressing.

In the above expressions, R_s is the design tensile strength of the steel, multiplied by all service factors, m_{sn}, except m_{s4}; and σ_0 is the prestress in the steel with allowance for appropriate losses at m_{ac} below unity.

If use is made of concrete with the service factor $m_{c1} = 0.85$ (Appendix II), the stress σ'_{com} in Eq. (III.4) is taken as $\sigma'_{com} = 500 - m_{ac}\sigma_0$, but not more than $R_{s,com}$, and 400 in Eq. (III.7) is replaced by 500.

If, however, the depth of the compression zone as determined from Eq. (III.3) exceeds $\xi_R h_0$, the member should be analysed using the expressions for the general case (see Sec. II.6). At $x = \xi_R h_0$, the analysis may be carried out according to Eq. (III.4).

III.3. NORMAL-SECTION STRENGTH ANALYSIS OF RECTANGULAR AND T-BEAMS

Rectangular Beams with Tensile Steel Only. A nonprestressed rectangular beam with tensile steel only has the following geometry (Fig. III.12)

$$F_c = bx \text{ and } z_c = h_0 - 0.5x \qquad \text{(III.9)}$$

where h_0 and b are the depth and width of the cross section, respectively.

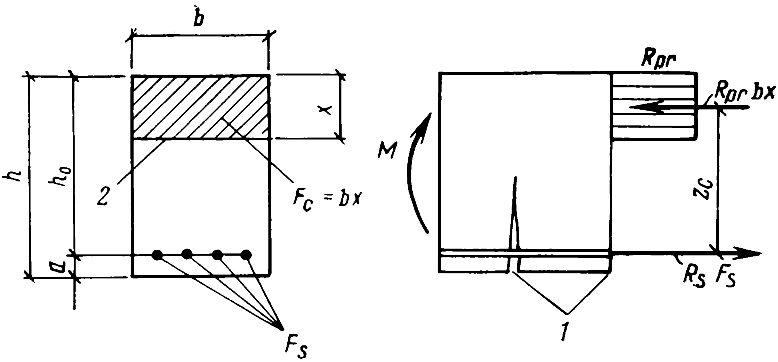


Fig. III.12. Rectangular beam with tensile steel only and the loading system for the normal-section strength analysis
1—normal cracks; 2—neutral axis

The depth of the concrete in compression, x, is determined on the basis of Eq. (III.3) from the following expression

$$bxR_{pr} = R_sF_s \qquad \text{(III.10)}$$

From Eq. (III.4), it follows that the condition for strength has the following form

$$M \leqslant R_{pr}bx\,(h_0 - 0.5x) \qquad \text{(III.11)}$$

This may also be conveniently expressed in terms of the moments about the centroid of the compression zone

$$M \leqslant R_sF_s\,(h_0 - 0.5x) \qquad \text{(III.12)}$$

Expressions (III.10) and (III.11) or (III.12) are used in combination. They are valid at $x < \xi_R h_0$, where ξ_R is determined from Eq. (III.7).

The reinforcement ratio

$$\mu = F_s/bh_0 \qquad \text{(III.13)}$$

and the percentage of reinforcement, $\mu \times 100$, may, in view of Eq. (III.10) and $\xi = x/h_0$, be expressed as

$$\left.\begin{aligned} \mu &= \xi R_{pr}/R_s \\ \mu &= 100\xi R_{pr}/R_s\ (\%) \end{aligned}\right\} \qquad \text{(III.14)}$$

Hence, we may find the maximum allowable steel area at the rectangular section from the ultimate values of ξ_R, subject to condition (III.7) (see Sec. II.6).

If $x > \xi_R h_0$, the bending moment is found by Eq. (III.11) or (III.12) at $x = \xi_R h_0$.

From the analysis of Eqs. (III.11) and (III.12) it follows that a member may be sufficiently strong with various combinations of sectional dimensions and steel area. Under real conditions, the cost of reinforced concrete members is close to optimal at

$\mu = 1$ to 2% and $\xi = 0.3$ to 0.4 for beams

$\mu = 0.3$ to 0.6% and $\xi = 0.1$ to 0.15 for slabs

A section with specified b, x and F_s (the materials and bending moment, M, are assumed to be known) is tested for srength in the following sequence: first, we find the depth of the compression zone, x, from Eq. (III.10), then, we test it to see if it statisfies condition (III.6), and finally use Eq. (III.11) or (III.12).

A section is considered to be chosen correctly if its load-bearing capacity expressed in terms of the moment is not more than 3 to 5% above the specified design moment.

Example III.1. Given: The design bending moment induced by dead, long- and short-time live loads is $M = 74$ kN m; the beam width is $b = 20$ cm and depth is $h = 40$ cm; the longitudinal steel consists of four class A-III deformed bars 16 mm in diameter, the concrete is M-200 heavy concrete (the service factor for the concrete is $m_{c1} = 0.85$).

Check the member for strength at a normal section.

Solution. From Appendices I, V and VI, we find $R_{pr} = 9$ MPa, $R_s = 340$ MPa and $F_s = 8.04$ cm^2.

The effective depth of the member at $a = 3.5$ cm is

$$h_0 = h - a = 40 - 3.5 = 36.5 \text{ cm}$$

From Eq. (III.10), we get the depth of the concrete in compression

$$x = R_s F_s / m_{c1} R_{pr} b = (340 \times 8.04)/(0.85 \times 9 \times 20) = 17.9 \text{ cm}$$

From formulas (III.7*a*) and (III.7), we find

$$\xi_0 = 0.85 - 0.008 R_{pr} = 0.85 - 0.008 \times 0.85 \times 9 = 0.79$$

$$\begin{aligned} \xi_R &= \xi_0/[1 + R_s(1 - \xi_0/1.1)/500] \\ &= 0.79/[1 + 340(1 - 0.79/1.1)/500] = 0.666 \end{aligned}$$

Condition (III.6) is met because

$$\xi = x/h_0 = 17.9/36.5 = 0.490$$

which is below $\xi_R = 0.666$.

According to formula (III.12), the load-bearing capacity of the section is

$$R_sF_s(h_0 - 0.5x) = 340 \times 8.04 \times (36.5-0.5 \times 17.9) = 75\,300 \text{ MPa cm}^3$$

which is 1.8% above the design moment $M = 74$ kN m (1 MPa cm² is equal to 100 N); so, the member meets the condition for strength.

Sections are chosen according to the specified moment with the help of Eqs. (III.10) and (III.11) or (III.12), with their left-hand sides set equal to the right-hand sides.

In practice, rectangular beams with tensile steel only are designed with reference to an auxiliary table (Table III.1). Formulas (III.11) and (III.12) are transformed as follows

$$M = A_0bh_0^2R_{\ r} \tag{III.15}$$

$$F_s = M/\eta h_0R_s \tag{III.16}$$

where

$$A_0 = x\,(1 - 0.5x/h_0)/h_0 = \xi\,(1 - 0.5\xi) \tag{III.17}$$

$$\eta = z_c/h_0 = 1 - 0.5x/h_0 = 1 - 0.5\xi \tag{III.18}$$

From Eq. (III.15), we find the effective depth of the section

$$h_0 = \sqrt{M/A_0bR_{pr}} \tag{III.19}$$

The coefficients A_0 and η derived from Eqs. (III.17) and (III.18) are presented in Table III.1 which significantly cuts down the computation.

The dimensions b and h are chosen as follows: we assume the section width b and the recommended ξ for which we look up the coefficient A_0 in Table III.1; then, using formula (III.19), we determine the effective depth h_0 and the overall depth $h = h_0 + a$, and assign the respective standard dimension. If the values of b and h thus found do not meet the required conditions of design or manufacture, they are refined by repeated calculations.

The necessary steel area, F_s, is found as follows: first, deduce A_0 from Eq. (III.15); then get η and ξ from Table III.1 for the A_0 thus found; determine F_s by formula (III.16); and, finally, check to see if condition (III.6) is satisfied.

Table III.1 may also be used to check a member for strength. In this case, we calculate $\mu = F_s/bh_0$ using the known data about the section, and also ξ from formula (III.14) checking it for compliance with condition (III.6). Then, using ξ, we find A_0 from Table III.1, and calculate the maximum bearable bending moment by formula (III.15).

Example III.2. Given: A rectangular beam subjected to a bending moment of 85 kN m induced by dead, long- and short-time live loads; concrete: M-200 heavy concrete ($m_{c1} = 0.85$); reinforcement: class A-II deformed bars.

To find: b, h and F_s.

Solution. From Appendices I and V, we find $R_{pr} = 9$ MPa and $R_s = 270$ MPa. Then, we assume $b = 20$ cm and $\xi = 0.35$. In Table III.1, $\xi =$

TABLE III.1. Auxiliary Table for Design of Bending Rectangular Beams with Tensile Steel Only

$\xi = x/h_0$	$\eta = z_c/h_0$	A_0	$\xi = x/h_0$	$\eta = z_c/h_0$	A_0
0.01	0.995	0.01	0.37	0.815	0.301
0.02	0.99	0.02	0.38	0.81	0.309
0.03	0.985	0.03	0.39	0.805	0.314
0.04	0.98	0.039	0.4	0.8	0.32
0.05	0.975	0.048	0.41	0.795	0.326
0.06	0.97	0.058	0.42	0.79	0.332
0.07	0.965	0.067	0.43	0.785	0.337
0.08	0.96	0.077	0.44	0.78	0.343
0.09	0.955	0.085	0.45	0.775	0.349
0.1	0.95	0.095	0.46	0.77	0.354
0.11	0.945	0.104	0.47	0.765	0.359
0.12	0.94	0.113	0.48	0.76	0.365
0.13	0.935	0.121	0.49	0.755	0.37
0.14	0.93	0.13	0.5	0.75	0.375
0.15	0.925	0.139	0.51	0.745	0.38
0.16	0.92	0.147	0.52	0.74	0.385
0.17	0.915	0.155	0.53	0.735	0.39
0.18	0.91	0.164	0.54	0.73	0.394
0.19	0.905	0.172	0.55	0.725	0.399
0.2	0.9	0.18	0.56	0.72	0.403
0.21	0.895	0.188	0.57	0.715	0.408
0.22	0.89	0.196	0.58	0.71	0.412
0.23	0.885	0.203	0.59	0.705	0.416
0.24	0.88	0.211	0.6	0.7	0.42
0.25	0.875	0.219	0.61	0.695	0.424
0.26	0.87	0.226	0.62	0.69	0.428
0.27	0.865	0.236	0.63	0.685	0.432
0.28	0.86	0.241	0.64	0.68	0.435
0.29	0.855	0.248	0.65	0.675	0.439
0.3	0.85	0.255	0.66	0.67	0.442
0.31	0.845	0.262	0.67	0.665	0.446
0.32	0.84	0.269	0.68	0.66	0.449
0.33	0.835	0.275	0.69	0.655	0.452
0.34	0.83	0.282	0.7	0.65	0.455
0.35	0.825	0.289			
0.36	0.82	0.295			

$= 0.35$ corresponds to $A_0 = 0.289$. From formula (III.19), we obtain

$$h_0 = \sqrt{M/A_0 b R_{pr} m_{c1}} = \sqrt{8\,500\,000/[0.289 \times 20 \times 9 \times 0.85\,(100)^*]} = 43.8 \text{ cm}$$

The overall depth is $h = h_0 + a = 43.8 + 3.5 = 47.3$ cm; we assign $h = 45$ cm (a multiple of 5 cm) and $h_0 = 45 - 3.5 = 41.5$ cm. From Eq. (III.15), we get

$$A_0 = M/bh_0^2 R_{pr} m_{c1} = 8\,500\,000/[20 \times 41.5^2 \times 9 \times 0.85\,(100^*)] = 0.323$$

In Table III.1, this corresponds to $\eta = 0.797$ and $\xi = 0.405$. From Eq. (III.16), we get

$$F_s = M/\eta h_0 R_s = 8\,500\,000/[0.797 \times 41.5 \times 270\,(100)] = 9.55 \text{ cm}^2$$

Formulas (III.7*a*) and (III.7) yield

$$\xi_0 = 0.85 - 0.008 R_{pr} = 0.85 - 0.008 \times 0.85 \times 9 = 0.79$$

$$\xi_R = \xi_0/[1 + R_s(1 - \xi_0/1.1)/500] = 0.79/[1 + 270\,(1 - 0.79/1.1)/500] = 0.687$$

Condition (III.6) is met, because $\xi = 0.405 < \xi_R = 0.687$.

So, we may take four class A-II bars 18 mm in diameter of area $F_s = 10.18$ cm^2 (see Appendix VI).

Example III.3. Given: A slab subjected to a design moment of 3 800 N m per metre of its cross-sectional length (due to dead, long- and short-time live loads); thickness $h = 8$ cm; reinforcement: standard welded-wire fabric 150/250/6/4 (to USSR State Standard GOST 8478-66) made of class B-I plain wire; concrete: M-150 heavy concrete ($m_{c1} = 1$).

Check the slab for strength.

Solution. From Appendices I, V and VII, $R_{pr} = 7$ MPa, $R_s = 315$ MPa, and the design cross-sectional area of the load-bearing (longitudinal) bars per metre run of the fabric is $F_s = 2.07$ cm^2.

The effective depth of the slab is $h = h_0 - a = 8 - 1.5 = 6.5$ cm.

The percentage of reinforcement is

$$\mu = 100 F_s/bh_0 = (100 \times 2.07)/(100 \times 6.5) = 0.32\%$$

From Eq. (III.14), we get

$$\xi = \mu R_s/100 R_{pr} = (0.32 \times 315)/(100 \times 7) = 0.144$$

which is obviously smaller than ξ_R.

In Table III.1, $\xi = 0.144$ corresponds to $A_0 = 0.134$. From Eq. (III.15), we obtain

$$A_0 b h_0^2 R_{pr} = 0.134 \times 100 \times 6.5^2 \times 7 = 3\,960 \text{ MPa cm}^3$$

that is, the load-bearing capacity of the section is greater than it is required by the design moment $M = 3\,800$ N m (as already noted, 1 MPa cm^2 is equal to 100 N).

Rectangular Members with Tensile and Compressive Steel. In practice, we may come across members congested with tensile and compressive steel (Fig. III.13), although the steel in the compression zone is less effective than that in the tension zone.

If in a member in bending the design provides for longitudinal steel (with $R_{s.com} \leqslant 400$ MPa) in the zone which is in compression

* The numerator is in N cm, and the denominator is in MPA cm^3, so the latter must be multiplied by 100.

under load, the longitudinal bars should be prevented from buckling by transverse reinforcement. The latter should be spaced not more than $20d$ apart in welded bar mats, and not more than $15d$ in tied bar mats (where d is the least diameter of the longitudinal compressive bars), nor more than 500 mm.

Substituting F_c and z_c from Eq. (III.9) into Eq. (III.4) gives the condition for the strength of a bending nonprestressed rectangular member with tensile and compressive steel

$$M \leqslant R_{pr}bx\,(h_0 - 0.5x) + R_{s,com}F_s'\,(h_0 - a') \tag{III.20}$$

and substituting F_c into Eq. (III.3) gives an equation for the depth of the compression zone

$$R_{pr}xb = R_sF_s - R_{s,com}F_s' \tag{III.21}$$

Here, x should not exceed $\xi_R h_0$. If, with tensile steel only, $x > \xi_R h_0$,

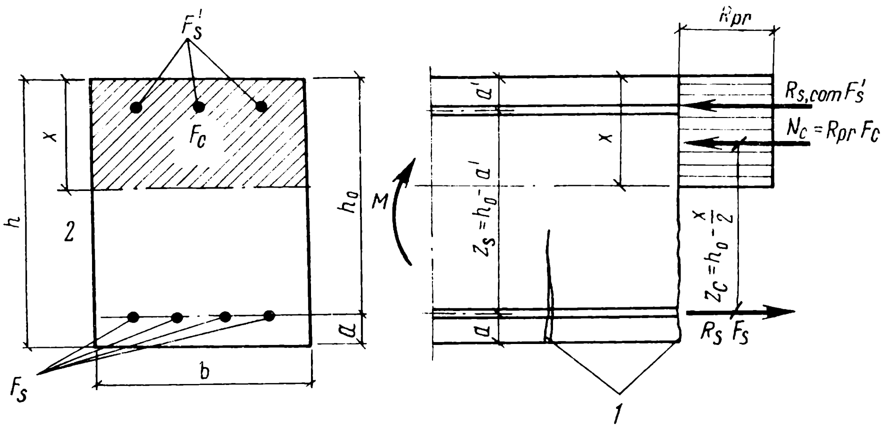


Fig. III.13. Rectangular beam with double reinforcement and the loading system for the normal-section strength analysis

1—normal cracks; *2*—neutral axis

compressive steel is required by the design. In this case, the condition for strength is as follows

$$M \leqslant A_R R_{pr} b h_0^2 + R_{s,com}F_s'\,(h_0 - a') \tag{III.22}$$

where $A_R = A_0$ is taken from Table III.1 for $\xi = \xi_R$ calculated by formula (III.7).

In the design of members with tensile and compressive steel according to the specified moment, concrete brand and steel class, two cases are possible.

Case 1. Given: b and h.

To find: F_s and F_s'.

Solution. Recalling Eq. (III.17), we find from condition (III.20) at $x = \xi_R h_0$

$$F'_s = (M - A_R R_{pr} bh_0^2)/_{s,} R_{com} z_s \qquad \text{(III.23)}$$

and from Eq. (III.21)

$$F_s = F'_s R_{s,com}/R_s + \xi_R R_{pr} bh_0/R_s \qquad \text{(III.24)}$$

Case 2. Given: b, h and F'_s.

To find: F_s.

Solution. Recalling Eq. (III.17), we find from condition (III.20) that

$$A_0 = (M - R_{s,com} F'_s z_s)/R_{pr} bh_0^2 \qquad \text{(III.25)}$$

If $A_0 \leqslant A_R$, we find ξ from Table III.1, and from Eq. (III.21), we get

$$F_s = R_{s,com} F'_s/R_s + \xi R_{pr} bh_0/R_s \qquad \text{(III.26)}$$

If $A_0 > A_R$, the assumed F'_s is insufficient.

In the strength analysis of a section (with all parameters known), we calculate the depth of the compression zone from Eq. (III.21) and then check to see if condition (III.20) is satisfied.

Prestressed members with prestressed steel of area F_{pr} and F'_{pr} are designed in a similar manner, using Eqs. (III.3) and (III.4) with all their terms remaining.

Example III.4. Given: The design bending moment due to dead, long- and short-time live loads is $M = 250$ kN m; $b = 25$ cm and $h = 50$ cm; concrete: M-200 heavy concrete ($m_{c1} = 0.85$); reinforcement: class A-II bars.

To find: F_s and F'_s.

Solution. This is a Case I problem. From Appendices I and V, we find that $R_{pr} = 9$ MPa and $R_s = R_{s,\,com} = 270$ MPa. We assume $a = 4$ cm and $a' = 3$ cm; then

$$h_0 = h - a = 50 - 4 = 46 \text{ cm}$$

$$z_s = h - a - a' = 50 - 4 - 3 = 43 \text{ cm}$$

From formula (III.15), we get

$$A_0 = M/m_{c1} R_{pr} bh_0^2$$
$$= 25\,000\,000/[0.85 \times 9 \times 25 \times 46^2\ (100)] = 0.619$$

For M-200 concrete and class A-II steel, $\xi_R = 0.687$ (see Example III.2). In Table III.1, this ξ_R corresponds to $A_R = 0.451$. Since $A_0 > A_R$, compressive steel is required.

Formula (III.23) gives

$$F'_s = (M - A_R m_{c1} R_p\, bh_0^2)/R_{s,\,com} z_s$$
$$= [25\,000\,000\ (0.01)^* - 0.451 \times 0.85 \times 9 \times 25 \times 46^2]/(270 \times 43)$$
$$= 5.8 \text{ cm}^2$$

* The multiplier (0.01) is included to convert N cm to MPa cm³.

According to Appendix VI, we may take two class A-II bars 20 mm in diameter ($F'_s = 6.28$ cm²). Expression (III.24) yields

$$F_s = F'_s R_{s,\,com}/R_s + \xi_R m_{c1} R_{pr} b h_0 / R_s$$
$$= 5.8 + (0.687 \times 0.85 \times 9 \times 25 \times 46)/270 = 28.2 \text{ cm}^2$$

So, we may take three class A-II bars 28 mm in diameter and two class A-II bars 25 mm in diameter ($F_s = 28.29$ cm²).

Example III.5. Given: reinforcement: class A-III steel ($R_s = R_{s,\,com} = 340$ MPa); compressive reinforcement: two class A-III bars 20 mm in diameter ($F'_s = 6.28$ cm²); all other parameters are the same as in Example III.4.

To find: F_s.

Solution. This is a Case II problem. Formula (III.25) gives

$$A_0 = (M - R_{s,\,com} F'_s z_s)/m_{c1} R_{pr} b h_0^2 = [25\,000\,000\,(0.01)$$
$$- (340 \times 6.28 \times 43)]/(0.85 \times 9 \times 25 \times 46^2) = 0.391$$

which is below $A_R = 0.451$ corresponding to $\xi_R = 0.687$. So, the assumed F'_s may be considered sufficient. In Table III.1, $A_0 = 0.391$ corresponds to $\xi = 0.532$.

From Eq. (III.26) we find

$$F_s = R_{s,\,com} F'_s / R_s + \xi m_{c1} R_{pr} b h_0 / R_s$$
$$= 6.28 + (0.532 \times 0.85 \times 9 \times 25 \times 46)/340 = 20 \text{ cm}^2$$

T-Section Members. These are used rather frequently in separate reinforced concrete T-beams (Fig. III.14*a* and *b*) and as parts of in-situ and precast panel floors (Fig. III.14*c* and *d*). A T-section consists of a flange and a rib.

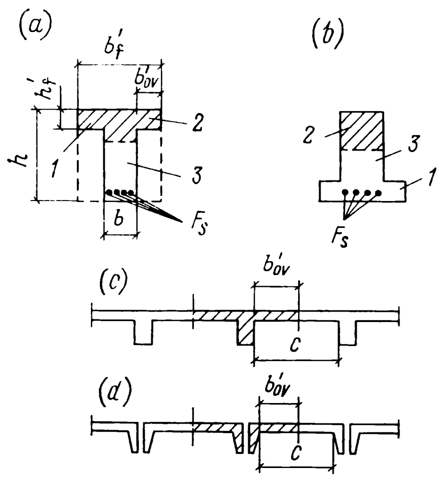


Fig. III.14. T-sections
(*a*) beam with the flange in compression; (*b*) beam with the flange in tension; (*c*) T-section in an in-situ floor; (*d*) T-section in a precast floor: *1*—flange; *2*—compression zone; *3*—rib

T-section members are more advantageous as compared with rectangular members (the dashed line in Fig. III.14*a*), because at the same load-bearing capacity (which does not depend on the cross-sectional area of the tension zone in a reinforced concrete member) they require less concrete due to the reduced size of the tension zone. For the same reason, T-section members with the flange in the compression zone (Fig. III.14*a*) are preferable because the flange located in the tension zone (Fig. III.14*b*) does not contribute to the bearing capacity of the member.

As a rule, T-members have tensile steel only.

With wide flanges, the overhang areas more distant from the rib are stressed less. So, in the design, use is made of the equivalent overhang, b'_{ov} (Fig. III.14c and d). It is taken equal to not more than one half of the clear space between the ribs, c, nor more than 1/6 of the span of the member in question on either side of the rib. In members with flanges having $h'_f < 0.1h$ without transverse ribs or

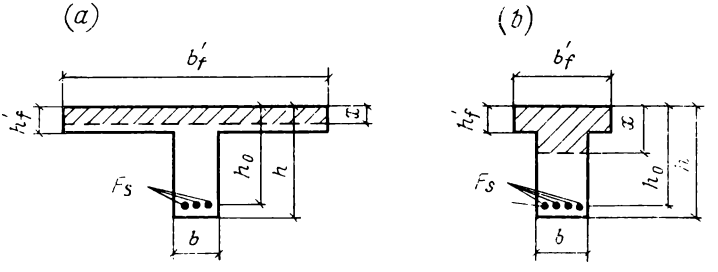


Fig. III.15. Two design cases for T-sections
(a) neutral axis within the flange; (b) neutral axis below the flange

with ribs spaced wider apart than the longitudinal ribs, b'_{ov} should not exceed $6h'_f$. For separate T-beams with overhanging flanges b'_{ov} (Fig. III.14a) should be

at $h'_f \geqslant 0.1h$ . . . . . . max $6h'_f$
at $0.05h \leqslant h'_f < 0.1h$ . . . max $3h'_f$

At $h'_f < 0.05h$, flange overhangs are not taken into account in the design.

In the design of T-section members, the neutral axis may be either within the flange (Fig. III.15a) or below the flange (Fig. III.15b).

The former case where $x \leqslant h'_f$ covers sections with large overhangs. Here, the T-section reduces to a rectangular section with the dimensions b'_f and h_0 (Fig. III.15a), because the concrete area in the tension zone does not affect the load-bearing capacity of the member.

The design formulas for nonprestressed members are

$$R_{pr}b'_f x = R_s F_s \qquad \text{(III.27)}$$

$$M \leqslant R_{pr}b'_f (h_0 - 0.5x) \qquad \text{(III.28)}$$

or

$$M \leqslant A_0 R_{pr} b'_f h_0^2 \qquad \text{(III.29)}$$

where A_0 is the coefficient taken from Table III.1.

The latter case where $x > h'_f$ covers members with narrow overhangs. Here, the compression zone of the section includes the compression zone of the rib and the overhangs.

The depth of the compression zone is found from the following equation

$$R_sF_s = R_{pr}bx + R_{pr}(b'_f - b)\, h'_f \qquad \text{(III.30)}$$

With the moments taken about the axis normal to the plane of flexure and passing through the point of application of the resultant force in the tensile steel, the condition for strength is as follows

$$M \leqslant R_{pr}bx\,(h_0 - 0.5x) + R_{pr}(b'_f - b)\, h'_f\,(h_0 - 0.5h'_f) \qquad \text{(III.31)}$$

For T-section members, x should not exceed $\xi_R b$.

Experience shows that the overall depth of a T-beam may approximately be determined from the following formula

$$h = (7 \text{ to } 9)\sqrt[3]{M} \qquad \text{(III.32)}$$

where h is in centimetres and M is in kN m. The width of the rib is usually taken as

$$b = (0.4 \text{ to } 0.5)h \qquad \text{(III.33)}$$

The flange dimensions b'_f and h'_f are most frequently assigned when proportioning a structure. The steel area F_s necessary to fit the design moment is found according to the position of the neutral axis. If the neutral axis lies within the flange, F_s is determined from Table III.1, assuming that the section is rectangular, with the width b'_f and depth h_0, and has tensile steel only.

Which of the two cases is involved may be found as follows:

(1) if all of the section parameters including F_s are known, then at

$$R_sF_s \leqslant R_{pr}b'_fh'_f \qquad \text{(III.34)}$$

the neutral axis lies within the flange; otherwise, it is located in the rib;

(2) if the dimensions b'_f, h'_f, b and h and the design bending moment are specified, but F_s is unknown, then at

$$M \leqslant R_{pr}b'_fh'_f\,(h_0 - 0.5h'_f) \qquad \text{(III.35)}$$

the neutral axis lies within the flange; otherwise, it crosses the rib.

Where the neutral axis lies below the flange, formulas (III.31) and (III.30) may be transformed, recalling that $x = \xi h_0$ and taking into consideration Eq. (III.17), thus

$$R_sF_s = \xi R_{pr}bh_0 + R_{pr}(b'_f - b)\, b'_f \qquad \text{(III.36)}$$

$$M \leqslant A_0R_{pr}bh_0^2 + R_{pr}(b'_f - b)\, h'_f\,(h_0 - 0.5h'_f) \qquad \text{(III.37)}$$

where the coefficients ξ and A_0 are taken from Table III.1.

These expressions may be used to choose a section. If it is necessary to determine F_s, from Eq. (III.37) we calculate

$$A_0 = [M - R_{pr}(b'_f - b)\, h'_f\,(h_0 - 0.5h'_f)]/R_{pr}bh_0^2 \qquad \text{(III.38)}$$

then we find ξ corresponding to the computed A_0 from Table III.1, and determine the steel area from formula (III.36)

$$F_s = [\xi b h_0 + (b'_f - b) h'_f] R_{pr}/R_s \tag{III.39}$$

If it is necessary to check a section for strength with all data known, which of the two cases applies is better determined from formula (III.35). Then (if the neutral axis lies below the flange), the depth of the compression zone is found from formula (III.30); further calculations are carried out according to Eq. (III.31).

Example III.6. Given: A T-beam subjected to a design bending moment of 450 kN m due to dead, long- and short- time live loads; $h = 70$ cm ($h_0 = 66$ cm); $b = 25$ cm; design flange width $b'_f = 60$ cm; $h'_f = 8$ cm; concrete: M-200 heavy concrete ($m_{c1} = 0.85$); reinforcement: class A-III deformed bars. To find: F_s.

Solution. From Appendices I and V, we find that $R_{pr} = 9$ MPa and $R_s = 340$ MPa.

Which of the two design cases applies is determined using formula (III.35):

$$m_{c1} R_{pr} b'_f h'_f (h_0 - 0.5h'_f) = 0.85 \times 9 \times 60 \times 8\,(66 - 0.5 \times 8)$$
$$= 228\,000 \text{ MPa cm}^3$$

This is below $M = 450$ kN m. So, the neutral axis crosses the rib. Equation (III.38) gives

$$A_0 = [M - m_{c1} R_{pr} (b'_f - b) h'_f (h_0 - 0.5h'_f)]/m_{c1} R_{pr} b h_0^2$$
$$= [4\,500\,000\,(0.01) - 0.85 \times 9\,(60 - 25)\,8\,(66 - 0.5 \times 8)]/(0.85 \times 9 \times 25 \times 66^2) = 0.381$$

In Table III.1, this corresponds to $\xi = 0.512$. According to formula (III.39), the necessary steel area is

$$F_s = [\xi b h + (b'_f - b) h'_f]\, m_{c1} R_{pr}/R_s$$
$$= [0.512 \times 25 \times 66 + (60 - 25)8]\, 0.85 \times 9/340 = 25.3 \text{ cm}$$

We may take (see Appendix VI) two class A-III bars 20 mm in diameter and four class A-III bars 25 mm in diameter, $F_s = 25.91$ cm^2.

The coefficient $\xi = 0.512$ is below the ultimate $\xi_R = 0.666$ (see Example III.1), so, the condition for the use of the design formulas is met.

III.4. NORMAL-SECTION STRENGTH ANALYSIS OF MEMBERS IN BIAXIAL BENDING

In practice, use is most often made of members with cross sections having at least one axis of symmetry. If, in this case, the plane of the external bending moment due to specified loads and support reactions lies at an angle to the plane of symmetry of the section, the member is said to be subjected to biaxial or unsymmetrical bending.

In the general case, members in biaxial bending may be reinforced with bars placed all over the cross section. If a member is intended to resist only biaxial bending, with the plane of flexure taking up a

permanent position, it is advisable to place longitudinal bars only in the tension zone of the section as far from the neutral axis as possible. Let us discuss the most commonly used rectangular members (Fig. III.16).

The design of a structure is aimed at finding the external bending moment and the position of the plane of flexure. As a rule, this plane passes through the geometric axis of the member, adopted for the design loading diagram. Obviously, the resultant of the forces in the tensile bars, N_s, lies in the same plane (Fig. III.16*a* and *b*). As a consequence, the resultant of the compressive forces in the compression zone of the concrete, N_c, should be in the same plane.

The resultant of the tensile forces, N_s, however, may lie at a certain distance, r, from the plane of the external moment (which may be the case when the same member is designed for a different load combination, or required for the sake of unification). In this case, the resultant of the forces in the concrete, N_c, will lie in a plane parallel to that of the external bending moment (Fig. III.16*c*).

The compression zone in the concrete may be triangular or trapezoidal in shape. The reinforcement of this zone would not serve any practical purpose.

The normal-section strength of a member in biaxial bending is determined in plane *III-III* at right angles to the neutral axis (see Fig. III.16*a*) according to the following condition

$$M \cos(\alpha - \varphi) \leqslant R_{pr} F_c z_c \tag{III.40}$$

For the meaning of α, φ and z_c see Fig. III.16*a*.

The concrete area in compression, F_c, is found from the equality of the resultant forces in the tension and compression zones

$$F_s R_s = F_c R_{pr} \tag{III.41}$$

In formulas (III.40) and (III.41), the stress in all reinforcing bars is taken to be the same because they are located approximately at the same distance from the neutral axis.

The position of the neutral axis is determined assuming that the plane of the internal couple of forces either coincides with or is parallel to the plane of the external bending moment. The other requirements placed upon the design of members in bending (such as the condition $\xi = x/h_0 < \xi_R$, and allowance for the increased strength of high-strength reinforcing steel) hold for biaxial bending as well.

Members in biaxial bending may also be designed by comparing the projections of the external moment, M_1, and the moment of the internal couple, M, on plane of symmetry *1*:

$$M_1 = M \cos\varphi \leqslant F_s R_s (h_{01} - x_0) \tag{III.42}$$

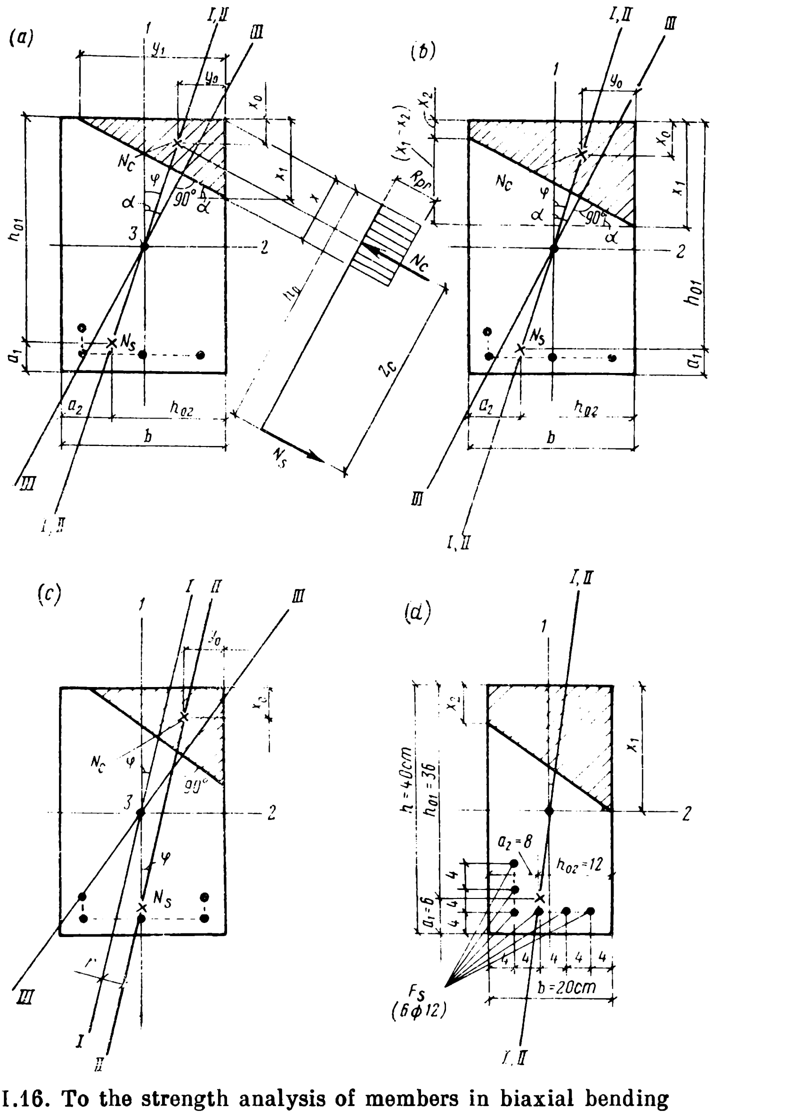


Fig. III.16. To the strength analysis of members in biaxial bending

(*a*) plane of the external moment, *I-I*, and the plane of the internal couple, *II-II*, coincide; triangular compression zone; (*b*) the same; trapezoidal compression zone; (*c*) planes *I-I* and *II-II* are parallel to each other; (*d*) to Example III.7; *1*, *2*—axes of symmetry of a rectangular section; *3*—geometrical axis of the member in the design load diagram; *III-III*—plane at right angles to the neutral axis; N_s—resultant of the forces in tensile bars; N_c—resultant of the forces in the concrete in compression (and also of the forces in the tensile steel if it is required by the design)

Finding the Dimensions of a Triangular Compression Zone. For the section of Fig. III.16*a* we may write

$$M_2/M_1 = F_sR_s\,(h_{02} - y_0)/F_sR_s\,(h_{01} - x_0)$$
$$= (h_{02} - y_0)/(h_{01} - x_0) \qquad \text{(III.43)}$$

where M_2 is the projection of the bending moment in plane *1* on plane *2*.

On setting

$$C_b = M_2/M_1 = \operatorname{tg}\varphi \qquad \text{(III.44)}$$

and taking into account that with a triangular compression zone

$$F_c = 1/2x_1y_1;\ x_0 = 1/3x_1;\ \ y_0 = 1/3y_1 \qquad \text{(III.44a)}$$

we may write on the basis of Eqs. (III.41) and (III.43) that

$$x_1^2 + 3\,(h_{02}/C_b - h_{01})\,x_1 - 2F_sR_s/C_bR_{pr} = 0 \qquad \text{(III.45)}$$

from which we find x_1. Then, we solve Eq. (III.41) for y_1.

If x_1 is negative or $y_1 > b$, the compression zone is not triangular but trapezoidal.

Determining the Dimensions of a Trapezoidal Compression Zone. Referring to Fig. III.16*b*, the dimensions of the compression zone, x_1 and x_2, may be found from the equation

$$F_sR_s = (x_1 + x_2)\,bR_{pr}/2 \qquad \text{(III.46)}$$

and Eq. (III.43) in which

$$\left.\begin{aligned} y_0 &= b\,(x_1 + 2x_2)/3\,(x_1 + x_2) \\ x_0 &= (x_1^2 + x_1x_2 + x_2^2)/3\,(x_1 + x_2) \end{aligned}\right\} \qquad \text{(III.47)}$$

These expressions yield the following equation

$$x_1^2 + (b/C_b - C_1)\,x_1 + C_1\,(3h_{02}/C_b - 2b/C_b - 3h_{01} + C_1) = 0 \qquad \text{(III.48)}$$

where

$$C_1 = 2F_sR_s/bR_{pr} \qquad \text{(III.49)}$$

The above expressions also hold when the plane of the resultants in the tension and compression zones is parallel to the plane of action of the bending moment (Fig. III.16*c*).

Example III.7. Given: A rectangular reinforced concrete member (Fig. III.16*d*) in biaxial bending; moments due to dead, long- and short-time live loads: $M_1 = 50$ kN m and $M_2 = 7$ kN m; concrete: M-200 heavy concrete ($m_{c1} = 0.85$, $R_{pr} = 9$ MPa); reinforcement: six class A-II bars 12 mm in diameter ($F_s = 6.79$ cm², $R_s = 270$ MPa).

Check the member for strength.

Solution. From Eq. (III.44), the ratio of the projections of the applied bending moment is

$$C_b = \operatorname{tg}\varphi = M_2/M_1 = 7/50 = 0.14$$

According to the position of the resultant in the tensile bars (see point N_s and its coordinates in Fig. III.16*d*):

$$\operatorname{tg}\varphi = 2/14 = 0.143$$

So, the planes of action of the applied bending moment and the internal couple practically coincide. By formula (III.49)

$$C_1 = 2F_sR_s/bm_{c1}R_{pr} = (2 \times 6.79 \times 270)/(20 \times 0.85 \times 9) = 24$$

Substituting the given values into Eq. (III.48) yields

$$x_1^2 + (20/0.14 - 24)\,x_1 + 24\,(3 \times 12/0.14 - 2 \times 20/0.14$$
$$- 3 \times 36 + 24) = 0$$

whence $x_1 = 20$ cm. Equation (III.46) yields

$$x_2 = 2F_sR_s/bm_{c1}R_{pr} - x_1 = 24 - 20 = 4 \text{ cm}$$

From Eq. (III.47), we have

$$x_0 = (x_1^2 + x_1x_2 + x_2^2)/3\,(x_1 + x_2)$$
$$= (20^2 + 20 \times 4 + 4^2)/3\,(20 + 4) = 6.9 \text{ cm}$$

According to condition (III.42)

$$M_1 \leqslant F_sR_s\,(h_{01} - x_0)$$

that is, 5 000 000 N cm $<$ 6.79 $\times$ 270 (36 — 6.9(100)) = 5 330 000 N cm.
As is seen, the member has an adequate strength.
Condition (III.6) is satisfied (the calculations are omitted).

III.5. NORMAL-SECTION STRENGTH ANALYSIS OF MEMBERS WITH STIFF REINFORCEMENT

1. Behaviour of Stiff-Reinforcement Members

During the erection of a structure, steel shapes encased in concrete behave as a steel structure until the concrete attains the required strength. So as regards erection loads (such as concrete and formwork self-weight, loads in transit, and wind force), the structure must be designed according to the design standards for metal structures.

In service, after the concrete has attained the necessary strength, the stiff reinforcement acts together with the concrete.

Experience shows that stiff reinforcement (rolled-steel sections and welded reinforcing cages) acts together with the concrete until the structure fails. Here, all of the strength of the reinforcement (with a definite yield point) and the concrete is utilized. The load-bearing capacity of members containing stiff reinforcement does not depend on the initial stresses in the reinforcement appearing during the erection.

The cross-sectional area of stiff reinforcement should be as small as erection loads permit. For service loads, it may be supplemented, if necessary, by flexible load-bearing reinforcement.

The strength analysis of reinforced concrete members containing stiff reinforcement does not differ from that of conventional reinforced concrete members.

2. Rectangular Members

In the analysis of members with stiff reinforcement, two cases may arise.

Case 1. The neutral axis does not cross the stiff reinforcement (Fig. III.17*a*). The stress distribution diagrams (compressive in the

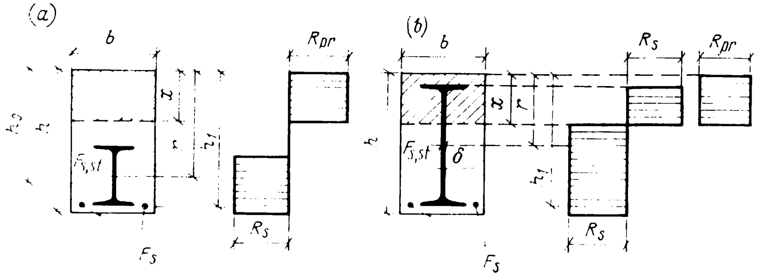


Fig. III.17. Rectangular members with encased steel shapes in bending
(*a*) neutral axis does not cross the steel beam; (*b*) neutral axis crosses the steel beam

concrete, and tensile in the steel) are assumed as rectangular. The strength of a member is considered adequate, if the following condition is met

$$M \leqslant 0.5R_{pr}bx^2 + R_{s,st}F_{s,st}\,(r - x) + R_sF_s\,(h_1 - x) \qquad \text{(III.50)}$$

Here, the moments are about the neutral axis; $F_{s,st}$ is the area of the stiff reinforcement; $R_{s,st}$ is the design strength of the stiff reinforcement; the remaining notation is explained in Fig. III.17*a*.

The position of the neutral axis is determined by the following equation

$$bxR_{pr} = R_{s,st}F_{s,st} + R_sF_s \qquad \text{(III.51)}$$

Case 2. The neutral axis crosses the web of the reinforcing beam (Fig. III.17*b*). The strength of the member should satisfy the following condition

$$M \leqslant 0.5R_{pr}bx^2 + R_{s,st}\,[T + (r - x)^2\delta] + R_sF_s\,(h_1 - x) \qquad \text{(III.52)}$$

where T is the plastic moment of resistance of the encased steel beam; and $(r - x)^2\delta$ is the correction factor applied because the moment in Eq. (III.52) is taken about the neutral axis of the entire section, and T is about the neutral axis of the steel.

For steel I-beams and channels, $T = 1.17W$ (where W is the elastic moment of resistance).

The position of the neutral axis is determined by the following equation

$$bxR_{pr} = 2R_{s,st}(r - x)\delta + R_sF_s \tag{III.53}$$

In either case, the depth of the compression zone should meet the condition

$$x \leqslant \xi_R h_0$$

where h_0 is the distance from the resultant of the tensile forces in the stiff and flexible steel to the compressive face; and ξ_R is the ultimate depth of the compression zone determined as prescribed in Sec. III.2.

3. T-Section Members

If $x \leqslant h'_f$ and the neutral axis does not cross the encased steel beam (Fig. III.18*a*), a section is regarded as a rectangular one with the dimensions b'_f and h; the stiff reinforcement is included in the analysis equally with the flexible steel.

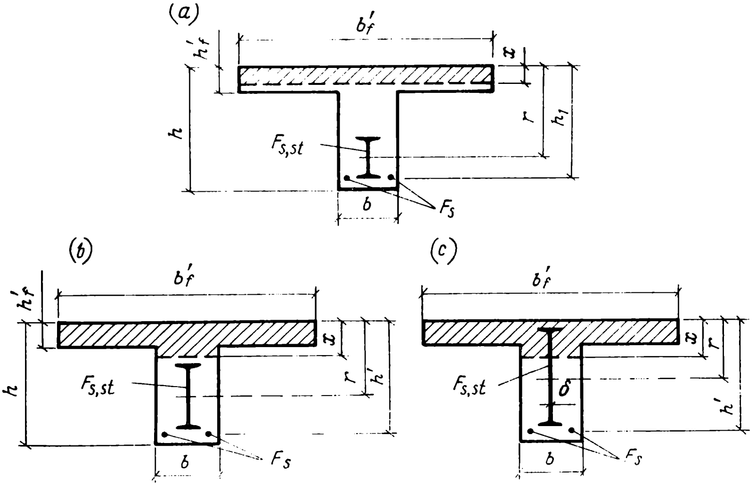


Fig. III.18. T-section members with stiff reinforcement in bending

(*a*) neutral axis lies within the flange and does not cross the steel beam; (*b*) neutral axis lies below the flange and does not cross the steel beam; (c) neutral axis lies below the flange and crosses the steel beam

If $x > h'_f$ but the neutral axis does not cross the encased steel beam (Fig. III.18*b*), the comparison of the moments about the neutral axis gives the following condition for strength

$$M \leqslant [(b'_f - b)\, h'_f\, (x - 0.5h'_f) + 0.5bx^2]\, R_{pr} + R_{s,st}F_{s,st}\,(r - x) + R_sF_s\,(h' - x) \qquad \text{(III.54)}$$

The position of the neutral axis is determined by the following equation

$$[(b'_f - b)\, h'_f + bx]\, R_{pr} = R_{s,st}F_{s,st} + R_sF_s \qquad \text{(III.55)}$$

If $x > h'_f$ and the neutral axis crosses the encased steel beam (Fig. III.18*c*), the above expressions take the form

$$M \leqslant [(b'_f - b)\, h'_f(x - 0.5h'_f) + 0.5bx^2]\, R_{pr} + R_{s,st}\,[T + (r - x)^2\delta] + R_sF_s\,(h' - x) \qquad \text{(III.56)}$$

$$[(b'_f - b)\, h'_f - bx]\, R_{pr} = R_{s,st}2\,(r - x)\,\delta + R_sF_s \qquad \text{(III.57)}$$

In the analysis according to Eqs. (III.54) through (III.57), x should not exceed $\xi_R h_0$.

III.6. INCLINED-SECTION SHEAR STRENGTH ANALYSIS

1. Basic Design Formulas

Bending moments and shearing forces applied simultaneously to members in bending give rise to inclined cracking which forces the members to fail. Consider the internal forces in the steel and the compression zone appearing at a crack. Figure III.19 shows a part of a reinforced concrete member at a support, containing longitudinal, transverse and diagonal reinforcement. This part is separated from the member by a section coinciding with the inclined crack.

The shear strength of a member should satisfy two conditions with regard to M and Q existing in the portion in question (see Fig. III.10, portion *I*). In the design force diagram (Fig. III.19), it is assumed that the member is subjected to the moment and the shearing force computed for design loads, and the stresses in the steel and the concrete are equal to their design strengths. Thus, the conditions for strength are as follows

$$M_D \leqslant R_sF_sz + \Sigma R_sF_bz_b + \Sigma R_sF_{tr}z_{tr} \qquad \text{(III.58)}$$

$$Q_D \leqslant \Sigma R_{s,tr}F_b \sin\alpha + \Sigma R_{s,tr}F_{tr} + Q_c \qquad \text{(III.59)}$$

Here, F_s, F_{tr} and F_b are the areas of the longitudinal reinforcement, transverse reinforcement (stirrups) and diagonal reinforcement (bent

bars), respectively; Q_c is the shearing force resisted by the concrete in the compression zone at the inclined section; α is the angle between the bent bars and the longitudinal axis of the member; z, z_b and z_{tr} are the lever arms for the forces in the longitudinal, transverse and diagonal bars; and M_D and Q_D are the design bending moment and shearing force determined for point D.

Equation (III.58) gives the condition for strength with regard to the bending moment. The shear strength of a member is considered adequate if the bending moment, M_D, due to all the forces applied to the part in question does not exceed the sum of the moments due to the internal design forces in the longitudinal, transverse and diagonal bars, taken about the same fulcrum.

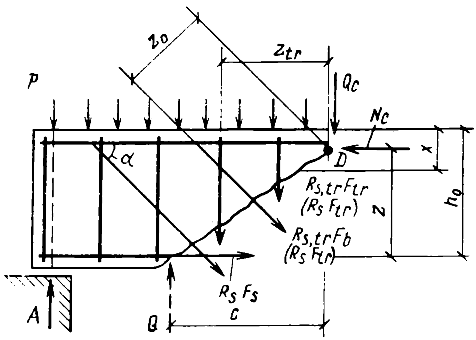


Fig. III. 19. Design loading diagram at an inclined section (quantities in the brackets give the forces in the transverse and diagonal bars, taken for the analysis of the inclined section for resistance to the applied bending moment)

Equation (III.59) gives the condition for the shear strength of a member. The shear strength of a member is considered adequate if the shearing force, Q_D, due to all the forces applied to the part in question does not exceed the sum of the projections of the internal design forces in the transverse and diagonal bars crossed by the inclined section on the axis normal to the longitudinal axis of the member and the shearing force carried by the concrete of the compression zone.

Some transverse and diagonal bars located at the inclined crack closer to the compression zone cannot act at their full strength because the crack is narrow there. As a consequence, the forces in the transverse and diagonal bars in Eq. (III.59) are determined for the design strength $R_{s,tr}$ which is somewhat smaller than R_s (Appendix V). In Eq. (III.58), the design strength is not reduced because the moments of the forces in the transverse and diagonal bars located close to the fulcrum are small and do not significantly affect the final results.

The design shearing force carried by the concrete in the compression zone is given by the following empirical formula

$$Q_c = k_2 R_{ten} b h_0^2 (100)/c = B/c \qquad \text{(III.60)}$$

where Q_c is in N; R_{ten} is in MPa; $B = k_2 R_{ten} b h_0^2$ (100); k_2 is an experimental coefficient, taken as 2 for heavy and cellular concrete,

1.75 for concrete using coarse porous aggregate, and 1.5 for concrete using fine and coarse porous aggregate; h_0 is the effective depth of the member; b is the width of a rectangular member, or rib or web width of a T- or I-beam; and c is the projection of the inclined section on the axis of the member.

The condition described by Eq. (III.58) is usually met without calculations by adequate proportioning which will be discussed later. The condition described by Eq. (III.59) generally requires an appropriate computation.

Practical recommendations require that the maximum shearing force for rectangular, T- and other similar sections should be

$$Q \leqslant 0.35 R_{pr} b h_0 \ (100) \tag{III.61}$$

where Q is in N and R_{pr} is in MPa. If the above condition is met, the web concrete between inclined cracks will stand well the inclined compressive stress.

For prestressed members, which, in the general case, contain prestressed longitudinal, transverse and diagonal reinforcement, Eqs. (III.58) and (III.59) should be extended to include the respective terms as it is done in Eqs. (III.3) and (III.4) for the normal-section strength analysis of a member.

If no inclined cracking is expected to occur in a member (according to the design calculations), no shear strength analysis is carried out. The required strength is given by the following empirical formula

$$Q \leqslant k_1 R_{ten} b h_0 \ (100) \tag{III.62}$$

where k_1 is equal to 0.6 for heavy and cellular concrete, and 0.4 for porous-aggregate concrete (Q is in N, and R_{ten} is in MPa).

2. Transverse Bar Design

Let us examine a member in bending containing transverse reinforcement without bent bars, which is the most commonly used type of reinforcement.

Of all the possible inclined sections originating from point B (Fig. III.20), we shall consider for design purposes the section with the least load-bearing capacity. We shall take into account that

$$\left.\begin{aligned} Q_D &= Q - pc \\ \Sigma R_{s,\,tr} F_{tr} &= q_{tr} c \end{aligned}\right\} \tag{III.63}$$

where Q is the shearing force at the beginning of the inclined section (see Fig. III.20), and q_{tr} is the force carried by the transverse bars per unit length of the member.

Substituting Eqs. (III.60) and (III.63) into Eq. (III.59) gives

$$Q \leqslant (q_{tr} + p)\, c + B/c \qquad \text{(III.64)}$$

The least load-bearing capacity of the inclined member will obviously be given by

$$dQ/dc = (q_{tr} + p) - B/c^2 = 0$$

Hence the projection of the design inclined section is

$$c = \sqrt{B/(q_{tr} + p)} = \sqrt{k_2 R_{ten} b h_0^2/(q_{tr} + p)} \qquad \text{(III.65)}$$

Substituting this value into Eq. (III.64) gives the condition for the shear strength in terms of the least load-bearing capacity of the inclined section

$$Q \leqslant 2\sqrt{B\,(q_{tr} + p)}$$

Substituting the value of B given by formula (III.60) into the above expression gives the shearing force carried by the stirrups and the concrete at the design inclined section

$$Q_{tr,\,c} = 2\sqrt{k_2 R_{ten} b h_0^2\,(q_{tr} + p)} \qquad \text{(III.66)}$$

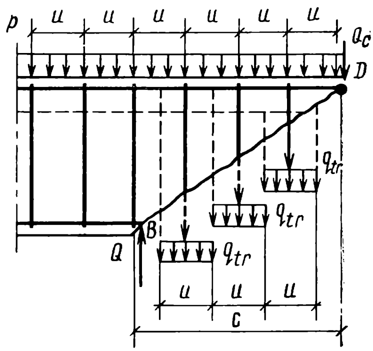


Fig. III.20. Forces in transverse bars, taken for the inclined-section analysis of a beam

In many design cases, p is taken as a uniformly distributed load although it is actually concentrated at several points. Indeed, it may so happen that it does not exist at all over the entire length of the inclined section. So, the load should be taken into account only when it is actually uniformly distributed (such as with ground or water pressure).

Putting $p = 0$ in Eqs. (III.65) and (III.66), we determine that the shearing force which the concrete of the compression zone and the stirrups may carry is

$$Q_{tr,\,c} = 2\sqrt{k_2 b h_0^2 R_{ten} q_{tr}} \qquad \text{(III.67)}$$

The projection of the design inclined section is

$$c_0 = \sqrt{k_2 R_{ten} b h_0^2/q_{tr}} \qquad \text{(III.68)}$$

Referring to the diagram of Fig. III.20, we may write

$$q_{tr} u = R_{s,tr} f_{tr} n \text{ or } q_{tr} u = R_{s,tr} F_{tr},\ F_{tr} = f_{tr} n \qquad \text{(III.69)}$$

where u is the spacing between the transverse bars (stirrups); f_{tr} is the cross-sectional area of one transverse bar (stirrup); and n is the number of transverse bars within the section of the member.

In the design calculations, the diameter of transverse bars and their number within the cross section of the member are usually specified in advance, so $f_{tr}n = F_{tr}$ is regarded as a term known beforehand.

An idea about the required amount of reinforcement may be formed from Eq. (III.67)

$$q_{tr} = Q^2/4k_2bh_0^2R_{ten} \text{ or } q_{tr} \geqslant R_{ten}b/2 \qquad \text{(III.70)}$$

whichever is the greater. This value of q_{tr} should be matched by the force in stirrups per unit length of the member

$$q_{tr} = R_{s,tr}F_{tr}/u \qquad \text{(III.71)}$$

It should be kept in mind that the spacing between the transverse bars ought not to exceed a value for which a likely crack would lie anywhere between two adjacent bars where the member owes its strength solely to the strength of the concrete in the compression zone. In this case, the condition $Q \leqslant Q_c$ holds. The design shearing force resisted by the concrete of the compression zone [see Eq. (III.60)] is multiplied by the coefficient 0.75 which takes care of possible departure of cracks from their design direction due to the nonuniformity of the concrete, and also of possible inaccuracy in the position of the stirrups. Then, the spacing between the transverse bars should not exceed

$$u_{\max} = 0.75k_2R_{ten}bh_0^2/Q \qquad \text{(III.72)}$$

When determining the spacing between the transverse bars, we should also provide for some constructional features (see Sec. III.1).

The spacing and diameter of transverse bars may vary for different parts of a beam. The origin of the design diagonal sections is chosen at the support face and where $Q = Q_{tr,cII}$ (Fig. III.21*a*), and the shearing force is assigned the design values applicable at these points. The member portion with q_{trI} runs from the support to the point where $Q = Q_{tr,cII}$ (Fig. III.21*a*).

If a member is subjected to a concentrated load P applied at a distance $a < c_0$ from the support, an inclined crack may occur between the support and the point where the load is applied (Fig. III.21*b*, section *I-I*). The condition for strength for this section is based on Eq. (III.64) in which $c = a$:

$$Q_1 \leqslant q_{tr}a + B/a$$

or, recalling Eq. (III.60),

$$Q_I \leqslant q_{tr}a + k_2R_{ten}bh_0^2/a \qquad \text{(III.73)}$$

Hence, the force per unit length of the beam, carried by the transverse reinforcement in section *I-I* is

$$q_{tr}^{I\text{-}I} = Q_I/a - k_2R_{ten}bh_0^2/a^2 \qquad \text{(III.74)}$$

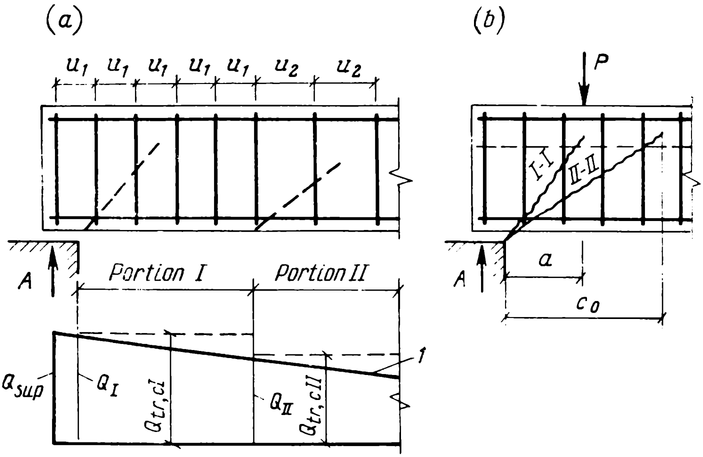


Fig. III.21. Design inclined sections at different beam portions

(a) with different spacings between the transverse bars; (b) between the support and the concentrated load, provided $a < C_0$; 1—distribution of Q

According to Eq. (III.70), the force in the stirrups for section *II-II* past the load P may be determined from the following formula

$$q_{tr}^{II\text{-}II} = (Q_I - P)^2/4k_2bh_0^2R_{ten} \qquad \text{(III.75)}$$

The transverse reinforcement is assigned to suit the largest of $q_{tr}^{I\text{-}I}$ and $q_{tr}^{II\text{-}II}$.

Example III.8. Given: A beam with the dimensions $b = 20$ cm and $h = 40$ cm ($h_0 = 36.5$ cm) subjected to a shearing force of 80 kN; concrete: M-200 heavy concrete ($m_{c1} = 1$); transverse reinforcement: class A-II bars.

To find: F_{tr} and u_{max}.

Solution. From Appendices I and V, we find that $R_{pr} = 9$ MPa, $R_{ten} = 0.75$ MPa and $R_{s,\,tr} = 215$ MPa.

A check to see if Eqs. (III.61) and (III.62) are satisfied gives

$$k_1R_{ten}bh_0\,(100) \leqslant Q \leqslant 0.35R_{pr}bh_0\,(100)$$

$$0.6 \times 0.75 \times 20 \times 36.5(100) = 33\,000 \text{ N} < Q$$

$$= 80\,000 \text{ N} < 0.35 \times 9 \times 20 \times 36.5(100) = 2\,300\,000 \text{ N}$$

As is seen, transverse reinforcement is necessary, and the cross-sectional dimensions of the beam are acceptable.

We take two transverse bars 6 mm in diameter ($f_{tr} = 0.283$ cm^2, $F_{tr} = 0.283 \times 2 = 0.566$ cm^2).

According to Eq. (III.70), the design force per unit beam length, carried by the transverse bars is

$$q_{tr} = 80\,000^2/[4 \times 2 \times 20 \times 36.5^2 \times 0.75\,(100)] = 400 \text{ N/cm}$$

$$< [0.75\,(100) \times 20]/2 = 750 \text{ N/cm}$$

Equation (III.71) gives the spacing between the transverse bars

$$u = R_{s,tr}F_{tr}/q_{tr} = [215\,(100) \times 0.566]/750 = 16.2 \text{ cm}$$

According to the condition (III.72)

$$u_{max} = 0.75k_2R_{ten}bh_0^2/Q = [0.75 \times 2 \times 0.75 \times 20 \times 36.5^2 (100)]/80\,000 = 37 \text{ cm}$$

According to the constructional requirements in Sec. III.1, $u \leqslant h/2 = 40/2 = 20$ cm and $u \leqslant 15$ cm. We take the least of these values, that is, $u = 15$ cm.

3. Bent Bars

Nowadays, bent bars are used rather seldom, and then mostly to strengthen separate parts of a beam subjected to large shearing forces. The locations for bends (places where some of the longitudinal reinforcement is carried from the tension to the compression zone)

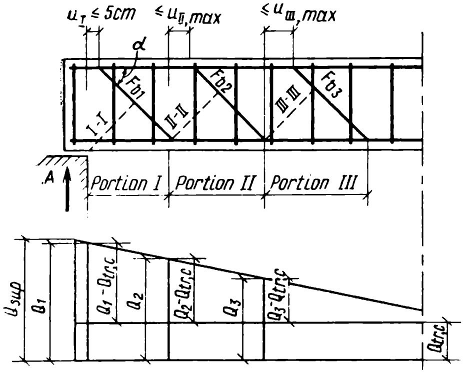


Fig. III.22. To bent bar design

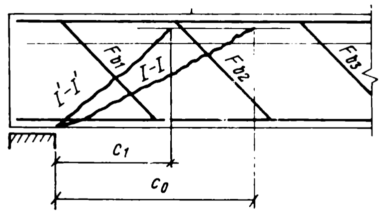


Fig. III.23. To bent bar design

are determined by the normal- and inclined-section analysis (Fig. III.22). Bends are provided where $Q > Q_{tr,c}$ [see Eq. (III.67)].

According to Eq. (III.59), the condition for the shear strength with allowance for bends may be written as

$$Q \leqslant R_{s,tr}F_b \sin\alpha + Q_{tr,c} \tag{III.76}$$

Hence, the cross-sectional area of the bends is

$$F_b = (Q - Q_{tr,c})/R_{s,tr} \sin\alpha \tag{III.77}$$

The trial section for each portion (see Fig. III.22) is that whose origin occurs at the maximum shearing force—this is the section at the face of the support and also where a bend starts. With uniform transverse reinforcement, the design ordinates of shearing forces for the bends are deduced by subtracting $Q_{tr,c}$ from Q. The cross-sectional areas of the bent bars for portions *I*, *II* and *III*, as determined by the analysis of inclined sections *I-I*, *II-II* and *III-III*, are as

follows

$$\left.\begin{aligned} F_{b1} &= (Q_1 - Q_{tr,c})/R_{s,tr}\sin\alpha \\ F_{b2} &= (Q_2 - Q_{tr,c})/R_{s,tr}\sin\alpha \\ F_{b3} &= (Q_3 - Q_{tr,c})/R_{s,tr}\sin\alpha \end{aligned}\right\} \quad \text{(III.78)}$$

The bends are placed as shown in Fig. III.22: the clear space between the inner face of the support and the upper end of the first bent bar, u_I, should not exceed 5 cm, those between the subsequent bent bars, u_{II} and u_{III}, should not be more than

$$u_n \leqslant 0.75k_2R_{ten}bh_0^2/Q_{n,\max} \quad \text{(III.79)}$$

where $Q_{n,\max}$ is the maximum shearing force within a given portion.

If the inclined section corresponding to the least shearing-force bearing capacity (with projection c_0), for example, section *I-I* (Fig. III.23), crosses two bent bars, the condition for strength is

$$Q_1 \leqslant Q_{tr,c} + (F_{b1} + F_{b2})\,R_{s,tr}\sin\alpha \quad \text{(III.80)}$$

whence

$$F_{b1} + F_{b2} = (Q_1 - Q_{tr,c})/R_{s,tr}\sin\alpha \quad \text{(III.81)}$$

In this case, section *I′-I′* with projection c_1 should also be checked for strength

$$F_{b1} = [Q_1 - (q_{tr}c_1 + B/c_1)]/R_{s,tr}\sin\alpha \quad \text{(III.82)}$$

Here, the cross-sectional area assigned to the first bent bar, F_{b1}, is the largest of the two found by Eqs. (III.81) and (III.82), rather than the one found by Eq. (III.78).

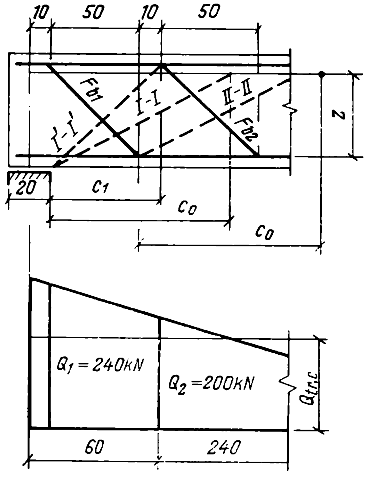


Fig. III.24. To Example III.9

Example III.9. Given: A rectangular beam, $b = 20$ cm and $h = 60$ cm ($h_0 = 56$ cm); concrete: M-200 heavy concrete ($m_{c1} = 1$), $R_{pr} = 9$ MPa, $R_{ten} = 0.75$ MPa; bent bars: class A-II steel, $R_{s,tr} = 215$ MPa; stirrups: class A-I steel, $R_{s,tr} = 170$ MPa; the position of the bent bars and the shearing force diagram are shown in Fig. III.24.

To find: transverse steel and bent bar area.

Solution: A check on Eqs. (III.61) and (III.62) gives

$$0.6 \times 0.75 \times 20 \times 56(100) = 50\,500 \text{ N} < 240\,000 \text{ N} < 0.35 \times 9 \times 20 \times 56\,(100) = 353\,000 \text{ N}$$

As is seen, the beam requires transverse reinforcement, and the dimensions of its cross section are adequate.

We shall use two bars with $d = 8$ mm ($f_{tr} = 0.503$ cm²) and $u = 20$ cm, which is in compliance with condition (III.72)

$$u = 20 \text{ cm} < u_{max} 0.75 k_2 R_{ten} bh_0^2/Q$$
$$= [0.75 \times 2 \times 0.75 \times 20 \times 56^2(100)]/240\,000 = 29.5 \text{ cm}$$

and the constructional requirement of Sec. III.1, $u = 20$ cm $= h/3$.

According to Eq. (III.71), the force per unit beam length, carried by the transverse bars is

$$q_{tr} = R_{s,tr} F_{tr}/u = [170(100) \times 0.503 \times 2]/20 = 855 \text{ N/cm}$$

Formula (III.67) gives the shearing force resisted by the transverse bars and the concrete

$$Q_{tr,c} = 2\sqrt{k_2 bh_0^2 R_{ten} q_{tr}}$$
$$= 2\sqrt{2 \times 20 \times 56^2 \times 0.75 \times 855\,(100)} = 179\,500 \text{ N}$$

From Eq. (III.68), we find the projection of the design diagonal section necessary for the calculation of the bent bars

$$c_0 = \sqrt{k_2 bh_0^2 R_{ten}/q_{tr}}$$
$$= \sqrt{[2 \times 0.75 \times 20 \times 56^2\,(100)]/855} = 105 \text{ cm}$$

So, inclined section *I-I* crosses two bent bars and section *II-II*, one bent bar.

The cross-sectional area required for the second bent bar is determined from Eq. (III.78)

$$F_{b2} = (Q_2 - Q_{tr,c})/R_{s,tr} \sin \alpha$$
$$= (200\,000 - 179\,500)/[215 \times 0.707(100)] = 1.35 \text{ cm}^2$$

For section *I-I*, formula (III.81) yields

$$F_{b1} + F_{b2} = (Q_1 - Q_{tr,c})/R_{s,tr} \sin \alpha$$
$$= (240\,000 - 179\,500)/[215 \times 0.707(100)] = 4 \text{ cm}^2$$

Hence, $F_{b1} = 4 - 1.35 = 2.65$ cm².

According to Eq. (III.82), for section *I-I* at $c_1 = 60$ cm

$$F_{b1} = [240\,000 - (855 \times 60 + 2 \times 0.75 \times 20 \times 56^2\,(100)$$
$$/60)]/[215 \times 0.707\,(100)] = 2.11 \text{ cm}^2$$

Of the two values, we shall take the largest, $F_{b1} = 2.65$ cm².

4. Variable-Depth Members

Variable-depth members are rather common. In particular, it is often advisable to increase the depth of a member in proportion to the increase in the applied bending moment, so that either the bottom or top face of the member is inclined (Fig. III.25).

If the angle between the tensile face of a member and its longitudinal axis is β (Fig. III.25*a*), the condition for the shear strength of the inclined section is described by the following expression

$$Q \leqslant \Sigma R_{s,tr} F_b \sin \alpha + \Sigma R_{s,tr} F_{tr} + Q_c + \sigma_s F_s \sin \beta \qquad \text{(III.83)}$$

where the forces in the tensile steel, as projected on the normal to the compression zone, are

$$\sigma_s F_s \sin\beta = (M - \Sigma F_b R_{s,tr} z_b - \Sigma F_{tr} R_{s,tr} z_{tr}) \sin\beta / z \cos\beta$$
$$= (M - \Sigma F_b R_{s,tr} z_b - \Sigma F_{tr} R_{s,tr} z_{tr}) \operatorname{tg}\beta / z \qquad \text{(III.84)}$$

where M and z are the design moment and the lever arm of the internal couple at a section normal to the compressive face and passing through the end of the diagonal section in question; Q_c is the load-bearing capacity of the concrete in the compression zone as given by

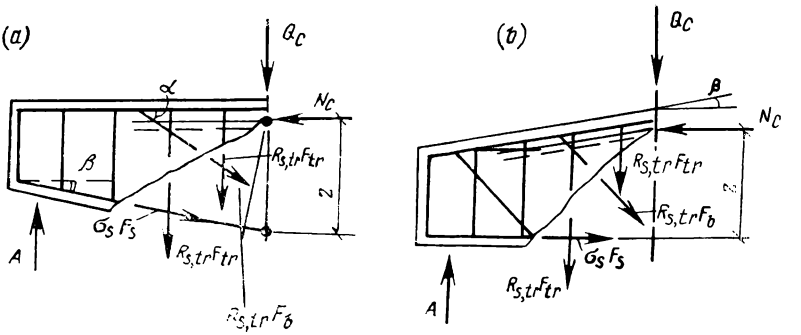


Fig. III.25. Loading systems for the inclined sections in variable depth members

Eq. (III.60) for the effective depth at the beginning of the inclined section (equal to the effective beam depth); here, the projection of the design inclined section is found from Eq. (III.68).

If it is the compressive face of the beam that makes an angle β with the longitudinal axis (Fig. III.25*b*), the strength of the beam is determined according to the same general expressions, but the force carried by the concrete, Q_c, is found from Eq. (III.60) for the effective depth at the compressive end of the inclined section (equal to the effective beam depth), h_0.

III.7. CONSTRUCTIONAL FEATURES ENSURING BENDING MOMENT STRENGTH FOR INCLINED SECTIONS

As regards the applied bending moment, the load-bearing capacity of an inclined section [see the right-hand side of inequality (III.58)] should not be less than that of the normal section passing through the same point D (Fig. III.26). Given certain constructional features (to be discussed later), this requirement can be met, and the analysis of inclined sections for bending moment may be omitted.

If the longitudinal reinforcement at a free support is anchored as advised in Sec. III.1, so that the longitudinal steel acts at full strength in the span, the conditions for the member to develop the necessary strength at any inclined section beginning at the support face are guaranteed.

Without anchorage, the strength of the longitudinal reinforcement at the support as found by Eq. (III.58) is reduced according to Sec. I.3.

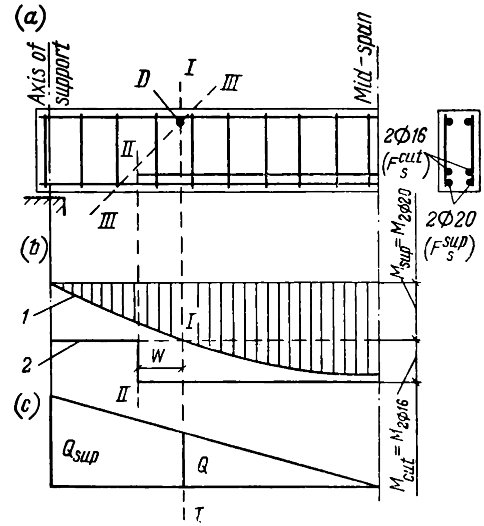


Fig. III.26. Location of the cutoff point in the span

(*a*) reinforcement scheme; (*b*) moment diagram; (*c*) shearing force diagram; *I-I*—theoretical cutoff point for two bars 16 mm in diameter; *II-II*—actual cutoff point of these bars; *1*—distribution of the moments due to the load; *2*—distribution of the moments resisted by the normal sections of the member (what is known as the material diagram)

If the longitudinal bars are insufficiently anchored for them to act at full strength at the section in question, they may be additionally reinforced in the anchorage zone: equipped with bearing plates or embedded parts welded to their ends, or have bent anchoring bars. In this case, the grip length of the bars should be not less than $10d$.

Beams are most often reinforced without bent bars. If all of the tensile longitudinal steel designed on the basis of the normal section carrying the maximum bending moment is extended as far as the support and properly anchored, any inclined section will resist the applied bending moment well, owing to the longitudinal reinforcement alone, without the transverse steel. Under the circumstances, the analysis of the inclined section for bending moment is unnecessary.

To reduce steel consumption, some longitudinal steel (not more than 50% of the design area) may be cut off in the span at a point where it is no longer required by the normal-section strength analysis.

These bars should be extended beyond their theoretical cutoff point determined from the bending moment diagram (section *I-I* in Fig. III.26) for a length w where (to guarantee the required resistance to bending moments) the transverse steel compensates for missing longitudinal bars at inclined sections (section *III-III* in Fig. III.26*a*). As has been shown by experiments and design practice, if the length

w is to meet the above considerations and anchorage conditions of cutoff bars, it must be taken equal to the greater of the two following values

$$\left.\begin{aligned} w &= (Q-Q_b)/2q_{tr,\,w}+5d \\ w &= 20d \end{aligned}\right\} \tag{III.85}$$

Here, Q is the design shearing force at the theoretical cutoff point (section *I-I* in Fig. III.26), corresponding to the load at which this point is determined; Q_b is the shearing force carried by the diagonal bars at the theoretical cutoff point, if the member contains bent bars in addition to transverse steel; $q_{tr,\,w}$ is the force per unit beam length, carried by the transverse bars, found from the bending moment at an inclined section (section *III-III* in Fig. III.26*a*); and d is the diameter of the cutoff bar.

The values of Q_b and $q_{tr,\,w}$ are determined from the following expressions

$$Q_b = R_s F_b \sin\alpha \tag{III.86}$$

$$q_{tr,\,w} = R_s F_{tr}/u \tag{III.87}$$

If there are no bent bars in the zone where the longitudinal steel is cut off, $Q_b = 0$ in the first line of Eqs. (III.85).

Figure III.26 illustrates the positioning of the cutoff bars in the span. The moment diagram plotted for the external design loads shows the ordinates of the moment resisted by the normal section of the reinforced concrete member containing the steel which is extended as far as the support without cutting off (in Fig. III.26, F_s^{sup} is for two bars 20 mm in diameter, $M_{sup} = M_{2\phi 20}$). This ordinate is found as

$$M_{sup} = R_s F_s^{sup} z_c \tag{III.88}$$

The intersections between the ordinate M_{sup} and the design moment diagram define the theoretical cutoff points, *I-I*. The real cutoff points, *II-II*, lie within the distance w of the theoretical cutoff points. The shearing force diagram shows the ordinate Q which is included in formula (III.85) to determine w.

Example III.10. Given: A reinforced concrete beam (Fig. III.27); concrete: M-200 ($m_{c1} = 0.85$, $R_{pr} = 9$ MPa); longitudinal steel: class A-II steel ($R_s = 270$ MPa); transverse steel: class A-I ($R_s = 210$ MPa) bars 8 mm in diameter ($f_{tr} = 0.503$ cm²) with the spacing $u = 20$ cm; span: $l = 6$ m; design load: $q = 43.2$ kN/m; beam dimensions: depth $h = 60$ cm ($h_0 = 55$ cm) and width $b = 20$ cm.

To find: the cutoff point for two longitudinal tensile bars in the second row.

Solution. Let us find the normal-section strength of the beam with the two longitudinal bars cut off. The cross-sectional area of the four remaining bars

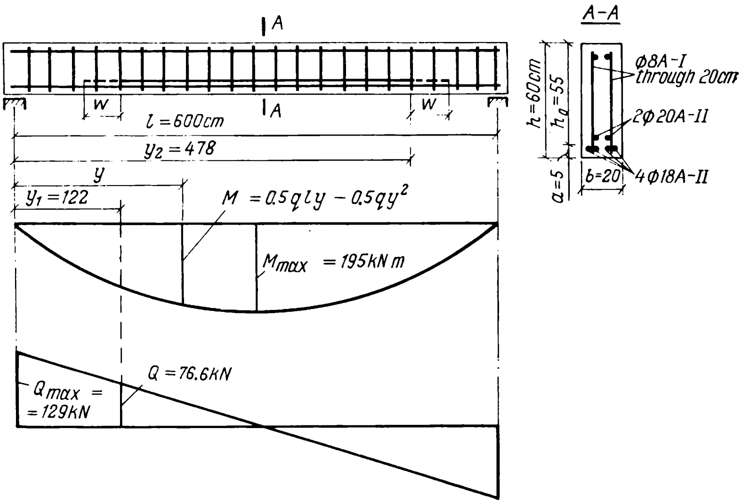


Fig. III.27. To Example III.10

18 mm in diameter is $F_s^{sup} = 10.18$ cm². The depth of the compression zone is

$$x = R_s F_s^{sup} / m_{c1} R_{pr} b = (270 \times 10.18)/(0.85 \times 9 \times 20) = 18 \text{ cm}$$

Formula (III.88) gives the moment carried by the four bars 18 mm in diameter

$$M = R_s F_s^{sup} (h_0 - 0.5x) = 270 \times 10.18 (55 - 0.5 \times 18) (100)$$
$$= 12\,600\,000 \text{ N cm} = 126 \text{ kN m}$$

The theoretical cutoff point is found from the condition

$$M = 0.5qly - 0.5qy^2$$
$$126 = 0.5 \times 43.2 \times 6y - 0.5 \times 43.2y^2$$

whence

$$y_1 = 1.22 \text{ m and } y_2 = 4.78 \text{ m}$$

Equation (III.87) gives

$$q_{tr,\,w} = R_s f_{tr} n/u = [210 \times 0.503 \times 2 (100)]/20 = 1\,050 \text{ N/cm}$$

The shearing force at the theoretical cutoff point is found from the similar triangles in the shearing force diagram

$$Q = Q_{max} (1 - 2y_1/l) = 129 (1 - 2 \times 1.22/6) = 76.6 \text{ kN}$$

At $Q_b = 0$ (no bent bars), formula (III.85) gives

$$w = Q/2q_{tr,w} + 5d = 76\,600/(2 \times 1\,050) + 5 \times 2$$
$$= 36 + 10 = 46 \text{ cm}$$

$$w = 20d = 20 \times 2 = 40 \text{ cm}$$

So, we take $w \geqslant 46$ cm.

III.8. INCLINED-SECTION ANALYSIS OF MEMBERS WITH ENCASED STEEL BEAMS

Experiments have shown that under the action of a shearing force reinforced concrete members with encased steel beams and conventional reinforcement subjected to bending fail in a similar manner. Before failure, inclined cracks reach a considerable width. This is an indication that the transverse bars and the steel beam web are in yield. The compression zone of the concrete (along an inclined

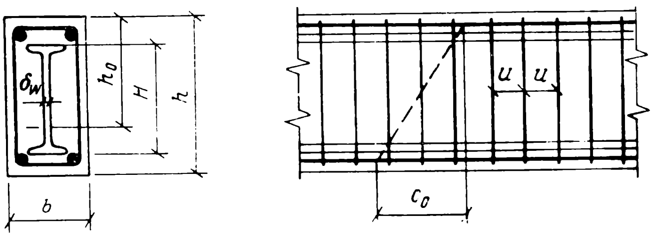


Fig. III.28. To the inclined-section analysis of bent rectangular members containing stiff reinforcement

crack) fails because of compression and shear applied together. The concrete may also fail along an inclined section due to the principal compressive stress.

The inclined-section load-bearing capacity of a bent member with stiff reinforcement (Fig. III.28) is constituted by the resistances of the transverse bars, the steel beam web, and the concrete in compression.

The condition for strength is as follows

$$Q \leqslant (H\delta_w R_{s,\,st}/h_0 + R_{s,\,tr} f_{tr} n/u)\, c_0 + k_2 R_{ten} b h_0^2/c_0$$
$$= q_s c_0 + B/c_0 \qquad \text{(III.89)}$$

where

$$q_s = H\delta_w R_{s,\,st}/h_0 + R_{s,\,tr} f_{tr} n/u;$$
$$B = k_2 R_{ten} b h_0^2 \qquad \text{(III.90)}$$

H is the depth of the encased steel beam; h_0 is the effective depth measured from the compressive face to the resultant of the forces in the tension zone in the flexible and stiff reinforcement; δ_w is the width of the steel beam web.

At the weakest diagonal section, the projection of the inclined crack within h_0 may be determined as it has been done earlier

$$c_0 = \sqrt{B/q_s} \tag{III.91}$$

Substituting c_0 into Eq. (III.89) gives

$$Q \leqslant 2\sqrt{k_2 b h_0^2 R_{ten} q_s} \tag{III.92}$$

When checking diagonal sections for shear strength, use is directly made of Eqs. (III.89) and (III.92). When determining the transverse steel area, we find

$$q_s = Q^2/4k_2 R_{ten} b h_0^2 \tag{III.93}$$

and then choose the necessary reinforcement according to Eq. (III.90).

CHAPTER IV

MEMBERS IN COMPRESSION

IV.1. CONSTRUCTIONAL FEATURES

Members in axial compression are assumed to include intermediate columns in buildings and other structures, top chords of trusses loaded at their joints, uprising diagonal and vertical web members (Fig. IV.1), and so on. In reality, however, the imperfect shape of structural members, departure of their actual dimensions from those specified by design, nonuniform inner structure of concrete, and some other factors prevent members from working in pure axial compression; instead, there exists what may be called eccentrical compression with accidental eccentricities.

As a rule, members intended to resist axial compression are quadrangular or rectangular in cross section. The cross-sectional dimensions of columns are determined by computation; for the standardization of formwork and reinforcing cages, they are assigned as multiples of 50 mm. For better concreting, insitu columns with a side of less than 25 cm are not recommended.

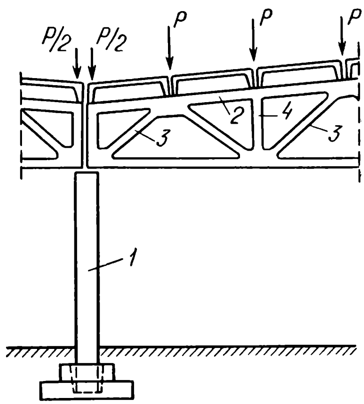


Fig. IV.1. Members in axial compression

1—intermediate columns (equally loaded from either side); *2*—top chord of a truss (with the load applied at the joints); *3*—uprising diagonals; *4*—verticals

Eccentric compression is induced in columns in one-storey industrial buildings subjected to crane load (Fig. IV.2*a*), top chords in trusses without diagonal web members (Fig. IV.2*b*), and walls of underground tanks rectangular in plan, which carry the lateral pres-

sure of ground or liquid and the vertical pressure of the roof (Fig. IV.2c). In this case, members are subjected to both the compressive force, N, and the bending moment, M.

The distance between the point of application of a direct load to a member and its longitudinal axis is termed the eccentricity*, designated e_0. In the general case, the eccentricity at any point of a

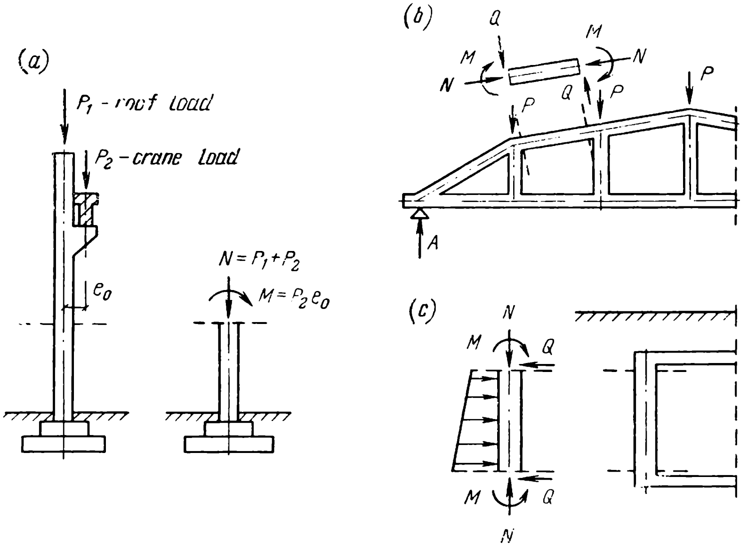


Fig. IV.2. Members in eccentrical compression

(a) column of an industrial building; (b) top chord of a truss without diagonals; (c) wall of a buried tank

member of statically determinate structures is defined as

$$e_0 = M/N + e_a \tag{IV.1}$$

where e_a is the accidental eccentricity (for detailed information see Sec. IV.2). For members of statically indeterminate structures, $e_0 = M/N$, but it should be not less than e_a.

It is advisable to proportion eccentrically loaded members so that their largest dimensions lie in the plane of the applied bending moment. They may be rectangular in cross section or have an I- or T-section.

The concrete brand for members in compression should be at least M-200, and at least M-300 for heavily loaded members. Columns are reinforced by longitudinal bars 12 to 40 mm in diameter (load-bearing reinforcement) mostly of class A-III hot-rolled steel and trans-

* Sometimes, it is called the arm.—Translator's note.

verse bars of class A-I hot-rolled steel or cold-drawn low-carbon wire (Fig. IV.3). The longitudinal and transverse reinforcement is combined into bar mats or reinforcing cages which may be either welded or tied (Fig. IV.4).

The amount of longitudinal steel in the cross section of members in axial compression is expressed in terms of the reinforcement ratio, μ, or the percentage of reinforcement

$$\mu 100 = (F_s/F)\,100\%$$

where F_s is the total longitudinal steel area, and F is the cross-sectional area of the member.

In practice, the percentage of reinforcement for members in axial compression does not exceed 3%.

In compression members with design eccentricities, longitudinal bars are distributed near the short sides of the cross section as shown

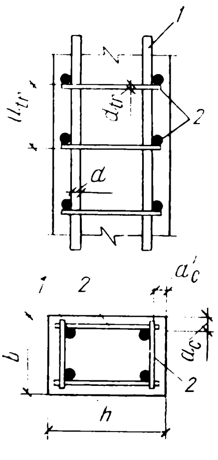


Fig. IV.3. Reinforcement of compression members

1—longitudinal bars; 2—transverse bars; a_c—concrete cover for longitudinal reinforcement; a'_c—concrete cover for transverse reinforcement

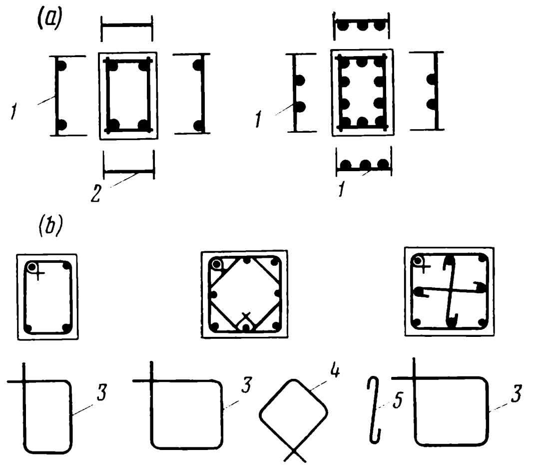


Fig. IV.4. Reinforcement of members in axial compression

(*a*) welded-bar mats; (*b*) tied mats; 1—welded-bar mats; 2—erection bars; 3—stirrups; 4— additional stirrups; 5—hooks

in Fig. IV.5: A with the cross-sectional area F_s is located near the face most distant from the compressive force, and A' with the cross-sectional area F'_s is located at the face closest to the compressive force. The amount of steel in eccentrically compressed members is expressed in terms of the longitudinal reinforcement ratio at one of the short sides. In practice, the steel area for eccentrically loaded

columns ranges between 0.5 and 1.2% of the cross-sectional area of the member.

If the cross-sectional areas of A and A' are not the same, we speak of asymmetrical reinforcement; symmetrical reinforcement is preferable.

According to relevant specifications, the minimum percentage of A and A' for members in compression is as follows:

0.05 for members with $l_0/r_g < 17$
0.1 for members with $17 \leqslant l_0/r_g \leqslant 35$
0.2 for members with $30 \leqslant l_0/r_g \leqslant 83$
0.25 for members with $l_0/r_g > 83$

Here, l_0/r_g is the slenderness ratio of a member, r_g is the radius of gyration of the cross section in the plane of the longitudinal force

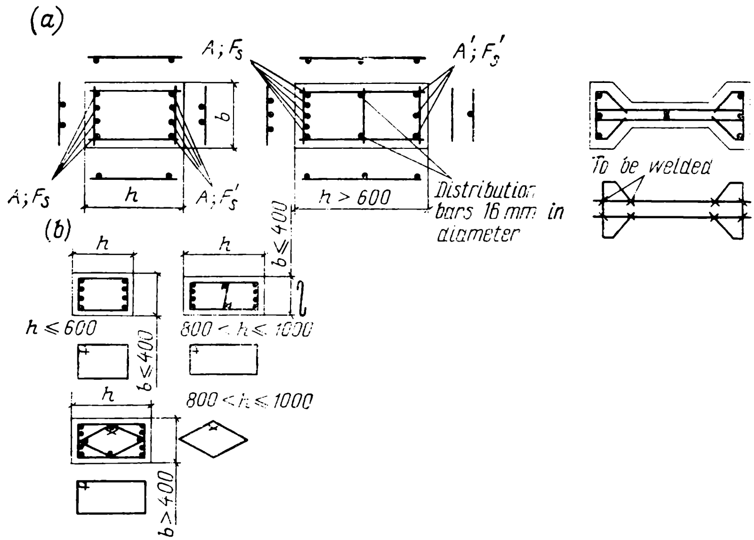


Fig. IV.5. Reinforcement of members in eccentrical compression
(a) welded-bar mats; (b) tied mats

eccentricity, and l_0 is the effective height of the compression member which will be discussed in Volume 2 of this book.

In the cross section of a column, the load-bearing bars should be placed as close to the surface as possible, that is, with a minimum concrete cover, a_c, which, according to specifications, should be not less than the bar diameter or 20 mm (see Fig. IV.3), whichever is the greater.

Columns up to 40 × 40 cm in cross section may be reinforced by four longitudinal bars (see Fig. IV.4), in which case, the clear space

between the load-bearing bars amounts to the maximum allowed value. The minimum clear space between these bars is 50 mm if the bars are vertical during concreting, and 25 mm for bottom and 30 mm for top bars if they are horizontal during concreting. In any case, the clear space should be not less than the maximum bar diameter. If load-bearing bars are spaced wider than 400 mm apart, intermediate bars should be distributed round the perimeter so that the clear space between the longitudinal bars will not exceed 400 mm.

Transverse bars are placed without design calculations, but subject to appropriate specifications. The maximum space between these bars at which the longitudinal bars would not buckle when loaded, u_{tr} (see Fig. IV.3), is $20d$ for welded reinforcing cages and $15d$ (but not more than 500 mm) for tied cages (where d is the least diameter of the longitudinal compression bars). The value of u_{tr} is rounded to a multiple of 50 mm.

The transverse bar diameter, d_{tr}, in welded reinforcing cages should meet the requirements for weldability (see Appendix IX). The stirrup diameter for tied cages should be not less than 5 mm, nor less than $0.25d$, where d is the maximum longitudinal bar diameter.

The concrete cover for transverse steel, a_c', should be at least 15 mm. It is not recommended to use spliced longitudinal bars within the length of a member.

At splices in reinforcing mats, the space between transverse bars within the overhang should not exceed $10d$ (where d is the diameter of the bars being spliced).

If the total percentage of reinforcement exceeds 3%, transverse bars should be placed not wider than $10d$ nor 300 mm apart.

Welded bar mats are combined into reinforcing cages by transverse bars spot-welded to the edge bars of the mats (see Fig. IV.5*a*). If the welded bar mats located near the larger faces of a member contain intermediate bars, the bars (belonging to the opposite mats) are welded together, using hooks disposed along the length of the member with a spacing equal to that between the transverse bars in the mats.

The longitudinal bars of tied reinforcing cages are held in place by stirrups placed at every other stirrup bend, as a minimum. If the face of a member is not wider than 400 mm, and the number of longitudinal bars located at this face does not exceed four, all the longitudinal bars may be embraced by a single stirrup (see Fig. IV.5*b*).

Prestress is used for eccentrically compressed members where the compressive force is applied with a large eccentricity and large bending moments induce tension in some part of the cross section. It is also used in very slender members. In the former case, prestress improves the crack resistance and stiffness of a member in service; in the latter case, it is used to improve the above properties during manufacture, transportation and erection.

It does not pay to use very slender members in axial compression because their load-bearing capacity is considerably reduced by poor stress-strain behaviour. In any case, the slenderness of heavy-concrete and porous-aggregate-concrete members in all directions should be $\lambda = l_0/r \leqslant 200$, and that of columns in buildings should be $\lambda = l_0/r \leqslant 120$.

IV.2. ACCIDENTAL ECCENTRICITY DESIGN

Experiments have shown that the strength of short axially compressed members (often called stub columns) is constituted by the strength of the concrete and longitudinal reinforcement. In the design calculations, the concrete is assumed to work at its ultimate strength, and the steel, at its yield point. This is so because the concrete subjected to high stresses undergoes sufficiently high non-elastic strains.

The load-bearing capacity of long (slender) axially compressed members is markedly affected by the eccentrical compression caused by accidental eccentricities, and also exposure to long-time loads and buckling. Appropriate specifications require that axially loaded members be analysed as eccentrically compressed members, with loads applied at accidental eccentricities. An accidental eccentricity is taken equal to the largest of the two following values: 1/30 of the depth of the member cross section, or 1/600 of the column length (or its part between two fixed points), but not less than 1 cm.

At $l_0 \leqslant 20h$ and $e_0 = e_0^a$, some members rectangular in cross section, namely, those symmetrically reinforced with class A-I, A-II or A-III bars, may be designed in terms of load-bearing capacity (the first group of limit states), assuming them to be axially loaded and subject to the following condition

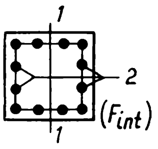
$$N \leqslant m\varphi \left(R_{pr}F + R_{s,\ com}F_s\right) \tag{IV.2}$$

Here, N is the longitudinal compressive force calculated for the design loads; $F = hb$ is the cross-sectional area of a member; h and b are the depth and width of the cross section, respectively; F_s is the cross-sectional area of all longitudinal bars; m is the service factor equal to 0.9 for $h \leqslant 200$ mm and 1 for $h > 200$ mm; φ is the coefficient derived from the empirical relationship taking care of the duration of loading, slenderness and type of member reinforcement

$$\varphi = \varphi_c + 2\left(\varphi_{st} - \varphi_c\right) R_{s,\ com}F_s/R_{pr}F \tag{IV.3}$$

but taken as not more than φ_{st}; here, the values of φ_c and φ_{st} are found from Table IV.1 where N_l is the longitudinal force due to the dead and long-time live loads.

TABLE IV.1. Values of φ_c and φ_{st}



l_0/h / N_l/N	≤ 6	8	10	12	14	16	18	20
				φ_c				
0	0.93	0.92	0.91	0.9	0.89	0.88	0.86	0.84
0.5	0.92	0.91	0.9	0.89	0.86	0.82	0.78	0.72
1	0.92	0.91	0.89	0.86	0.82	0.76	0.69	0.61
				φ_{st}				
A. With the cross-sectional area of the intermediate bars at the faces parallel to the plane in question, F_{int}, less than $1/3F_s$								
0	0.93	0.92	0.91	0.9	0.89	0.88	0.86	0.84
0.5	0.92	0.92	0.91	0.89	0.88	0.86	0.83	0.79
1	0.92	0.91	0.9	0.89	0.87	0.84	0.79	0.74
B. With F_{int} not less than $1/3\ F_s$								
0	0.92	0.92	0.91	0.89	0.87	0.85	0.82	0.79
0.5	0.92	0.91	0.9	0.88	0.85	0.81	0.76	0.71
1	0.92	0.91	0.89	0.86	0.82	0.77	0.70	0.63

1-1 — plane being examined
2 — intermediate bars
F_s — cross-sectional area of all longitudinal bars

With all the data about the dimensions of the cross section, reinforcement, materials and load known, the load-bearing capacity of a member in axial compression is checked for compliance with the condition of Eq. (IV.2), for which purpose we first need to find φ from Eq. (IV.3) and Table IV.1.

If the cross-sectional dimensions of a member have already been assigned and it is only necessary to find the steel area, use should be made of Eq. (IV.2), from which the required steel area is

$$F_s = N/m\varphi R_{s,\ com} - FR_{pr}/R_{s,\ com} \qquad \text{(IV.4)}$$

where φ is determined by successive approximation.

With the load, effective length and materials specified in advance, the cross-sectional dimensions of a member are determined by assu-

ming $\varphi = m = 1$ and $F_s = \mu F = 0.01\ F$. From Eq. (IV.2), we find

$$F = N/m\varphi\,(R_{pr} + \mu R_{s,\ com}) \qquad \text{(IV.5)}$$

and assign suitable standard cross-sectional dimensions. Then, we calculate l_0/h and select F_s in the above manner. If it turns out that the percentage of reinforcement of the designed cross section fails to meet the condition $\mu_{min}\% \leqslant \mu\% \leqslant \mu_{max}\%$ (3%), the dimensions should be altered, and φ and F_s calculated again. The cross-sectional dimensions are considered to be selected properly if $\mu = 1$ to 2%.

Example IV.1. Given: a column in axial compression; effective length $l_0 = 6.4$ m; design longitudinal force $N = 1\ 500$ kN; force induced by the dead and long-time live loads $N_l = 1\ 000$ kN; concrete: M-200 heavy concrete ($m_{c1} = 0.85$); reinforcing steel: class A-II hot-rolled deformed bars.

To find: The cross-sectional dimensions and steel area.

Solution. According to Appendices I and V, $R_{pr} = 9$ MPa and $R_{s,\ com} = 270$ MPa. At $\varphi = m = 1$ and $\mu = 0.01$, Eq. (IV.5) yields

$$F = N/(m_{c1}R_{pr} + \mu R_{s,\ com})$$
$$= 1\ 500\ 000\ (0.01)/(0.85 \times 9 + 0.01 \times 270) = 1\ 450\ \text{cm}^2$$

We adopt a column 40×40 cm in cross section, so $F = 1\ 600\ \text{cm}^2$. The necessary ratios are

$$l_0/b = 640/40 = 16$$
$$N_l/N = 100/150 = 0.667$$

The service factor $m = 1$, because $h = 40$ cm which is greater than 20 cm.

Interpolating from Table IV.1 gives $\varphi_c = 0.8$ and $\varphi_{st} = 0.853$ (assuming that $F_{int} < 1/3F_s$). From Eq. (IV.3), we get

$$\varphi = \varphi_c + 2\,(\varphi_{st} - \varphi_c)\,R_{s,\ com}\mu/m_{c1}R_{pr}$$
$$= 0.78 + [2\,(0.853 - 0.8)\,270 \times 0.01]/(0.85 \times 9)$$
$$= 0.78 + 0.04 = 0.82$$

Since φ should not exceed φ_{st}, we take $\varphi = 0.82$. Equation (IV.4) gives

$$F_s = N/m\varphi R_{s,\ com} - Fm_{c1}R_{pr}/R_{s,\ com}$$
$$= 1\ 500\ 000\ (0.01)/(1 \times 0.82 \times 270) - (1\ 600 \times 0.85 \times 9)/270$$
$$= 22.4\ \text{cm}^2$$

For these cross-sectional dimensions, the percentage of reinforcement is $\mu = (22.4 \times 100)/(40 \times 40) = 1.4\%$

Using $\mu = 0.014$ thus found and Eq. (IV.3), we obtain a refined value of $\varphi = 0.83$. The re-calculation gives $F_s = 21.6\ \text{cm}^2$. The obtained percentage of reinforcement ($\mu = 1.4$) lies within the prescribed limits. So, we adopt a column 40×40 cm in cross section reinforced by four class A-II longitudinal bars 20 mm in diameter and four class A-II longitudinal bars 18 mm in diameter ($F_s = 22.74\ \text{cm}^2$). On distributing the bars over the cross section we can see that the condition $F_{int} < 1/3F_s$ is met; so, no further calculation is required.

The diameter of the transverse bars in welded bar mats, d_{tr}, is taken as 8 mm (see Appendix IX, bar mats with single-side longitudinal bars); the spacing is $u_{tr} \leqslant 35$ cm because $u_{tr} \leqslant 20d = 20 \times 1.8 = 36$ cm.

IV.3. DESIGN OF MEMBERS OF AN ARBITRARY SYMMETRICAL CROSS SECTION, ECCENTRICALLY COMPRESSED IN THE PLANE OF SYMMETRY

When loaded to the limit of load-bearing capacity, members of an arbitrary cross section subjected to eccentrical compression in the plane of symmetry may fail in any one of two ways during Stage III.

Case 1. This applies to eccentrically loaded members to which a longitudinal force is applied at a relatively large eccentricity. The stress-strain state (as well as the failure of a member) is close in nature to that of conservatively reinforced members in bending (Fig. IV.6*a*). The part of the section most distant from the point at which the force is applied is in tension and contains cracks normal to the longitudinal axis of the member. The tensile force in this zone is carried by the reinforcing steel. The part of the section closer to the compressive force is in compression together with its reinforcing steel. The member begins to fail as soon as the stress in the tensile steel reaches the yield point (or proof yield stress). The failure is complete when the stresses in the concrete and steel of the compression zone reach their ultimate strengths, with the stress in the tensile steel remaining constant, if the latter has a definite yield point, or rising if it has no definite yield point. The failure is gradual in nature.

Case 2. This refers to eccentrically loaded members to which a compressive force is applied at a relatively small eccentricity. This case covers two types of the stress-strain state, namely with all of the cross section in compression (Fig. IV.6*b*, diagram *1* shown with the dashed line), or with most of the section closer to the longitudinal force in compression and the opposite part exposed to a relatively weak tension (Fig. IV.6*b*, diagram *2*). The member fails as the stress in it exceeds the ultimate strength of the concrete and steel in the part of the section closer to the applied force. As this takes place, the stress (compressive or tensile) in the distant part of the section remains low, and the material does not act at its full strength.

In the plane of the bending moment, eccentrically compressed members are designed with allowance for the design eccentricity of longitudinal forces and accidental eccentricity, e_a [see Eq. (IV.1)].

In the plane normal to the plane of flexure, members are checked to see if they resist the longitudinal force applied only at accidental eccentricity e_a.

Figure IV.6 shows the loading diagrams for the strength analysis of members having an arbitrary cross section, subjected to compression with an eccentricity in the plane of symmetry in Cases 1 and 2. For members falling in Case 1, the design strength of the concrete in

the compression zone is taken to be constant and equal to R_{pr}, and that in the tensile and compressive steel is taken equal to R_s and $R_{s,\ com}$, respectively. For members falling in Case 2, the actual compressive stress diagram shown by the dashed line in Fig. IV.6*b* is

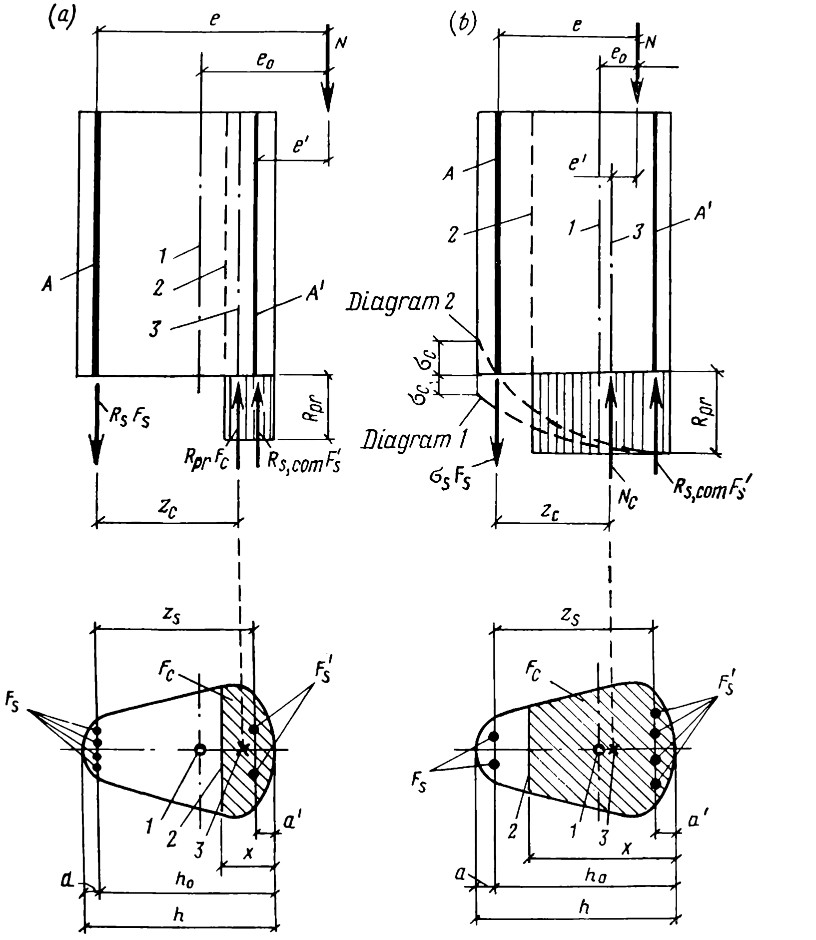


Fig. IV.6. Disposition of loads for members in eccentrical compression

(*a*) at $\xi = x/h_0 \leqslant \xi_R$; (*b*) at $\xi = x/h_0 > \xi_R$; *1*—geometrical axis of the member in the design loading system; *2*—neutral axis; *3*—centroid of the concrete in the compression zone; *A*—steel more distant from the compressive force; *A'*—steel closer to the compressive force

replaced by a rectangular diagram with the ordinate equal to R_{pr}, and the design strength of the compressive steel A' of cross-sectional area F'_s is taken to be $R_{s,\ com}$. The stress, σ_s in the steel A of cross-sectional area F_s is below the design value.

The loading diagram of Fig. IV.6*a* is valid when $\xi = x/h_0 \leqslant \xi_R$, and that of Fig. IV.6*b*, when $\xi = x/h_0 > \xi_R$ [where ξ_R is the ultimate relative depth of the compression zone, defined by Eq. (III.7)].

When $\xi = x/h_0 \leqslant \xi_R$ (see Fig. IV.6*a*), the position of the neutral axis is located by equating the design longitudinal force due to the external design loads, N, with the sum of the projections of all the internal design forces in the steel and the concrete in compression on the longitudinal axis of the member

$$N = R_{pr}F_c + R_{s,\ com}F'_s - R_sF_s \tag{IV.6}$$

The load-bearing capacity of member is said to be adequate if the bending moment, $M = Ne$, due to the external design loads does not exceed the sum of the moments due to the above internal forces about the axis normal to the plane of the bending moment and passing through the point at which the tensile resultant is applied to the steel A

$$Ne \leqslant R_{pr}F_cz_c + R_{s,\ com}F'_sz_s \tag{IV.7}$$

In Eq. (IV.7)

$$z_s = h_0 - a' \tag{IV.8}$$

In Fig. IV.6*a*, e and e' are the distances between the longitudinal force, N, and the centroids of the tensile steel, F_s, and compressive steel, F'_s, respectively.

When $\xi = x/h_0 > \xi_R$ (Fig. IV.6*b*), the strength of compression members is calculated according to Eq. (IV.7), and the depth of the compression zone is determined from the following equation

$$N = R_{pr}F_c + R_{s,\ com}F'_s - \sigma_sF_s \tag{IV.9}$$

Here, the stress in the less stressed reinforcing steel, σ_s, is specified according to the materials used. For example, for members made of M-400 (or lower brand number) concrete and using class A-I, A-II or A-III nonprestressed steel, σ_s is determined from the following relation

$$\sigma_s = [2\,(1 - x/h_0)/(1 - \xi_R) - 1]\,R_s \tag{IV.10}$$

in which the stress in the steel is linearly related to ξ in the range from $\xi = x/h_0 = 1$ to $\xi = \xi_R$, which has been proved experimentally. For members made of M-400 (and higher brand number) concrete and using class A-III prestressed or nonprestressed steel, σ_s in MPa is found from the empirical expression

$$\sigma_s = 400\,(\xi_0/\xi - 1)/(1 - \xi_0h_0/h) + \sigma_0 \tag{IV.11}$$

where ξ_0 is found by Eq. (III.7*a*).

The stress σ_s determined by Eqs. (IV.10) and (IV.11) is taken with the sign obtained: it ranges between R_s and $-R_{s,\ com}$ multiplied by

the appropriate service factors except m_{s4} [see Eq. (III.2)]. In prestressed members, σ_s should be not less than σ_{com}, that is, the stress in the steel equal to the prestress σ_0 reduced by 400 MPa, or 500 MPa, if use is made of the concrete service factor $m_{c1} = 0.85$.

For high-strength reinforcing steel having no definite yield, Eq. (IV.11) holds up to $\sigma_s =$ (approx.) $0.8R_s$. So, if σ_s calculated by Eq. (IV.11) exceeds $0.8R_s$, the increasing value of σ_s is found by linear interpolation between $0.8R_s$ and R_s according to the following expression

$$\sigma_s = [0.8 + 0.2\,(\xi_{el} - \xi)/(\xi_{el} - \xi_R)]\,R_s \qquad \text{(IV.12)}$$

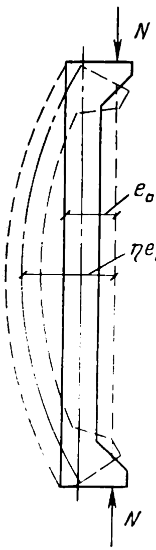


Fig. IV.7. Eccentricity due to buckling

Here, ξ_R is calculated by Eq. (III.7) at $\sigma_s = R_s + 400$ (or 500)$-\sigma_0$ (in MPa), and ξ_{el} at $\sigma_s = 0.8R_s - \sigma_0$. If $\sigma_s > R_s$, R_s is multiplied by the service factor m_{s4} calculated by Eq. (III.2). For members with $l_0/r > 35$, m_{s4} is neglected.

Under the action of the bending moment, a slender member buckles, thereby increasing the initial eccentricity, e_0, of the longitudinal force, N (Fig. IV.7). As this takes place, the bending moment rises, too, and the member fails at a lower longitudinal force than does a short (stub) column.

If the slenderness ratio is $l_0/r > 14$, slender members subjected to eccentrical compression may be designed by the above expressions, but taking into consideration the increased eccentricity which is obtained by multiplying the initial eccentricity, e_0, by the coefficient η (>1). The value of this coefficient is found from the following relation

$$\eta = 1/(1 - N/N_{cr}) \qquad \text{(IV.13)}$$

Here,

$$N_{cr} = (6.4E_c/l_0^2)\,\{I\,[0.11/(0.1 + t/k_{pr}) + 0.1]/k_l + I_{s,\,tr}\} \qquad \text{(IV.14)}$$

Equation (IV.14) takes care of the distinctions of reinforced concrete, namely presence of reinforcing steel in the cross section, inelastic properties of concrete in compression, cracks in the tension zone, and the effect of the long-time loading on the stiffness of a member in the limit state.

In Eq. (IV.14), E_c is the tangent modulus of elasticity of concrete; l_0 is the effective length of the member (the necessary calculations for which will be found in Volume 2 of this book); I is the moment

of inertia of the concrete section; and $I_{s,\ tr}$ is the transformed moment of inertia of the steel about the centroid of the concrete section.

The coefficients k_l (taking care of the effect which the long-time loading has on the buckling of a member in the limit state) and k_{pr} (taking care of the effect that the prestress in the steel has on the stiffness of a member in the limit state; the steel is assumed to be uniformly prestressed) are found from the following empirical relations

$$k_l = 1 + \beta M_l/M \qquad \text{(IV.15)}$$

$$k_{pr} = 1 + 40\sigma_{c,\ pr}e_0/R_{prII}h \qquad \text{(IV.16)}$$

In the general case, M and M_l in Eq. (IV.15) are the moments about the axis parallel to the neutral axis and passing through the centre of the most tensioned or least compressed reinforcing bar (with the section completely in compression); they are due to the combination of all loads and the dead and long-time live loads, respectively; the coefficient β is taken from Table IV.2.

TABLE IV.2. Coefficient β in Eq. (IV. 15)

Concrete	β
Heavy and cellular	1
Manufactured porous-aggregate:	
ceramsite, agloporite, slag pumice, with quartz sand	1
same with porous sand	1.5
Natural porous-aggregate:	
tuff, pumice, lava slag, shell rock limestone (irrespective of sand)	2.5

In Eq. (IV.16), $\sigma_{c,\ pr}$ is the prestress in the concrete with all losses taken into account, and $R_{pr\,II}$ is the prism crushing strength of the concrete adopted for the design in terms of Group II limit states.

In Eq. (IV.14), t is taken equal to

$$t = e_0/h \qquad \text{(IV.17}a\text{)}$$

but not smaller than that calculated from the empirical formula

$$t_{\min} = 0.5 - 0.01l_0/h - 0.01R_{pr} \qquad \text{(IV.17}b\text{)}$$

where R_{pr} is in MPa.

At $l_0/r_g < 14$, η is taken equal to unity.

If N turns out to exceed N_{cr}, the cross-sectional dimensions should be increased.

The transverse reinforcement in members subjected to eccentrical compression is designed to resist shearing forces by the formulas for members in bending given in Sec. III.6.

IV.4. DESIGN OF ECCENTRICALLY LOADED RECTANGULAR MEMBERS

For the rectangular section of Fig. IV.8

$$\left.\begin{aligned} F_c &= bx \\ N_c &= R_{pr}bx \\ z_c &= h_0 - x/2 \end{aligned}\right\} \qquad \text{(IV.18)}$$

Taking into consideration the above expressions, we may rewrite Eq. (IV.7) for the load-bearing capacity as follows

$$\begin{aligned} Ne \leqslant\ & R_{pr}bx\,(h_0 - x/2) \\ & + R_{s,\,com}F'_s\,(h_0 - a') \end{aligned} \qquad \text{(IV.19)}$$

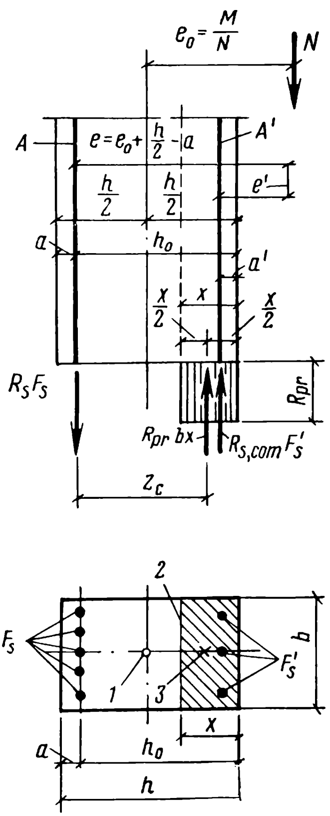


Fig. IV.8. To the design of rectangular members in eccentrical compression

1—geometrical axis of the member; *2*—neutral axis; *3*—centroid of the concrete in the compression zone

The depth of the compression zone is given by any one of the following equations:

(a) at $\xi = x/h_0 \leqslant \xi_R$

$$N = R_{pr}bx + R_{s,\,com}F'_s - R_sF_s \qquad \text{(IV.20)}$$

(b) at $\xi = x/h_0 > \xi_R$

$$N = R_{pr}bx + R_{s,\,com}F'_s - \sigma_sF_s \qquad \text{(IV.21)}$$

where σ_s is determined from Eq. (IV.10) or (IV.11), depending on the materials used.

1. Load-Bearing Capacity Check

In checking a member for load-bearing capacity, when its characteristics are known, the depth of the compression zone is found from Eq. (IV.20) on the assumption that $\xi = x/h_0 \leqslant \xi_R$

$$x = (N - R_{s,\,com}F'_s + R_sF_s)/R_{pr}b \qquad \text{(IV.22)}$$

Then, ξ_R is found from Eq. (III.7). After that, the condition $x \leqslant \xi_R h_0$ is checked; if it is met, the member is checked for load-bearing capacity by Eq. (IV.19) for the found value of x.

If the condition $\xi = x/h_0 \leqslant \xi_R$ is not satisfied, x should be determined from Eq. (IV.21) at $\xi = x/h_0 > \xi_R$.

When $x > \xi_R h_0$ and use is made of concrete with a brand number of not higher than M-400 and class A-I, A-II or A-III nonprestressed steel, σ_s found by Eq. (IV.10) should be substituted into Eq. (IV.21), whence

$$x = (N + C - R_s F_s - R_{s,\,com} F_s')/(R_{pr} b + C/h_0) \tag{IV.23}$$

where

$$C = 2R_s F_s/(1 - \xi_R) \tag{IV.24}$$

The value of x found from the above expression should be substituted into Eq. (IV.19) to check the member for load-bearing capacity.

If $x > \xi_R h$, and use is made of concrete with a brand number of higher than M-400 and class A-IV and higher nonprestressed reinforcing steel, σ_s found by Eq. (IV.11) (with $\sigma_0 = 0$) should be substituted into Eq. (IV.21), whence we have the following quadratic equation

$$\xi^2 - (N - R_{s,\,com} F_s' - DF_s)\,\xi/R_{pr} bh_0 - DF_s \xi_0/R_{pr} bh_0 = 0 \tag{IV.25}$$

where

$$D = 400/(1 - \xi_0 h_0/h) \text{ [MPa]} \tag{IV.26}$$

On finding ξ by Eq. (IV.25), we may calculate $x = \xi h_0$ which is finally used in Eq. (IV.19) to check the member for load-bearing capacity.

2. Steel Area Calculation

When calculating the necessary steel area F_s and F_s' (N, l_0, b and h are assumed to be known), the design formulas should be transformed as follows.

The condition governing steel area is $\xi = x/h_0 \leqslant \xi_R$. It is obvious that the steel A' will be required by design only if the relative depth of the compression zone calculated with only the tensile steel A present exceeds the limiting value of ξ_R. Using this depth of the compression zone and the respective value of A_R from Table III.1, we obtain on the basis of Eqs. (IV.19) and (IV.20)

$$F_s' = (Ne - A_R R_{pr} bh_0^2)/R_{s,\,com} z_s \tag{IV.27}$$

$$F_s = (\xi_R R_{pr} bh_0 - N)/R_s + F_s' R_{s,\,com}/R_s \tag{IV.28}$$

The steel area F'_s should not be less than the minimum value given in Sec. IV.1.

When the steel area F'_s is specified in advance (from constructional or any other considerations, for example, in the presence of moments of both signs), Eq. (IV.19) yields

$$x(h_0 - x/2) = [Ne - R_{s,\,com}F'_s\,(h_0 - a')]/R_{pr}b$$

All terms on the right-hand side of this expression are known. On the other hand, recalling Eq. (III.17), we find that

$$A_0 = \xi\,(1 - \xi/2) \text{ where } \xi = x/h_0 \tag{IV.29}$$

is known, so

$$A_0 = [Ne - R_{s,\,com}F'_s\,(h_0 - a')]/R_{pr}bh_0^2 \tag{IV.30}$$

With A_0 known, we can look up ξ in Table III.1 or calculate it from the following expression

$$\zeta = 1 - \sqrt{1 - 2A_0} \tag{IV.31}$$

Thus, knowing $x = \xi h_0$, we can deduce the necessary steel area from Eq. (IV.20) as

$$F_s = (\xi R_{pr}bh_0 - N)/R_s + F'_sR_{s,\,com}/R_s \tag{IV.32}$$

In practice, members, especially those subjected to bending moments differing in sign but close in value, are often reinforced symmetrically.

With symmetrical reinforcement where $F_s = F'_s$ and $R_{s,\,com} = R_s$, that is, $R_{s,\,com}F'_s = R_sF_s$, we may get from Eq. (IV.20)

$$x = N/R_{pr}b \tag{IV.33}$$

then, using this x, we get from Eq. (IV.19)

$$F_s = F'_s = N\,(e - h_0 + N/2R_{pr}b)/R_{s,\,com}\,(h_0 - a') \tag{IV.34}$$

The condition governing steel area is $\xi = x/h_0 > \xi_R$. We shall discuss the calculation of steel area for concrete with a brand number of not higher than M-400 reinforced with class A-I, A-II or A-III nonprestressed bars (because these are materials most commonly used in practice). Equation (IV.10) suggests that the stresses in the steel A are as follows: $\sigma_s = R_s$ at $\xi = \xi_R$, and $\sigma_s = -R_{s,\,com}$ at $\xi = 1$, that is, they change sign fairly rapidly with increasing ξ.

When ξ has its limiting value, ξ_R, the necessary steel areas are determined from Eqs. (IV.27) and (IV.28).

When the design eccentricities are equal to zero (that is, the member is subjected to axial compression), the relative depth of the compression zone is

$$\xi = h/h_0 = (1 + a/h_0) = \text{(approx.) } 1.1$$

In this case, the stress in the steel A is $\sigma_s = -R_{s,\ com}$. For such a stress, Eqs. (IV.19) and (IV.21) give the necessary steel areas as

$$F'_s = (Ne - 0.5R_{pr}bh^2)/R_{s,\ com}\,(h_0 - a') \tag{IV.35}$$

$$F_s = (N - R_{pr}bh)/R_{s,\ com} - F'_s \tag{IV.36}$$

In contrast to the foregoing expressions, these formulas include the overall depth of the section, h, instead of h_0.

According to Eq. (IV.10), $\sigma_s = 0$ when $\xi = 0.5\,(1 + \xi_R)$, that is, $x = 0.5\,(1 + \xi_R)\,h_0$. Then, Eq. (IV.21) gives

$$F'_s = [N - 0.5R_{pr}bh_0\,(1 + \xi_R)]/R_{s,\ com} \tag{IV.37}$$

Here, F_s may be chosen without calculations, to provide the minimum percentage of reinforcement specified in Sec. IV.1.

The above expressions serve as approximate guides in assigning steel areas F_s and F'_s.

For prestressed members, high-strength reinforcing steel, or members made of concrete with a brand number of M-500 or higher, the necessary steel area should be found by successive approximation (trial calculations).

To sum up, it is recommended to calculate the steel area for rectangular members with asymmetrical reinforcement in the following order:

1. Write down the design values of R_{pr}, R_s, $R_{s,\ com}$, E_s and E_c, and calculate h_0, z_s, $e_0 = M/N$, e_0/h, l_0/h and n.

2. Assume a trial reinforcement ratio

$$\mu = (F_s + F'_s)/bh_0$$

anywhere between 0.005 and 0.035 and determine t, k_l and N_{cr} from Eqs. (IV.17), (IV.14) and (IV.15).

If $N_{cr} < N$, the cross-sectional area of the member should be increased.

3. Find the coefficient η from Eq. (IV.13), and the distances from the force N to the steel A

$$e = e_0\eta + h/2 - a \tag{IV.38}$$

where e_0 is given by Eq. (IV.1).

4. Assuming the expected value of F_s/F'_s, find the depth of the compression zone, x, from Eq. (IV.22) and then $\xi = x/h_0$. Calculate the steel areas F_s and F'_s by Eqs. (IV.27) through (IV.31), but take them as not less than the minimum values specified in Sec. IV.1.

5. Using the steel areas thus found, calculate the reinforcement ratio μ. If it differs from the initial value by not more than 0.005, the problem may be considered solved. Otherwise, the calculations should be carried out again, assuming a new trial reinforcement ratio.

If the design calculations yield $\mu > 0.035$, the cross-sectional dimensions b and h should be changed, or use should be made of other types of concrete and reinforcing steel.

6. Check the member for strength with allowance for the effect of buckling in the plane normal to the plane of flexure, treating it as for a member subjected to compression at accidental eccentricities.

7. If necessary, check the member for load-bearing capacity using Eqs. (IV.22) through (IV.26) and condition (IV.19).

Example IV.2. Given: design longitudinal compressive force $N = 600$ kN; design bending moment due to dead, long- and short-time live loads $M = 162$ kN m; longitudinal compressive force due to dead and long-time live load $N_l = 400$ kN; bending moment due to the same loads $M_l = 100$ kN m; dimensions of the section: $b = 30$ cm and $h = 45$ cm; effective length of the member (between the hinged ends) $l_0 = 6.3$ m; concrete: M-200 heavy concrete ($m_{cl} = 0.85$); reinforcing steel: class A-II deformed bars.

To find: F_s and F'_s.

Solution. From Appendices I, IV and V, we find the necessary design data

$R_{pr} = 9$ MPa; $R_s = R_{s,\,com} = 270$ MPa

$E_s = 2.1 \times 10^5$ MPa; $E_c = 2.15 \times 10^4$ MPa (heat-cured concrete)

Calculate the necessary additional quantities

$h_0 = h - a = 45 - 4 = 41$ cm

$z_s = h - a - a' = 45 - 4 - 4 = 37$ cm

$l_0/h = 630/45 = 14$

$n = E_s/E_c = (2.1 \times 10^5)/(2.15 \times 10^4) = 9.75$

Determine the accidental eccentricity. It should be at least 1/30 of the cross-section side in the direction of bending, that is, $1/30 \times 45 = 1.5$ cm, and also not less than 1/600 of the member length, that is, $1/600 \times 630 = 1.05$ cm, but not less than 1 cm. We shall take the largest value $e_a = 1.5$ cm.

The design eccentricity calculated from Eq. (IV.1) is

$e_0 = M/N + e_a = 16\,200\,000/600\,000 + 1.5 = 28.5$ cm

The relative eccentricity is

$e_0/h = 28.5/45 = 0.633$

Use a trial longitudinal reinforcement ratio (for all the steel in the member) of 0.02.

According to Eqs. (IV.17*a* and *b*) and (IV.15)

$t = e_0/h = 0.633$

but it should be not less than

$$t_{min} = 0.5 - 0.01 l_0/h - 0.01 R_{pr}$$
$$= 0.5 - 0.01 \times 14 - 0.01 \times 9 \times 0.85 = 0.27$$

$k_l = 1 + \beta M_l/M = 1 + (1 \times 100)/162 = 1.617$

where $\beta = 1$ according to Table IV.2.

Equation (IV.14) gives

$$N_{cr} = 6.4 E_c \{I\,[0.11/(0.1 + t/k_{pr}) + 0.1]/k_l + I_{s,\,tr}\}/l_0^2$$
$$= [6.4 \times 2.15 \times 10^4 (100)\,\{22.8 \times 10^4\,[0.11/(0.1 + 0.633)$$
$$+ 0.1]\}/630^2 \times 1.617) + 9 \times 10^4 = 4\,350\,000 \text{ N} = 4\,350 \text{ kN}$$

where

$$I = bh^3/12 = (30 \times 45^3)/12 = 22.8 \times 10^4 \text{ cm}^4$$

$$I_{s,tr} = n\mu F\,(z_s/2)^2 = 9.75 \times 0.02\,(30 \times 45)\,(37/2)^2 = 9 \times 10^4 \text{ cm}^4$$

$k_{pr} = 1$ according to Eq. (IV.16) where $\sigma_{c,\,pr} = 0$ because the member is not prestressed.

Equation (IV.13) gives η, and Eq. (IV.32) gives the distance e:

$$\eta = 1/(1 - N/N_{cr}) = 1/(1 - 600/4\,350) = 1.16$$

$$e = e_0\eta + h/2 - a = 28.5 \times 1.16 + 45/2 - 4 = 51.6 \text{ cm}$$

Assuming that $F_s =$ (approx.) F_s', we get from Eq. (IV.22)

$$x = (N - R_{s,\,com}F_s' + R_sF_s)/m_{c1}R_{pr}b = N/m_{c1}R_{pr}b$$
$$= 600\,000\,(0.01)/(0.85 \times 9 \times 30) = 26.2 \text{ cm}$$

whence

$$\xi = x/h_0 = 26.2/41 = 0.639$$

Because in the conditions of the problem $\xi_R = 0.687$ (see Example III.2), we have $\xi < \xi_R$. According to Table III.1, $\xi_R = 0.687$ corresponds to $A_R = 0.451$.

Equation (IV.27) gives

$$F_s' = (Ne - A_R m_{c1}R_{pr}bh_0^2)/R_{s,\,com}z_s$$
$$= [600\,000 \times 51.6\,(0.01) - 0.451 \times 0.85 \times 9 \times 30 \times 41^2]/(270 \times 37)$$
$$= 13.5 \text{ cm}^2$$

This value exceeds the minimum safe percentage of reinforcement equal to 0.2%, because at

$$r_g = \sqrt{I/F} = \sqrt{bh^3/12bh} = 0.289h = 0.289 \times 45 = 13 \text{ cm}$$

and

$$l_0/r_g = 630/13 = 48 \text{ (see Sec. IV.1)}$$

we get

$$F_{s,\,\min} = 0.002 \times 30 \times 45 = 2.7 \text{ cm}^2$$

So, F_s may be determined from Eq. (IV.28)

$$F_s = (\xi_R m_{c1}R_{pr}bh_0 - N)/R_s + F_x'R_{s,\,com}/R_s$$

$$= [0.687 \times 0.85 \times 9 \times 30 \times 41 - 600\,000\,(0.01)]/270$$

$$+ 13.5 = 15.2 \text{ cm}^2$$

The reinforcement ratio determined as follows

$$\mu = (F_s + F_s')/bh = (15.2 + 13.5)/30 \times 41 = 0.021$$

is close to the trial $\mu = 0.02$.

So, the required steel area is calculated correctly.

IV.5. DESIGN OF T- AND I-SECTION MEMBERS

T- and I-section members subjected to eccentrical compression are often used in arches, columns, and similar applications.

In T-section members, the flange is usually placed near the face which carries the greater compressive force (Fig. IV.9*a*). Here, two design cases are of interest, namely with the neutral axis lying within the flange, and with the neutral axis crossing the rib. In the former case, the section is regarded as rectangular with the width b'_f;

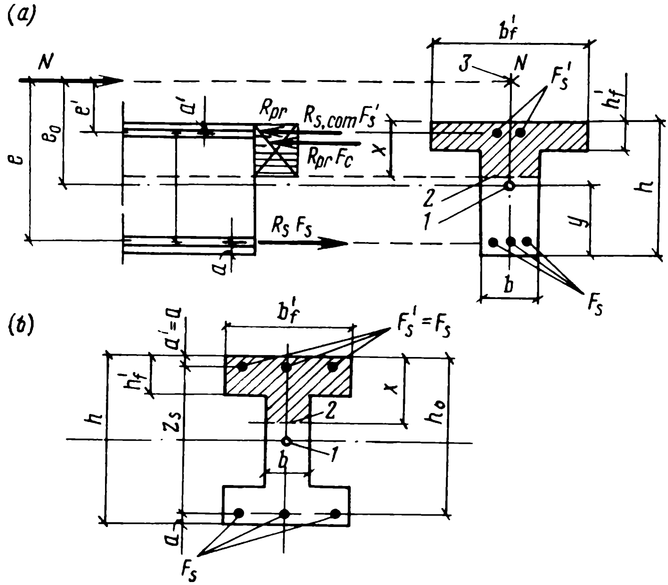


Fig. IV.9. To the design of T- and I-section members

(*a*) T-section member (with the flange in the compression zone); (*b*) symmetrical I-section member; *1*—geometrical axis of the member; *2*—neutral axis; *3*—longitudinal compressive force

in the latter, the compression in the rib is taken into account. The flange width used in the design calculations is assumed to be the same as in bending (see Sec. III.3).

If the flange of a T-section is located near the tensile (or less compressed) face, it is not taken into consideration, and the section is regarded as rectangular, having the design width equal to the width of the rib.

I-section members are treated as T-sections with the flange in the compression zone, since the flange in the tension zone does not contribute to the strength of the member (Fig. IV.9*b*).

Eccentrically loaded T-section members with the flange in the compression zone, as well as any other members with a symmetrical cross section, are treated, according as the following condition

$$\xi = x/h_0 \leqslant \xi_R$$

is satisfied or not.

At first, it is necessary to locate the position of the neutral axis. If the condition

$$N > R_{pr} b'_f h'_f \tag{IV.39}$$

is met, the neutral axis lies below the flange.

If $x > h'_f$, the strength analysis of the section is carried out on] ‹basis of the following condition

$$Ne \leqslant R_{pr} bx (h_0 - 0.5x) + R_{pr} (b'_f - b) h'_f (h_0 - 0.5h'_f) + R_{s,\,com} F'_s (h_0 - a') \tag{IV.40}$$

The depth of the compression zone is found from the following expressions

(a) at $\xi = x/h_0 \leqslant \xi_R$

$$N = R_{pr} bx + R_{pr} (b'_f - b) h'_f + R_{s,\,com} F'_s - R_s F_s \tag{IV.41}$$

(b) at $\xi = x/h_0 > \xi_R$

$$N = R_{pr} bx + R_{pr} (b'_f - b) h'_f + R_{s,\,com} F'_s - \sigma_s F_s \tag{IV.42}$$

where σ_s is determined from Eq. (IV.10) or Eq. (IV.11), depending on the materials used.

The same expressions hold for symmetrically reinforced I-section members (Fig. IV.9*b*).

In the design expressions, the distance e (see Fig. IV.9*a*) is determined as

$$e = \eta e_0 + y - a \tag{IV.43}$$

where y is the distance from the centroid of the section to the tensile face of the rib. We may take $y = \theta h$, with the coefficient θ looked up in Table IV.3.

TABLE IV.3. Coefficients ν and θ for T-Section Members

h'_f/h	Coeffi-cient	b'_f/b ratio				
		2	3	5	10	15
0.1	ν	0.3	0.33	0.32	0.31	0.29
	θ	0.54	0.58	0.63	0.71	0.76
0.2	ν	0.3	0.31	0.29	0.26	0.23
	θ	0.57	0.61	0.68	0.76	0.79
0.3	ν	0.3	0.3	0.27	0.23	0.2
	θ	0.58	0.63	0.69	0.76	0.78
0.4	ν	0.29	0.28	0.25	0.21	0.19
	θ	0.58	0.63	0.68	0.74	0.76
0.5	ν	0.27	0.26	0.23	0.2	0.19
	θ	0.58	0.62	0.67	0.7	0.72

With allowance for the slenderness of a member, the radius of gyration in the plane of flexure may be determined as

$$r_g = \nu h$$

where ν is the coefficient looked up in Table IV.3 for T-section members, and in Table IV.4 for I-section symmetrical members.

TABLE IV.4. Coefficient ν for I-section Symmetrical Members

h'_f/h	b'_f/b ratio				
	2	3	5	10	15
0.1	0.32	0.34	0.37	0.4	0.42
0.15	0.33	0.35	0.36	0.39	0.41
0.2	0.33	0.35	0.36	0.38	0.39
0.25	0.32	0.34	0.35	0.37	0.37
0.3	0.32	0.33	0.34	0.35	0.35
0.35	0.31	0.32	0.33	0.33	0.34

IV.6. DESIGN OF CIRCULAR MEMBERS

Circular sections are encountered in columns, overhead line supports, chimneys, and so on. As a rule, circular-section members are reinforced with longitudinal bars uniformly distributed around the circumference (Fig. IV.10).

The standard formulas used in the analysis of such members are derived on the basis of the general expressions for members with an arbitrary cross section, extended to include empirical coefficients.

At $r_1/r_2 \geqslant 0.5$, the strength of circular compression members (see Fig. IV.10) is designed on the basis of the following condition

$$Ne_0 \leqslant (R_{pr}Fr_{av} + R_{s,com}F_{s,l}r_s)\sin(\pi\alpha_{cir})/\pi + R_sF_{s,l}k_sz_s \qquad \text{(IV.44)}$$

The relative area of the concrete in compression is found as

$$\alpha_{cir} = [N + (\sigma_0 + A_sR_s)F_{s,l}]/[R_{pr}F + (R_{s,com} + B_sR_s)F_{s,l}] \qquad \text{(IV.45)}$$

if $\alpha_{cir} > 0.15$.

In Eqs. (IV.44) and (IV.45),

$$r_{av} = (r_1 + r_2)/2 \qquad \text{(IV.46)}$$

where r_s is the radius of the circle passing through the centroids of

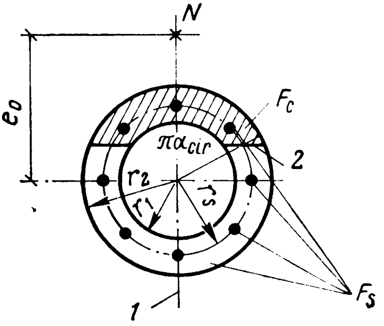


Fig. IV.10. To the design of circular members
1—plane of flexure; *2*—neutral axis

the reinforcing bars; $F_{s,l}$ is the cross-sectional area of all the longitudinal bars; σ_0 is the prestress in the steel as determined with the tension accuracy factor $m_{ac} > 1$; z_s is the distance from the tensile resultant in the steel to the centroid of the section

$$z_s = (0.2 + 1.3\alpha_{cir})r_s \qquad \text{(IV.47)}$$

but not more than r_s; k_s is the coefficient determined from the following relation

$$k_s = A_s - B_s\alpha_{cir} \qquad \text{(IV.48)}$$

where

$$A_s = m_{s,cir} - \sigma_0/R_s \qquad \text{(IV.49)}$$

(for class A-I, A-II and A-III steel, $m_{s,\ cir} = 1$, for steel of other classes, $m_{s,\ cir} = 1.1$);

$$B_s = A_s\,(1.5 + 60R_s \times 10^{-5}) \qquad \text{(IV.50)}$$

where R_s is in MPa.

If α_{cir} calculated from Eq. (IV.45) is below 0.15, then α_{cir} substituted in Eq. (IV.44) should be found as follows

$$\alpha_{cir} = [N + (\sigma_0 + k_sR_s)\,F_{s,\,l}]]/(R_{pr}F + R_{s,\ com}F_{s,\ l}) \qquad \text{(IV.51)}$$

Here, z_s and k_s are determined from Eqs. (IV.47) and (IV.48) at $\alpha_{cir} = 0.15$.

If Eq. (IV.48) yields $k_s \leqslant 0$, then $k_s = 0$ and α_{cir} calculated by Eq. (IV.45) at $A_s = B_s = 0$ must be substituted in Eq. (IV.44).

IV.7. MEMBERS IN BIAXIAL ECCENTRICAL COMPRESSION

Let us examine rectangular members subjected to biaxial eccentrical compression. Here, the compressive force, N, is applied at two eccentricities with respect to the axes of symmetry, e_1^0 and e_2^0 (Fig. IV.11). Members in biaxial eccentrical compression may be reinforced with longitudinal bars distributed around the entire perimeter of the section or concentrated at the most stressed parts of the section. In the latter case, less reinforcing steel is used, therefore, it is preferable. We shall deal only with such members.

The external compressive force, N, and the resultant internal forces, N_s (in the steel placed in the zone more distant from N) and N_{com} (in the concrete and steel in the compression zone which is closer to N), should be in a common plane which may (Fig. IV.11*a*) or may not (Fig. IV.11*b*) pass through the geometrical axis of the member.

The compression zone may be either triangular or trapezoidal in shape. As often as not, it is useful to reinforce the compression zone with the steel whose area is allowed for in the strength analysis of the member.

The disposition of internal loads at a trial normal section is shown in Fig. IV.11*a*. It is similar to that for a member subjected to eccentrical compression in the plane of symmetry (see Fig. IV.6*a*).

The member is analysed for strength in plane *I-I* normal to the neutral axis (the dimensions of the compression zone being x and h_0) subject to the following condition

$$Ne \leqslant (F_cR_{pr} + F_s'R_{s,\ com})\,z_{N\ com} \qquad \text{(IV.52)}$$

where $z_{N\ com}$ is the distance from the resultant of the forces in the concrete and the steel in the compression zone to the resultant of

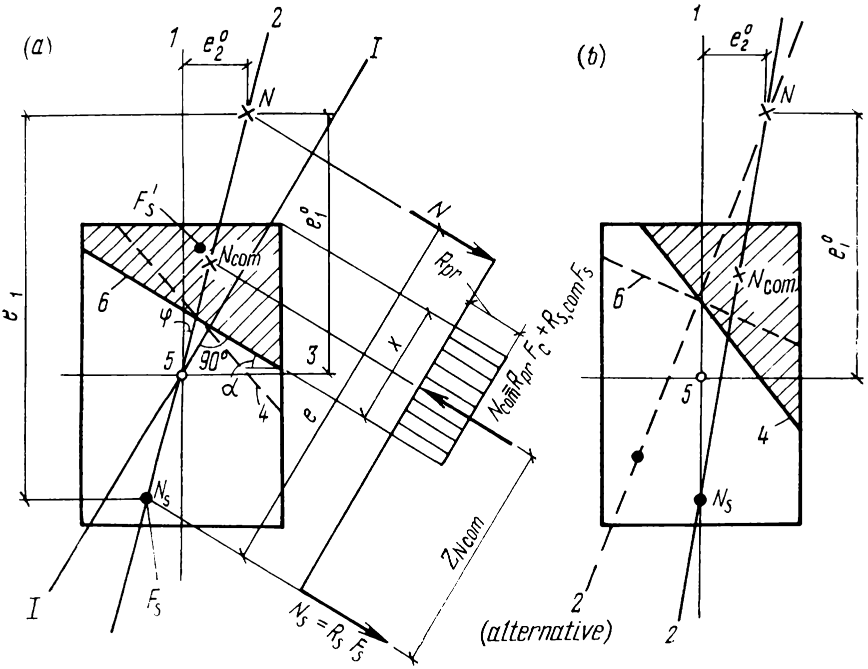


Fig. IV.11. To the strength analysis of members in biaxial compression

(*a*) plane of forces N, N_s and N_{com} coincides with the geometrical axis of the member; (*b*) plane of forces N, N_s and N_{com} does not coincide with the geometrical axis of the member; *1*—axis of symmetry of the rectangular member; *2*—plane of forces, N, N_s and N_{com}; *3*—axis of symmetry of the rectangular member; *4*—triangular compression zone; *5*—geometrical axis of the member in the design loading system; *6*—trapezoidal compression zone; N—external compressive force; N_s—resultant in the steel more distant from N; N_{com}—resultaat in the concrete (and steel, if necessary) of the compression zone; *I-I*—plane normal to the neutral axis

the forces in the steel located in the zone more distant from the force N.

The area of concrete in compression, F_c, is found by equating the sum of projections of the internal forces at the section with the external compressive force to zero. Thus,

— at $\xi = x/h_0 \leqslant \xi_R$

$$N + F_sR_s - F_s'R_{s,\,com} - F_cR_{pr} = 0 \tag{IV.53}$$

— at $\xi = x/h_0 > \xi_R$

$$N + F_s\sigma_s - F_s'R_{s,\,com} - F_cR_{pr} = 0 \tag{IV.54}$$

where σ_s is determined by Eq. (IV.10) or (IV.11) depending on the materials used.

In Eqs. (IV.52) through (IV.54), the stress in all bars of the compressive or tensile steel is assumed to be the same because in each group the bars are within approximately the same distance of the neutral axis. Members may also be analysed in the plane of symmetry, *1*; in this case (Fig. IV.12), the condition for strength is

$$Ne_1 \leqslant (F_cR_{pr} + F'_sR_{s,\,com})(h_{01} - x_0) \tag{IV.55}$$

Determining the Dimensions of a Triangular Compression Zone (Fig. IV.12*a*). Let us assume that the steel areas, F_s and F'_s, and steel location in the section are fixed by the conditions of the problem. According to Fig. IV.12*a*, we may write (with the equals sign remaining) similarly to Eqs. (IV.54) and (IV.55)

$$N + F_sR_s - F'_sR_{s,\,com} - x_1y_1R_{pr}/2 = 0 \tag{IV.56}$$

$$\left.\begin{aligned} Ne_1 &= R_{pr}x_1y_1(h_{01} - x_0)/2 + F'_sR_{s,\,com}(h_{01} - x_s) \\ Ne_2 &= R_{pr}x_1y_1(h_{02} - y_0)/2 + F'_sR_{s,\,com}(h_{02} - y_s) \end{aligned}\right\} \tag{IV.57}$$

On putting

$$\begin{aligned} C_{com} &= [Ne_1 - F'_sR_{s,\,com}(h_{01} - x_s)]/[Ne_2 \\ &\quad - F'_sR_{s,\,com}(h_{02} - y_s)] \end{aligned} \tag{IV.58}$$

and recalling Eqs. (IV.57), we may write

$$\begin{aligned} C_{com} &= [R_{pr}x_1y_1(h_{01} - x_0)/2]/[R_{pr}x_1y_1(h_{02} - y_0)/2] \\ &= (h_{01} - x_0)/(h_{02} - y_0) \end{aligned} \tag{IV.59}$$

The coordinates of the resultant in a triangular compression zone with dimensions x_1 and y_1 and steel area F'_s are as follows (see Fig. IV.12*a*)

$$\left.\begin{aligned} x_0 &= (x_1^2y_1 + 6F'_sx_sR_{s,\,com}/R_{pr})/3(x_1y_1 + 2F'_sR_{s,\,com}/R_{pr}) \\ y_0 &= (x_1y_1^2 + 6F'_sy_sR_{s,\,com}/R_{pr})/3(x_1y_1 + 2F'_sR_{s,\,com}/R_{pr}) \end{aligned}\right\} \tag{IV.60}$$

Using the above expressions in Eq. (IV.59) and setting

$$x_1y_1/2 = (N + F_sR_s - F'_sR_{s,\,com})/R_{pr} = C_2 \tag{IV.61}$$

we obtain an equation for x_1:

$$\left.\begin{aligned} &x_1^2 + 3[(1 + F'_sR_{s,\,com}/C_2R_{pr})(-h_{01} + C_{com}h_{02}) \\ &\quad - F'_sR_{s,\,com}(C_{com}y_s - x_s)/C_2R_{pr}]\,x_1 - 2C_2C_{com} = 0 \end{aligned}\right\} \tag{IV.62}$$

On solving the above equation for x_1, we find from Eq. (IV.61)

$$y_1 = 2C_2/x_1$$

Then, we solve Eq. (IV.60) for x_0 and use the result in Eq. (IV.55).

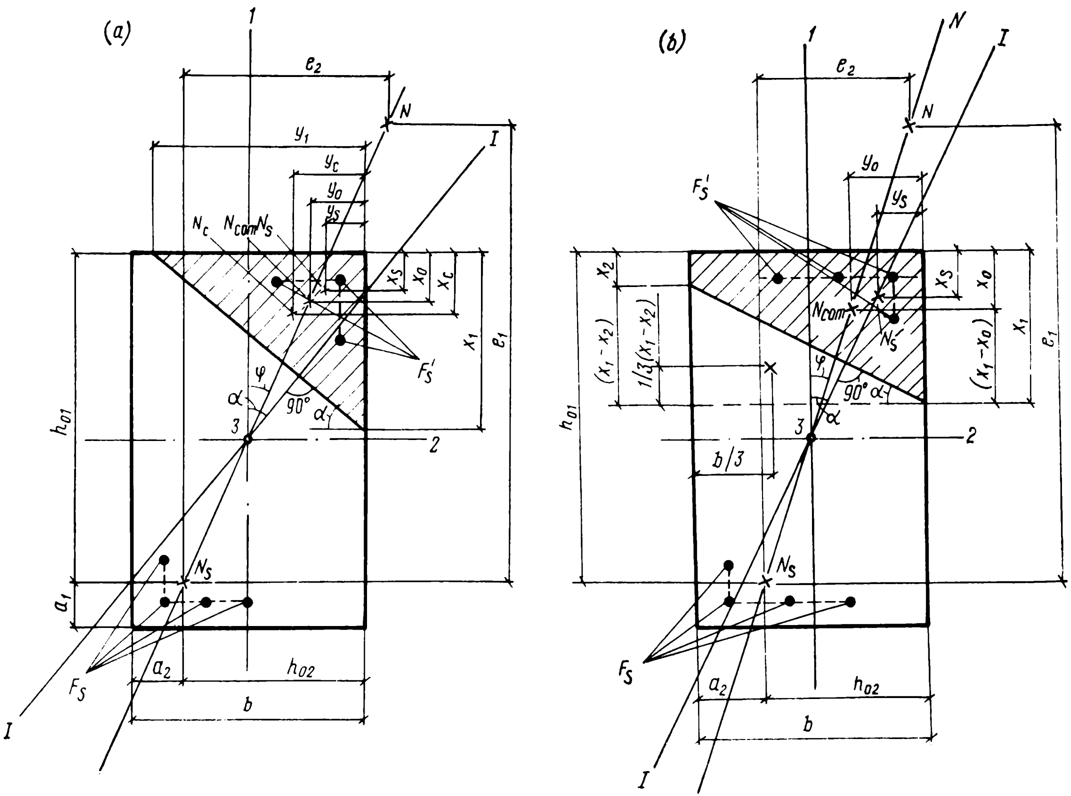


Fig. IV.12. To the strength analysis of members in biaxial compression; plane of forces N, N_s and N_{com} coincides with the geometrical axis of the member

(*a*) triangular compression zone; (*b*) trapezoidal compression zone; *1* and *2*—axes of symmetry of the rectangular member; *3*—geometrical axis of the member; N—external compressive force; N_s—resultant in the steel more distant from N; N_s'—resultant in the steel of the compression zone; N_c—resultant in the concrete of the compression zone; *I-I*—plane normal to the neutral axis

Determining the Dimensions of a Trapezoidal Compression Zone (Fig. IV.12*b*). On the basis of Eq. (IV.53), we may write

$$N + F_sR_s - F'_sR_{s,\ com} - b\,(x_1 + x_2)\,R_{pr}/2 = 0 \tag{IV.63}$$

The coordinates of the resultant in a trapezoidal compression zone with dimensions x_1 and x_2 and steel area F'_s are as follows (see Fig. IV.12*b*)

$$\left.\begin{aligned} x_0 &= [x_1\,(x_1 + x_2)\,b - bx_1^2 + b\,(x_1 - x_2)^2/3 \\ &\quad + 2F'_sx_sR_{s,\ com}/R_{pr}]/[(x_1 + x_2)\,b \\ &\quad + 2F'_sR_{s,\ com}/R_{pr}] \\ y_0 &= (b^2x_1/3 + 2b^2x_2/3 + 2F'_sy_sR_{s,\ com}/R_{pr})/[(x_1 \\ &\quad + x_2)\,b + 2F'_sR_{s,\ com}/R_{pr}] \end{aligned}\right\} \tag{IV.64}$$

Substituting these expressions into Eq. (IV.59) and putting

$$b\,(x_1 + x_2)/2 = (N + F_sR_s - F'_sR_{s,\ com})/R_{pr} = C_3 \tag{IV.65}$$

gives an equation for x_1:

$$x_1^2 + Ax_1 + B = 0 \tag{IV.66}$$

where

$$\left.\begin{aligned} A &= C_{com}b - 2C_3/b \\ B &= 6\,[(C_{com}h_{02} - 2C_{com}b/3 - h_{01} + 2C_3/3b)\,C_3 \\ &\quad + F'_sR_{s,\ com}\,(C_{com}h_{02} - C_{com}y_s - y_{01} + x_s)/R_{pr}]/b \end{aligned}\right\} \tag{IV.67}$$

Substituting x_1 from the above equation into Eq. (IV.65) gives

$$x_2 = 2C_3/b - x_1$$

Now, we may calculate x_0 by Eq. (IV.64), which is necessary in order to check the member for strength on the basis of Eq. (IV.55).

Accidental eccentricities are taken care of as prescribed in Sec. IV.1, and the effect of buckling, as prescribed in Sec. IV.3.

IV.8. TIED AND SPIRAL COMPRESSION MEMBERS

If we supply a short axially loaded column with lateral reinforcement which may effectively restrain lateral deformation, the load-bearing capacity of the member may be considerably increased. In Soviet practice, this is known as indirect reinforcement.

Various types of indirect reinforcement have been investigated. Spiral and welded-ring reinforcement have found the widest application in round and polygonal columns (Fig. IV.13*a*). Closely spaced lateral welded-wire fabric is used in rectangular members (Fig. IV.13*b*). The latter is also often used for the local reinforcement of precast

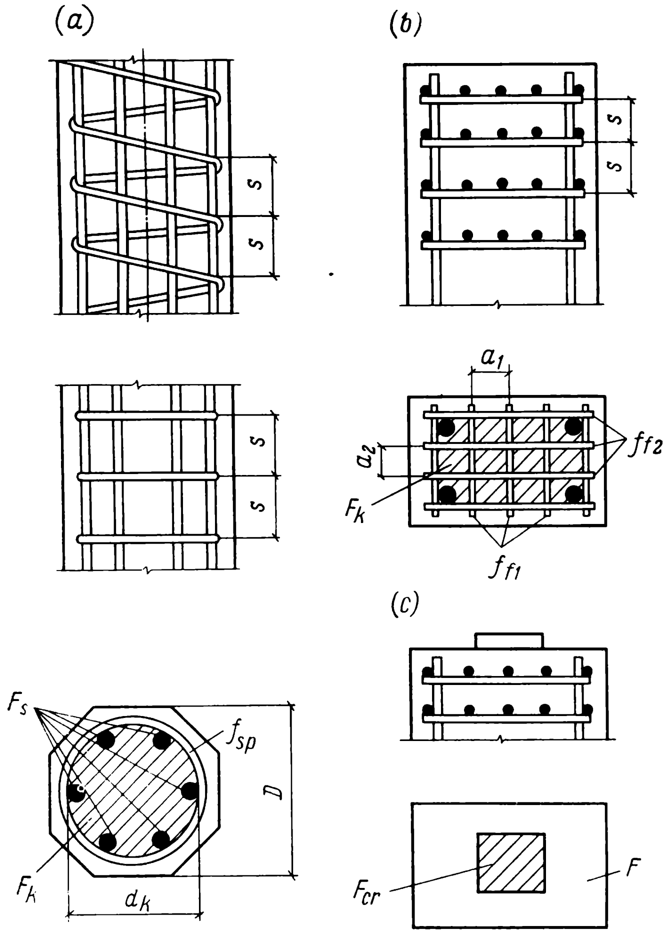


Fig. IV.13. Members in axial compression

(*a*) with spiral or welded-ring reinforcement; (*b*) with lateral welded-wire fabric; (*c*) with welded-wire fabric placed under the centering pad

reinforced concrete columns near the joints (Fig. IV.13*c*), under anchors and also in the anchorage zone of prestressed steel (see Fig. III.8).

Experiments have shown that concrete resists compression better in the kern within a spiral or welded-wire fabric. Acting much like a clip, spirals and rings restrain concrete from lateral deformation occurring in longitudinal compression. In this way, they contribute to resistance against longitudinal compression including that which arises after the first longitudinal cracks have appeared in the con-

crete. Within the kern, the concrete offers a considerable resistance even after the outer concrete layer has spalled (Fig. IV.14), and keeps going so until the stress in the lateral reinforcement reaches the yield point.

The longitudinal strain in tied and spirally reinforced members is rather great and increases with the amount of lateral reinforcement.

The strength of tied and spirally reinforced compression members is calculated by the expressions already analysed, with the section dimensions taken around the periphery of the fabric, rings or spiral, whereas R_{pr} is replaced by a modified strength determined from empirical expressions as follows

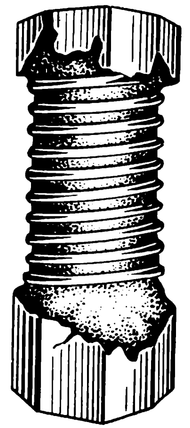


Fig. IV.14. Test specimen with spiral reinforcement after exposure to axial compression

—for welded-wire fabric reinforcement

$$R_{pr}^{*} = R_{pr} + k\mu_{ind}^{f} R_{s}^{f} \qquad \text{(IV.68)}$$

—for spiral and welded-ring reinforcement

$$R_{pr}^{*} = R_{pr} + 2\mu_{ind}^{sp} (-7.5e_0/d_k) \qquad \text{(IV.69)}$$

The above expressions hold for heavy-concrete members having a slenderness ratio of

$$l_0/r_{g,\,k} \leqslant 35$$

where $r_{g,\,k}$ is the radius of gyration of the kern bounded by the extreme lateral bars.

In Eq. (IV.68), R_s^{sp} is the design tensile strength of the fabric wires; μ_{ind}^{f} is the fabric reinforcement ratio, defined as

$$\mu_{ind}^{f} = (f_{f1}a_1 + f_{f2}a_2)/a_1a_2s \qquad \text{(IV.70)}$$

where f_{f1} and a_1 are the cross-sectional area of one fabric wire and the distance between the wires spreading in one direction, respectively; f_{f2} and a_2 are the cross-sectional area of one fabric wire and the distance between the wires spreading in the other direction, respectively; s is the pitch, i.e. distance apart, of the lateral ties; k is the efficiency of lateral ties, defined as

$$k = (5 + \alpha)/(1 + 4.5\alpha) \qquad \text{(IV.71)}$$

where

$$\alpha = \mu_{ind}^{f} R_s^{f}/R_{pr}$$

In Eq. (IV.69), R_s^{sp} is the design strength of the spiral or ring reinforcement; d_k is the diameter of the concrete kern; μ_{ind}^{sp} is the spiral or ring reinforcement ratio, defined as

$$\mu_{ind}^{sp} = 4f_{sp}/d_k s \qquad \text{(IV.72)}$$

where f_{sp} is the cross-sectional area of the spiral or ring, and s is the pitch of the spiral or rings.

The value of ξ_R is calculated by Eq. (III.7) in which ξ_0 is found with allowance for ties or spiral reinforcement from the following empirical relation

$$\xi_0 = 0.85 - 0.008R_{pr} + b$$

Here, ξ_0 is assumed to be not more than 0.9, R_{pr} is in MPa, and b is equal to $10\mu_{ind}^{f}$ or $10\mu_{ind}^{sp}$, but not more than 0.15.

Lateral ties or spiral reinforcement may be provided if the load-bearing capacity of a member calculated by the above expressions (with F_k and R_{pr}^{*}) exceeds that calculated on the basis of the entire cross-sectional area and R_{pr}, without taking into account lateral ties or spiral reinforcement.

To prevent the column cover from cracking in service, members with lateral ties or spiral reinforcement are additionally checked to see if they meet the following condition

$$N \leqslant 1.8R_{pr}F_{tr}/(1 + e_0y/r_{tr}^2) \qquad \text{(IV.73)}$$

where F_{tr} and r_{tr} are the cross-sectional area and radius of gyration of the common transformed section, respectively; e_0 is the eccentricity of the longitudinal force relative to the centroid of the transformed section, determined with allowance for accidental eccentricity as prescribed in Sec. IV.1; y is the distance from the centroid of the transformed section to the most compressed edge.

In this case, the transformed area is determined as usual, but the reduction factor (R_s/R_{pr}) is taken equal to $n = 0.65$ (here, R_s is assumed to be not more than 350 MPa).

The extreme wires of welded-wire fabric, rings and spirals should embrace all longitudinal bars.

Columns with ring and spiral reinforcement are most advantageous where members with the least possible cross-sectional area are to support heavy loads. The effect of indirect reinforcement sharply diminishes in slender columns because of buckling. So, it is most often confined to members for which $l_0/D \leqslant 10$.

Experiments have shown that the transformed spiral area

$$F_{sp} = \pi 2r_{sp}f_{sp}/s$$

should be at least 25% of the longitudinal steel area, since otherwise spiral reinforcement would be ineffective. In Soviet practice, reinforcing spirals (or rings) are made of class A-I, A-II or A-III steel bars 6 to 14 mm in diameter or class B-I wire, with a pitch of not less than 40 mm and not more than 1/5 of the member diameter, nor more than 100 mm. Spirals and rings less than 200 mm in diameter are not recommended for use.

If a force is transferred through a joint between two members over a part of the end face surface, known as a centering pad, rather than the entire end face (Fig. IV.13c), the strength of the members under the pad should meet the following condition

$$N \leqslant R^*_{pr} F_{cr} \tag{IV.74}$$

where F_{cr} is the area of crushing, and R^*_{pr} is the strength of the concrete found as

$$R^*_{pr} = R_{pr}\gamma_c + k\mu^f_{ind} R^f_s \gamma_{ind} \tag{IV.75}$$

Here, γ_c is the factor which takes care of the increase in the load-bearing capacity of the concrete caused by local crushing and assumed, according to an empirical relation, to be equal to

$$\gamma_c = \sqrt[3]{F/F_{cr}} \tag{IV.76}$$

but not more than 3.5; F is the entire cross-sectional area of the member; γ_{ind} is the same factor for tied or spirally reinforced members

$$\gamma_{ind} = 4.5 - 3.5F_{cr}/F_k \tag{IV.77}$$

μ^f_{ind}, k and R^f_s have already been explained. For the areas F, F_k and F_{cr}, see Fig. IV.13c.

The difference in amount of fabric reinforcement per unit length in either direction should not exceed 50%. Welded fabric is made of the same steel as spiral reinforcement. The mesh should be not less than 45 mm and not more than 1/4 of the least side of the cross section, nor more than 100 mm. The pitch of the fabric wires, s, should be not less than 60 mm and not more than 1/3 of the section width, nor more than 150 mm.

The ends of compression members should be reinforced with at least four fabric pieces (see Fig. IV.13c). The length of the reinforcement zone should be not less than $10d$ for deformed longitudinal bars and $20d$ for plain bars (or wire strands).

IV.9. COMPRESSION MEMBERS WITH LOAD-BEARING REINFORCEMENT

These are used in in-situ reinforced concrete structures requiring complex falsework for their manufacture, such as the framework for civil high-rise buildings. In the course of erection, the load-bearing reinforcement acts as the falsework supporting the formwork, freshly placed concrete, and all erection devices. After the concrete has attained a sufficient strength, the load-bearing reinforcement begins to work together with the concrete in the structure.

Load-bearing reinforcement is most advantageous in structures whose self-weight does not exceed 25% of the total load; in this case, excessive steel consumption either does not exist altogether or is compensated for by economy in falsework.

Load-bearing reinforcement may be made of rolled-steel I-sections, channels or large-size angles (in which case we speak of "stiff reinforcement") or large bars and small-size angles in the form of welded reinforcing cages.

Figure IV.15 illustrates columns containing stiff reinforcement. The sections are joined together by straps or webs. The stiff steel

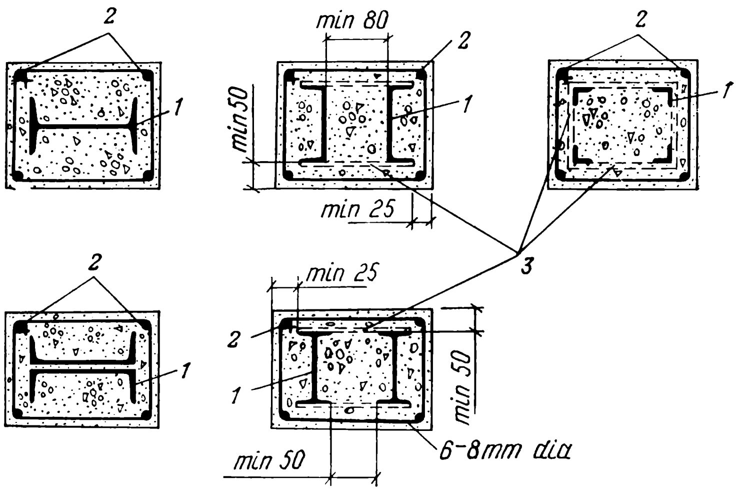


Fig. IV.15. Columns with stiff reinforcement
1—steel shape encased in the concrete; 2—reinforcing bars; 3—connecting straps

area is kept to the minimum value necessary to carry erection loads, and usually ranges between 3 and 8% of the concrete cross-sectional area. To prevent members from spalling, the percentage of reinforcement should not exceed 15%. With a higher percentage, the concrete is regarded as being able to act only as the protective encasement carrying no load. The brand number of concrete for such members should not be less than M-200. Here, lateral reinforcement is also required.

If the design calls for additional flexible reinforcement, the latter should be distributed around the perimeter and arranged according to the usual rules. Flexible reinforcement may be in the form of separate bars or welded bar mats. If the design does not call for flexible reinforcement, light welded-wire fabric distributed around the

circumference is attached to erection bars at the corners of the section.

The concrete cover for rolled-steel shapes and the spacings between them are assigned according to Fig. IV.15.

Load-bearing reinforcement in the form of cages is made of round-bar or rolled-steel section mats welded together (Fig. IV.16). Here, the main longitudinal bars are braced by lateral or inclined bars (Fig. IV.16*b*) spaced not wider than $20d$ apart (double-fillet welds must be used throughout), and additional round bars, not

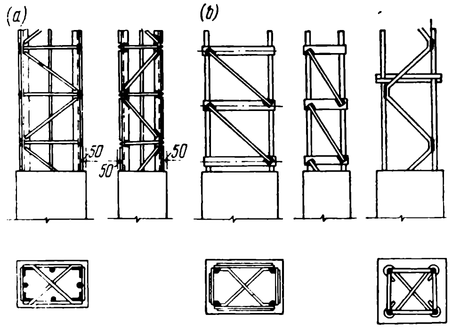


Fig. IV.16. Columns with load-bearing reinforcing cages

(*a*) with longitudinal cage members made of shaped and round rolled-steel sections; (*b*) with longitudinal cage members made of round rolled-steel sections

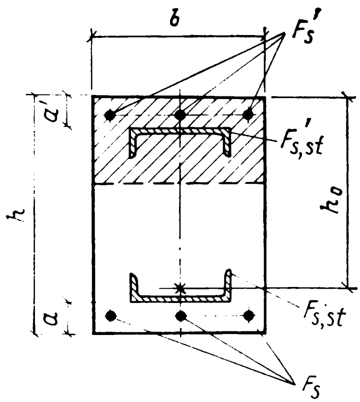


Fig. IV.17. To the design of eccentrically loaded members with stiff reinforcement in the compression and tension zones

wider than $15d$ apart (Fig. IV.16*a*). The latter may be either welded to the reinforcing cage or fastened by additional stirrups.

Load-bearing reinforcement is designed according to specifications and standards applicable to steel work and for loads arising in erection before the concrete has attained its strength (these loads are regarded as special short-time live loads). Afterwards, the steel and the concrete work together. Total service load may be transferred to a structure only after the concrete has reached its design strength. For total design load, a structure containing load-bearing reinforcement is designed in the usual manner, with allowance for all of the stiff and additional flexible steel.

Experiments have shown that in correctly designed structures the stiff reinforcement and the concrete may work together until the structure fails. Here, the stress in the stiff reinforcement reaches the yield point, and the initial stress arising in the steel in the course

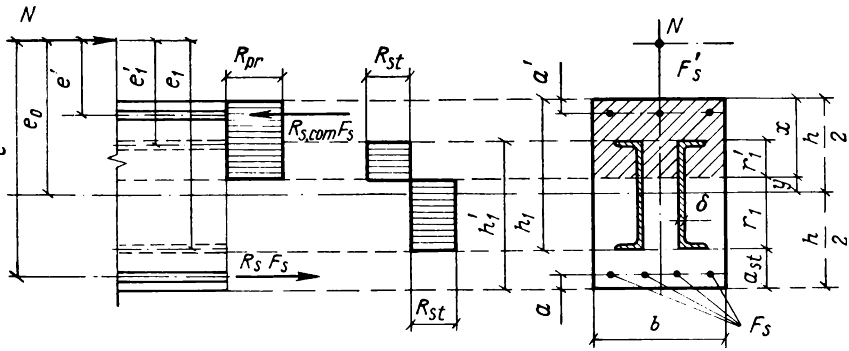


Fig. IV.18. To the design of members with stiff reinforcement in eccentrical compression; the neutral axis crosses the webs of the encased steel beams

of erection does not affect the final strength of the reinforced concrete member.

In the design calculations of eccentrically loaded members containing stiff reinforcement, the compression zone area is taken without the steel area, which is equivalent to the reduction in the design strength of the stiff reinforcement in that zone to $R_{s,\ st} - R_{pr}$.

Eccentrically loaded members containing stiff reinforcement in the compression and tension (or less compressed) zones (Fig. IV.17) are designed in much the same way as members reinforced with flexible steel. In this case, the effective depth of the section, h_0, is taken as the distance from the most compressed face to the common centroid of the stiff and flexible steel at the opposite face.

Eccentrically loaded members reinforced with rolled-steel sections whose webs are parallel to the plane of flexure and occupy a considerable part of the section depth (Fig. IV.18) may be designed as prescribed in the above sections. If the stiff reinforcement is made of a steel having a definite yield point, it is legitimate to assume that the stress is the same over the entire steel area (including the webs of the encased steel beams) and equal to the design strength, R_{st}, as shown in Fig. IV.18.

CHAPTER V

MEMBERS IN TENSION

V.1. CONSTRUCTIONAL FEATURES

The most common reinforced concrete members in axial tension are arch strings, bottom chords and descending diagonals in trusses, and walls of round tanks for liquids (Fig. V.1).

As a rule, members subjected to axial tension are prestressed, so that their crack resistance is considerably increased.

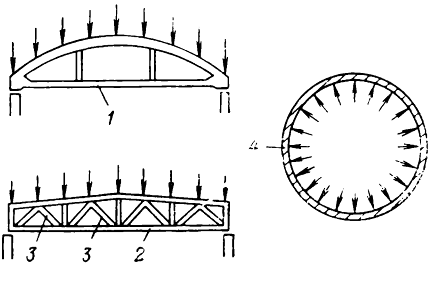


Fig. V.1. Members in axial tension

1—arch string; *2*—bottom chord of a truss; *3*—descending diagonal of a truss; *4*—wall of a round tank

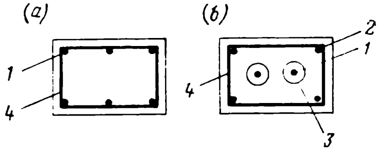


Fig. V.2. Distribution of reinforcing steel in prestressed linear members subjected to axial tension

(*a*) with pretensioning; (*b*) with posttensioning; *1*—prestressed steel (bars, wire strands, and cables); *2*—nonprestressed steel; *3*—duct for prestressed steel; *4*—transverse bars

The basic principles of arrangement of reinforced concrete members discussed in Chapter II also apply to members in axial tension. Main nonprestressed reinforcing bars are generally held together by welds made along their length; lap splicing without welding is only allowed in slabs and walls.

No joints are allowed in tensile prestressed reinforcement (bars, wire strands, and cables) used in linear members (such as arch

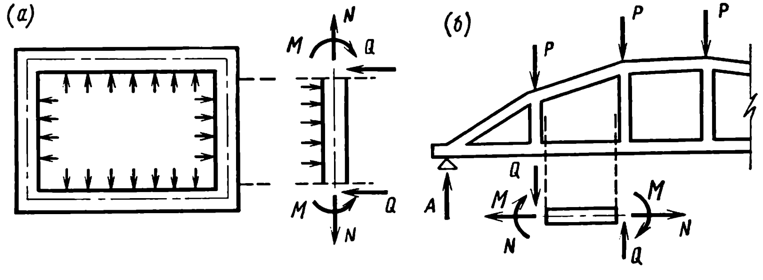


Fig. V.3. Members in eccentrical tension
(a) wall of a tank (bin); (b) bottom chord of a truss without diagonals

strings and bottom truss chords). Prestressed steel should be distributed symmetrically over the cross section of a member (Fig. V.2) in order to avoid, as far as possible, an eccentrical prestress in the member during the transfer of the prestress (which may be done by releasing all bars at the same time or by gradually releasing separate groups of bars).

In posttensioning, prestressed steel located in ducts is not able to contribute to the strength of the members during the transfer of the prestress. In the circumstances, it is advisable to provide prestressed members with a small amount of nonprestressed reinforcement (Fig. V.2*b*). It should be distributed closer to the outer faces, so as to protect the member against the likely eccentrical effects of the prestress.

Members subjected to eccentrical tension are the walls of rectangular tanks (bins) carrying the inner pressure of the materials they contain (Fig. V.3*a*), the bottom chords of trusses without diagonals (Fig. V.3*b*), and the like. Such members are simultaneously subjected to a longitudinal tensile force, N, and a bending moment, M, which is equivalent to eccentrical tension brought about by the force, N, applied at the eccentricity $e_0 = M/N$ about the longitudinal axis of the member.

Two cases of eccentrical tension may be distinguished. Case 1 refers to the external longitudinal tensile force, N, applied between the resultants in the steel A and A' (Fig. V.4*a*), and Case 2 refers to N applied outside this range (Fig. V.4*b*).

Eccentrically loaded members falling under Case 2 are reinforced with longitudinal and lateral bars similarly to members in bending. Those falling under Case 1 are reinforced as members designed to carry axial tension.

Eccentrically and axially loaded members are usually prestressed to increase their crack resistance.

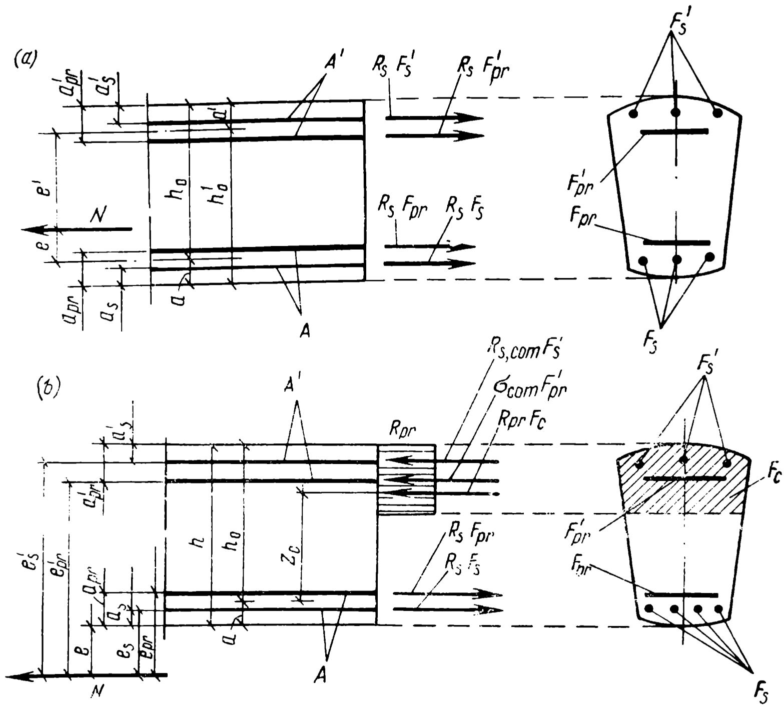


Fig. V.4. Loading configurations for members in eccentrical tension
(a) with the longitudinal tensile force N applied between the resultants in the steel A and A'; (b) with N applied outside this range

The percentage of longitudinal reinforcement in members under eccentrical tension should be at least 0.05%; this refers to the steel A in Case 2, and $A + A'$ in Case 1.

The anchorage of tensile bars in concrete subjected to tension or compression, and the splicing of welded-wire and tied fabric have been discussed in Secs. I.2 and I.3.

The joints transmitting tensile force between tension members are made either by welding together reinforcing steel stick-outs or embedded steel items, or by using posttensioned joint-overlapping wire strands, cables or bars placed in ducts or slots.

V.2. STRENGTH ANALYSIS OF MEMBERS IN AXIAL TENSION

Axially loaded members fail after through cracks have appeared in the concrete so that it no longer resists the load at the cracks, and the stress in the steel has reached the yield strength (if the steel has

a definite yield) or the ultimate strength. The load-bearing capacity of a member subjected to axial tension is determined solely by the ultimate tensile strength of the steel; the concrete does not contribute to the total strength of the member.

Accordingly, the strength of axially loaded members which, in the general case, contain prestressed steel of cross-sectional area F_{pr}, and nonprestressed steel of cross-sectional area F_s, should meet the following condition

$$N \leqslant m_{s4} R_s F_{pr} + R_s F_s \tag{V.1}$$

where m_{s4} is the service factor taking care of the steel behaviour when the stress in it exceeds the proof yield strength. This factor is taken as 1.2 for class A-IV and Aт-IV steel; 1.15 for class A-V, Aт-V, B-II, Bp-II and K-7 steel; and 1.1 for class Aт-VI steel.

Members containing prestressed reinforcing steel without anchorage should be checked for strength within the prestress transmission length. Here, the design strength of the steel is reduced by multiplying R_s by the following service factor

$$m_{s3} = l_x / l_{tr}$$

where l_x is the distance from the beginning of the transmission zone to the steel section in question within the transmission length; l_{tr} is the total transmission length assumed as prescribed in Sec. I.3.

V.3. STRENGTH ANALYSIS OF SYMMETRICAL MEMBERS SUBJECTED TO ECCENTRICAL TENSION IN THE PLANE OF SYMMETRY

In Case 1 (see Fig. V.4*a*), eccentrically loaded members of any symmetrical cross section reach the limit state in terms of load-bearing capacity when through transverse cracks appear in the concrete, so that only the longitudinal steel resists tension at normal sections containing the cracks. A member fails when the stresses in the longitudinal steel A and A' reach the ultimate strength.

In Case 2 (see Fig. V.4*b*), the limit state in terms of load-bearing capacity is similar to that for members in bending. The part of the section near the side farther away from the force, N, is in compression; the opposite part is in tension. Because of cracks in the concrete of the tension zone, the tensile force there is taken by the steel. As a result, the member owes its load-bearing capacity to the ultimate tensile strength of the steel in the tension zone and the ultimate compressive strength of the concrete and nonprestressed steel in the compression zone. If the compression zone is reinforced with prestressed steel, the stress in the latter is assumed as σ_{com} which is determined as prescribed for members in bending (see Sec. III.2).

The load-bearing capacity of members in eccentrical tension is adequate if the following conditions are met:

Case 1

$$Ne \leqslant m_{s4}R_sF'_{pr}(h_0 - a'_{pr}) + R_sF'_s(h_0 - a'_s) \quad (V.2)$$

$$Ne' \leqslant m_{s4}R_sF_{pr}(h_0 - a_{pr}) + R_sF_s(h'_0 - a_s) \quad (V.3)$$

Case 2

$$Ne \leqslant R_{pr}F_cz_c + R_{s,\,com}F'_s(h_0 - a'_s) + \sigma_{com}F'_{pr}(h_0 - a'_{pr}) \quad (V.4)$$

In Eq. (V.4), the compression zone area, F_c, is found from the following expression

$$N = m_{s4}R_sF_{pr} + R_sF_s - R_{pr}F_c - \sigma_{com}F'_{pr} - R_{s,\,com}F'_s \quad (V.5)$$

In Case 2, the condition $\xi = x/h_0 \leqslant \xi_R$ should be satisfied. If this condition is not met, we assume $\xi = x/h_0 = \xi_R$ in Eq. (V.4). The value of ξ_R is found as prescribed for members in bending (see Sec. III.2).

1. Rectangular Members

Case 1. The load-bearing capacity of a member is checked and the necessary steel area is calculated directly by Eqs. (V.2) and (V.3).

Case 2. Equation (V.4) is re-written thus

$$Ne \leqslant R_{pr}\,bx\,(h_0 - 0.5x)$$
$$+ R_{s,\,com}F'_s(h_0 - a'_s) + \sigma_{com}F'_{pr}(h_0 - a'_{pr}) \quad (V.6)$$

The depth of the compression zone may be deduced from Eq. (V.5)

$$x = (m_{s4}R_sF_{pr} + R_sF_s - \sigma_{com}F'_{pr} - R_{s,\,com}F'_s - N)/R_{pr}b \quad (V.7)$$

It should be kept in mind that Eq. (V.6) holds if

$$x \leqslant \xi_Rh_0$$

To determine the steel areas F_s and F'_s, Eqs. (V.6) and (V.7) are transformed as

$$F'_s = [Ne - A_RR_{pr}bh_0^2 - \sigma_{com}F'_{pr}(h_0$$
$$- a'_{pr})]/R_{s,\,com}(h_0 - a'_s) \quad (V.8)$$

$$F_{pr} = (\xi_RR_{pr}bh_0 - R_sF_s + R_{s,\,com}F'_s$$
$$+ \sigma_{com}F'_{pr} + N)/m_{s4}R_s \quad (V.9)$$

Here, ξ_R and A_R are looked up in Table III.1.

If the design calculations reveal that F'_s is negative or less than the minimum permissible value (according to Sec. V.1), F'_s is assigned the minimum permissible value. In such a case and also when F'_s is

specified beforehand for some other reason, we should first calculate

$$A_0 = [Ne - R_{s,\,com}F'_s\,(h_0 - a'_s) - \sigma_{com}F'_{pr}\,(h_0 - a'_{pr})]/F_{tr}bh_0^2 \tag{V.10}$$

then look up the respective ξ in Table III.1 and, finally, determine

$$F_{pr} = (\xi R_{pr}bh_0 - R_sF_s + R_{s,\,com}F'_s + \sigma_{com}F'_{pr} + N)/m_{s4}R_s \tag{V.11}$$

2. Inclined-Section Shear Resistance Analysis

The analysis procedure is the same as for members in bending; inclined-section strength analysis is carried out by the formulas given in Sec. III.6. Here, the value of Q_c is determined by Eq. (III.60) with the additional coefficient, k_N, applied. This coefficient taking care of the reduced shear strength of the concrete in the compression zone due to longitudinal tension is defined by the following empirical relationship

$$k_N = 1 - 0.2N\,(0.01)/R_{ten}bh_0 \tag{V.12}$$

Here, N is in N, and R_{ten} in MPa; k_N is taken as not less than 0.2.

Shear strength analysis may be omitted if the condition described by Eq. (III.62), with its right-hand side multiplied by k_N as given by Eq. (V.12), is met.

CHAPTER VI

MEMBERS IN COMBINED BENDING AND TORSION

VI.1. GENERAL

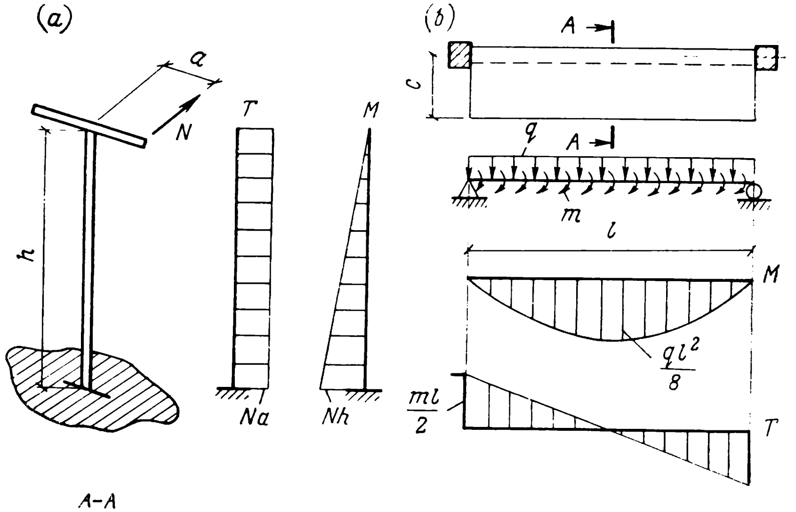
There is almost no such thing as pure torsion in reinforced concrete structural members, whereas torsion combined with bending is rather common. The torsional strength of reinforced concrete members is considerably smaller than their resistance to bending. So in some structures, torsional moments, however small, should be included in the design calculations.

Examples of reinforced concrete members subjected to combined bending and torsion are furnished by a pole acted upon by an external lateral force applied at a certain distance from its axis (Fig. VI.1*a*), or a beam with the flange loaded on one side (Fig. VI.1*b*).

Torsion imposed on a reinforced concrete member gives rise to principal compressive and tensile stresses at 45° to the longitudinal axis. The onset of cracking and crack slope depend on the value and direction of the principal tensile stresses. In a member subjected to torsion, cracks are spiral (Fig. VI.2*a*). They occur already at an early stage of loading. After the advent of cracking, the forces in the direction of the principal tensile stresses are taken by the steel, and those in the direction of the principal compressive stresses by the concrete. The member begins to fail when considerable nonelastic elongation occurs in the tensile steel (Fig. VI.2*b*).

A rectangular reinforced concrete member subjected to combined bending and torsion fails at one of the through cracks (Fig. VI.3). The opposite ends of such a crack crossing three faces of the member come closely to the compression zone which is located near the fourth face.

Members in bending combined with torsion should contain reinforcing steel capable of resisting the bending moment, shearing force and torque. Within areas of pure torsion, members may be reinforced with spiral steel (Fig. VI.4*a*), or transverse and longitu-



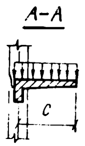


Fig. VI.1. Reinforced concrete members in combined bending and torsion

(a) pole acted upon by a horizontal force N applied at a distance a from its axis; (b) beam with the flange loaded on one side

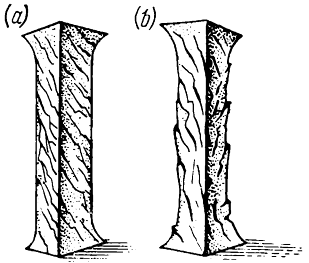


Fig. VI.2. Specimen after exposure to torsion

(a) after the advent of cracking (intermediate stage of loading); (b) final stage of loading

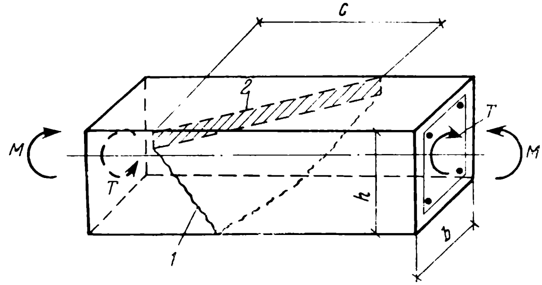


Fig. VI.3. Failure of a rectangular member subjected to combined bending and torsion

1—through crack; 2—compression zone in a cracked section

dinal bars (Fig. VI.4b). Spiral reinforcement is more effective because it is better aligned with the principal tensile stresses; it is advantageous, however, only when a member is subjected to torques of the same sign. Longitudinal and transverse bars are more handy than spiral reinforcement in manufacture and erection.

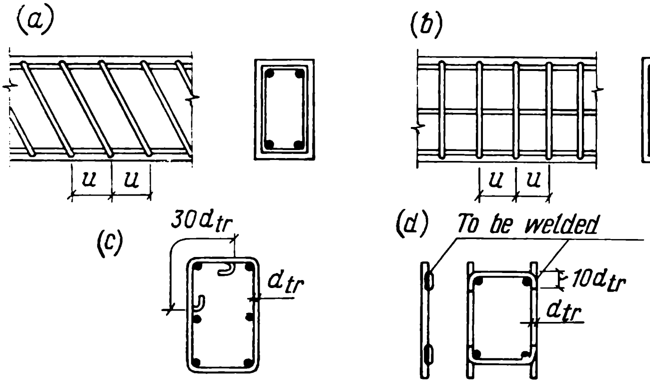


Fig. VI.4. Reinforcement of rectangular members subjected to combined bending and torsion

(*a*) longitudinal bars and spiral reinforcement; (*b*) longitudinal bars and closed stirrups; (*c*) tied reinforcing cage; (*d*) welded reinforcing cage

All longitudinal bars introduced in design for torsion with their design strength unreduced should be carried beyond the face of the support for at least l_{an} (see Sec. I.3), or safely anchored.

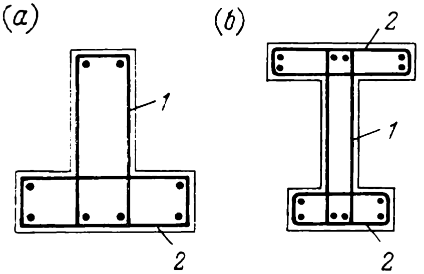


Fig. VI.5. Reinforcement of members subjected to combined bending and torsion

(*a*) T-sections; (*b*) I-sections; *1*—closed stirrups in the rib and web; *2*—closed stirrups in the flange

The behaviour of reinforced concrete members under torsion requires that the stirrups in tied reinforcing cages should be closed and have an overlap of $30d_{tr}$ (Fig. VI.4*c*). In welded cages, all vertical and horizontal bars should be resistance spot-welded to the longitudinal bars at the corners to form closed contours, or joined together by arc-welded hooked stirrup ends with welds at least $10d_{tr}$ long (Fig. VI.4*d*).

In members having a complex cross section (such as I- and T-sections), each part (flanges, ribs, webs, etc.) should contain closed reinforcement of its own (Fig. VI.5).

VI.2. DESIGN OF RECTANGULAR MEMBERS

The state of stress produced by combined bending and torsion is one of the most complex in reinforced concrete practice, and has yet to be studied. In any case, there is no consensus among specialists about its nature, so methods of design significantly differ from co-

untry to country. We shall examine the method developed in the Soviet Union on the basis of many years' experiments, and adopted in relevant standards.

The load-bearing capacity of a member is determined by the limit-state equilibrium method with allowance for through cracking and on the assumption that the ultimate strength of the steel crossed by the crack is equal to its yield strength, and that of the concrete to its compressive strength.

A member will fail as shown in Fig. VI.6 if it is subjected predominantly to bending and torsion with no (or small) shearing force.

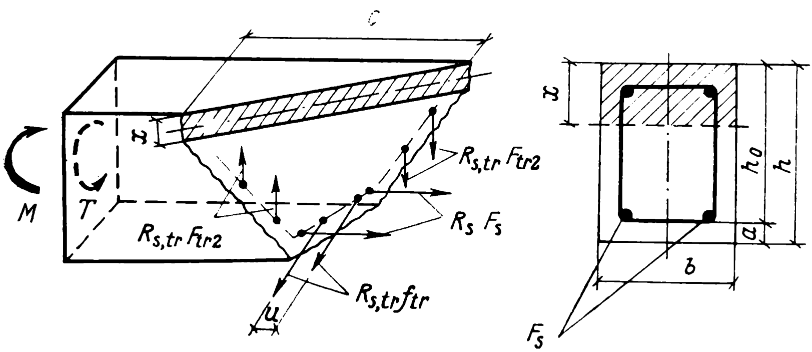


Fig. VI.6. To the design of rectangular members subjected to combined bending and torsion (Case 1)

In the general case, a member should be analysed for strength on the assumption that the steel located at any face of the member is in the state of yield, with the through crack and the compression zone properly positioned. So, in addition to the case shown in Fig. VI.6, we should examine those of Fig. VI.7.

The disposition shown in Fig. VI.7*a* applies to the case where a member is subjected to a torque and shearing force with no or small bending moment. A distinction of this scheme is that inclined cracks open at one of the sides of the member due to the yield of the stirrups. Experiments have shown that the shear strength of a member in bending combined with torsion is considerably less than that of a member subjected to bending alone.

Figure VI.7*b* illustrates the case where the effect of the torque is predominant in comparison with the bending moment, and the zone compressed by bending has much less steel than the opposite part of the section.

A member should preferably be checked for strength in each of all the three cases. The strength may be taken as adequate if the external moment taken about the axis passing through the centre and in the plane of the compression zone does not exceed the sum of the

moments taken about the same axis and due to the ultimate forces in the longitudinal and transverse reinforcement crossed by the through crack. The least of the three values should then be adopted in the design calculations. Relevant standards recommend the following generalized expression for this check, derived with certain simplifying assumptions

$$T \leqslant R_s F_s (1 + \gamma\delta\beta^2)(h_0 - 0.5x)/(k\beta + \varkappa) \qquad \text{(VI.1)}$$

where

$$\beta = c/b; \quad \delta = b/(2h + b) \qquad \text{(VI.2)}$$

$$\gamma = R_{s,\,tr} f_{tr} b / R_s F_s u \qquad \text{(VI.3)}$$

$$\varkappa = M/T; \; k = 1 + Qh/2T \qquad \text{(VI.4)}$$

Here, M, T and Q are the bending moment, torque and shearing force calculated for the normal section of the member, coinciding with

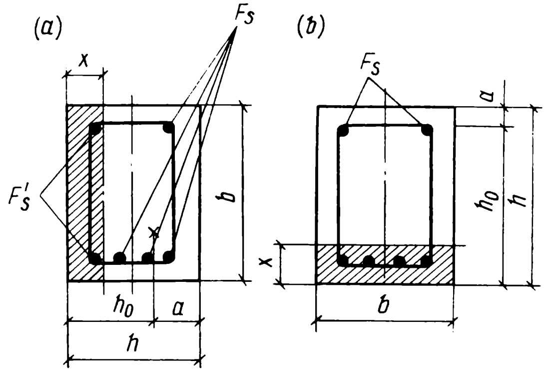


Fig. VI.7. To the design of rectangular members subjected to combined bending and torsion
(a) Case 2; (b) Case 3

the centroid of the compression zone of the trial section containing the through crack; F_s and F_s' are the cross-sectional areas of the longitudinal steel in the tension and compression zones in the appropriate loading configurations of Figs. VI.6 and VI.7; b and h are the width and depth of the cross section taken according to the configuration in question; c is the neutral axis as projected on the longitudinal axis of the member; and x is the depth of the compression zone found from the following equation

$$R_s F_s - R_{s,\,com} F_s' = R_{pr} bx \qquad \text{(VI.5)}$$

When no bending moment and shearing force are applied, $\varkappa = 0$ and $k = 1$. For the loading configuration of

Fig. VI.6

$$\varkappa = M/T; \; k = 1$$

Fig. VI.7*a*

$$\varkappa = 0; \quad k = 1 + Qh/2T$$

Fig. VI.7*b*

$$\varkappa = -M/T; \quad k = 1$$

The critical cross-sectional area of a member, that is, one corresponding to the least load-bearing capacity, is described in terms of the parameter c. The value of this parameter may be determined by substituting trial values in the design formulas, but, as has been proved experimentally, it ought not to exceed $c = 2h + b$.

Present evidence shows that the ratio of the transverse steel area to the longitudinal steel area, designated γ, should be within the limits

$$\gamma_{min} < \gamma < \gamma_{max} \tag{VI.6}$$

where

$$\left.\begin{aligned} \gamma_{min} &= 0.5/(1 + 2\varkappa \sqrt{\delta}) \\ \gamma_{max} &= 1.5/(1 + 2\varkappa \sqrt{\delta}) \end{aligned}\right\} \tag{VI.7}$$

If γ calculated by Eq. (VI.3) is below γ_{min}, then the force R_sF_s in Eqs. (VI.4) and (VI.5) should be multiplied by the reduction factor γ/γ_{min}.

The above limitation is included to ensure the desired stress-strain behaviour of members and the width of cracks in the concrete under service conditions, because limit-state design in terms of the second group of limit states has not yet been developed and included in appropriate standards for members under combined bending and torsion.

If, in the scheme of Fig. VI.7*a*, $T \leqslant 0.5Qh$, then the design on the basis of the second scheme may be replaced by the design on the basis of the following condition

$$Q \leqslant Q_{tr,c} - 3T/b \tag{VI.8}$$

where Q and T are assumed to have maximum values in the portion in question, and $Q_{tr,c}$ is found by Eq. (III.67).

The overall cross-sectional dimensions of members subjected to combined bending and torsion should be assigned so that the following condition would be satisfied

$$T \leqslant 0.1 R_{pr} b^2 h \ (0.01) \tag{VI.9}$$

where $h > b$, T is in N, and R_{pr} is in MPa. In the above expression, R_{pr} for concretes with a brand number of higher than M-400 is taken equal to that of M-400 concrete. This has been verified experimentally and introduced in design so as to prevent failure of the concrete at a side due to compression, or at a face due to compression as a result of bending.

CHAPTER VII

CRACK RESISTANCE AND DEFLECTION OF REINFORCED CONCRETE MEMBERS

As already defined, the crack resistance of members refers to their ability to resist incipient cracking in Stage I or opening of cracks in Stage II of the stress-strain state. Members are checked for crack resistance at sections normal to their longitudinal axis, and also at inclined sections if they are subjected to shearing forces. Analysis for crack resistance and deflection belongs to analysis on the basis of the second group of limit states. Load combinations are taken as prescribed in Chapter II (see Tables II.2 and II.3).

The design calculations are carried out on the following assumptions: (1) the stress in the concrete of the tension zone prior to cracking is equal to $R_{ten\,II}$; (2) the stress in the prestressed steel is equal to $\sigma_0 + 2nR_{ten\,II}$, that is, to the sum of the prestress (with allowance for losses and multiplied by the tensioning accuracy factor) and the increment in the stress caused by the increment in the strain of the surrounding concrete after all of the prestress has been cancelled; (3) the stress in the nonprestressed steel of prestressed members is equal to the sum of the compressive stress due to shrinkage and creep in the concrete, and the increment in the tensile stress due to the increment in the strain of the concrete.

VII.1. INCIPIENT-CRACKING RESISTANCE OF MEMBERS IN AXIAL TENSION

1. Incipient-Cracking Resistance Analysis

The analysis is based on the assumption that no cracks form at sections normal to the longitudinal axis of a member if the longitudinal force, N, induced by the external loading does not exceed the longitudinal cracking force, N_{cr}, that is

$$N \leqslant N_{cr} \tag{VII.1}$$

2. Determining N_{cr}

The longitudinal force, N_{cr}, is found from the stresses existing in the material immediately before cracking

$$N_{cr} = R_{ten\,II}\,(F + 2nF_s) + N_0 \qquad \text{(VII.2)}$$

where F is the cross-sectional area of the member; F_s is the total cross-sectional area of the prestressed and nonprestressed steel; and N_0 is the prestressing force defined by Eq. (II.26).

When determining N_{cr} for nonprestressed members, N_0 should be taken as $-\sigma_9 F_s$. The stress in the nonprestressed steel, σ_9, induced by shrinkage in the concrete reduces the incipient-cracking resistance of the member.

VII.2. INCIPIENT-CRACKING RESISTANCE OF MEMBERS IN BENDING, ECCENTRICAL COMPRESSION AND ECCENTRICAL TENSION

1. Normal Cracking Analysis

This is based on the assumption that no cracks will form at sections normal to the longitudinal axis of a member if the external

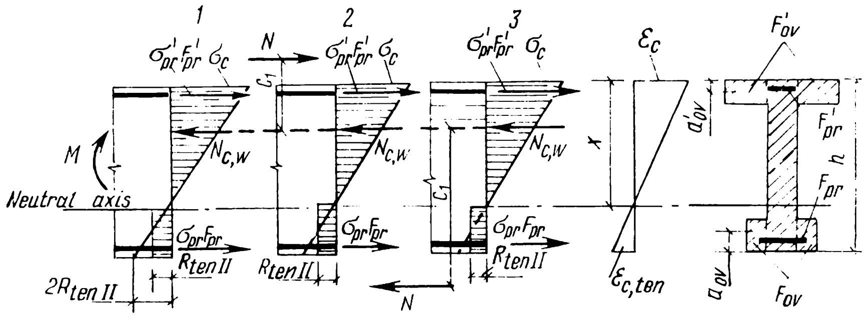


Fig. VII.1. To determining the cracking resistance of prestressed members in (*1*) bending, (*2*) eccentrical compression and (*3*) eccentrical tension in terms of Stage I with the concrete of the compression zone behaving elastically

moment, M, does not exceed the internal or cracking moment, that is,

$$M \leqslant M_{cr} \qquad \text{(VII.3)}$$

In bending, the external moment is determined in the regular way; whereas in eccentrical compression and eccentrical tension with

some of the cross section in compression, it is

$$M = Nc_1 \tag{VII.4}$$

where c_1 is the distance from the external longitudinal force, N, to the same axis about which the moment due to the internal forces is taken (Fig. VII.1).

2. Determining M_{cr} with the Concrete of the Compression Zone Behaving Elastically

Before the onset of cracking, members subjected to bending, eccentrical compression and eccentrical tension so that their sections are in compression and tension at the same time, are in Stage I of the stress-strain state. To determine M_{cr} in the general form, we shall examine a prestressed I-section and introduce the following notation: F'_{ov} for the cross-sectional area of the flange overhangs in the compression zone, and F_{ov} for the cross-sectional area of the flange overhangs in the tension zone.

In design calculations, we shall assume that: (1) sections which are plane before bending will be plane after bending (that is, Bernoulli's assumption holds); (2) there is inelastic strain in the concrete of the tension zone, so that $\nu_{ten} = 0.5$, and the normal stress distribution diagram is rectangular; and (3) strain in the concrete of the compression zone is elastic, so that $\nu = 1$, and the normal stress distribution diagram is triangular in shape.

The concrete in the compression zone is assumed to behave elastically if the stress-strength ratio is

$$k = \sigma_c/R_{pr\,II} < 0.7$$

The limiting value of k depends on the type of concrete, eccentricity of the longitudinal compressive force, duration of loading and some other factors. Let us express the stresses in the materials of both zones of the section in terms of $R_{ten\,II}$. As is seen from the strain diagram of the section (see Fig. VII.1), the marginal strain of the concrete in the compression zone is defined as

$$\varepsilon_c = \varepsilon_{c,ten}x/(h - x)$$

and the marginal stress is

$$\sigma_c = \varepsilon_c E'_c = \varepsilon_{c,ten} x \nu E_c/(h - x) \tag{VII.5}$$

where x is the depth of the compression zone (in Stage I before the advent of cracking).

Since the strain is defined as

$$\varepsilon_{c,ten} = R_{ten\,II}/E'_{c,ten} = R_{ten\,II}/\nu_{ten}E_c$$

the marginal stress is

$$\sigma_c = (R_{ten\,II}/\nu_{ten}E_c)\,[x\nu E_c/(h-x)] \tag{VII.6}$$

Recalling that $\nu_{ten} = 0.5$ and $\nu = 1$, we get

$$\sigma_c = 2R_{ten\,II}x/(h-x) \tag{VII.7}$$

The stress in the concrete of the compressed flanges at the centroid of the overhangs, that is, within a'_{ov} of the section edge, is defined as

$$\sigma'_{c,ov} = 2R_{ten\,II}\,(x - a'_{ov})/(h-x) \tag{VII.8}$$

The stresses in the prestressed steel in the tension and compression zones of the section are

$$\sigma_{ten} = \sigma_0 + 2nR_{ten\,II} \tag{VII.9}$$

$$\sigma'_{ten} = \sigma'_0 + 2nR_{ten\,II}\,(x - a')/(h-x) \tag{VII.10}$$

The force, $N_{c,w}$, in the concrete of the compression zone in the web of an I-section is applied at the point within $x/3$ of the section edge. In Eq. (VII.3), the internal moment, M_{cr}, and the external moment, M, are taken about the axis passing through this point. Then,

$$\begin{aligned} M_{cr} = {} & R_{ten\,II}\{b\,(h-x)\,(h/2 + x/6) + F_{ov}\,(h - a_{ov} - x/3) \\ & + 2F'_{ov}\,(x - a'_{ov})\,(x/3 - a'_{ov})/\,(h-x) \\ & + F_{pr}\,(2n + \sigma_0/R_{ten\,II})\,(h_0 - x/3) \\ & - F'_{pr}\,[\sigma'_0/R_{ten II} - 2n\,(x - a')/(h-x)] \\ & (\times\, x/3 - a')\} = R_{ten\,II}W_{cr} \end{aligned} \tag{VII.11}$$

where W_{cr} is the elastic-plastic moment of resistance of the tension zone of the prestressed section; it has the same dimension as the elastic moment of resistance, cm^3.

Before the advent of cracking, the depth of the compression zone is deduced from the equation of equilibrium for the external force, N, and the internal forces in the steel and concrete

$$\begin{aligned} & \pm N + R_{ten\,II}b\,(h-x) + R_{ten\,II}F_{ov} \\ & + (\sigma_0 + 2nR_{ten\,II})\,F_{pr} + [\sigma'_0 - 2nR_{ten\,II} \\ & (x - a')/(h-x)]\,F'_{pr} - R_{ten\,II}bx^2/(h-x) \\ & -2R_{ten\,II}\,(x - a'_{ov})\,F'_{ov}/(h-x) = 0 \end{aligned} \tag{VII.12}$$

In the above equation, the "plus" sign is taken if N is compressive, and the "minus" sign, if N is tensile. For members in bending, $N = 0$.

Equation (VII.12) is linear in x, so after multiplication by $(h - x)$ and transformation, we get the relative depth of the compression

zone

$$\xi = x/h = 1 - [bh + 2(1-\delta'_{ov})F'_{ov} + 2(1-\delta')nF'_{pr}]/[2F_{tr} - F_{ov} + (N_0 \pm N)/R_{ten\,II}] \qquad \text{(VII.13)}$$

Here, $\delta'_{ov} = a'_{ov}/h$; $\delta' = a'/h$; and F_{tr} is the transformed area.

$$F_{tr} = bh + F_{ov} + F'_{ov} + n(F_{pr} + F'_{pr}) \qquad \text{(VII.14)}$$

$$N_0 = \sigma_0 F_{pr} + \sigma'_0 F'_{pr} \qquad \text{(VII.15)}$$

It should be mentioned that in prestressed members the depth of the compression zone prior to cracking is greater than in nonprestressed members, being as large as $x = \xi h = (0.7 \text{ to } 0.9)\, h$.

Formula (VII.11) is common to the crack resistance analysis of prestressed and nonprestressed members and also concrete members of various cross sections, such as I-, T-section and rectangular members. For example, for an I-section nonprestressed member in bending, that is, with $N_0 = 0$, the elastic-plastic moment of resistance of the tension zone is

$$W_{cr} = b(h-x)(h/2 + x/6) + F_{ov}(h - a_{ov} - x/3) + 2F'_{ov}(x - a'_{ov})(x/3 - a'_{ov})/(h-x) + 2nF_s(h_0 - x/3) + 2nF'_s(x - a')(x/3 - a')/(h-x) \qquad \text{(VII.16)}$$

and the relative depth of the compression zone, according to Eq. (VII.13), is

$$\xi = 1 - [bh + 2(1-\delta'_{ov})F'_{ov} + 2(1-\delta') \times nF'_{pr}]/(2F_{tr} - F_{ov}) \qquad \text{(VII.17)}$$

When determining W_{cr} for a T-section with its flange in the compression zone, F_{ov} should be taken as zero, for a T-section with its flange in the tension zone, $F'_{ov} = 0$, and for a rectangular section, $F_{ov} = F'_{ov} = 0$. For a rectangular reinforced concrete member with tensile steel only

$$W_{cr} = b(h-x)(h/2 + x/6) + 2nF_s(h_0 - x/3) \qquad \text{(VII.18)}$$

$$\xi = x/h = 1 - bh/2(bh + nF_s) = 1 - 1/2(1 + n\mu_1) \qquad \text{(VII.19)}$$

where $\mu_1 = F_s/bh$.

If we assume that $F_s = F'_s = 0$, the elastic-plastic moment of resistance for, say, a rectangular concrete section with $\xi_c = 1/2$ is defined as

$$W_{c,cr} = 7bh^2/24 \qquad \text{(VII.20)}$$

that is, it is 7/4 times the elastic moment of resistance.

When determining the crack-inducing moment for nonprestressed members, we may assume that $\xi = 1/2$; then, at $\delta_1 = a/h = $ (ap-

prox.) 0.08, Eq. (VII.16) takes the following form

$$W_{cr} = [0.292 + 0.75(\gamma_1 + 2\mu_1 n) + 0.15\gamma_1'] bh^2 \qquad \text{(VII.21)}$$

where

$$\gamma_1 = (b_f - b) h_f'/bh, \quad \gamma_1' = [(b_f' - b) h_f' + hF_s']/bh \qquad \text{(VII.22)}$$

With $\mu_1 n \leqslant 0.25$ and $\gamma_1' \leqslant 0.3$, Eq. (VII.21) gives W_{cr} slightly in error.

3. Determining M_{cr} with the Concrete of the Compression Zone Behaving Inelastically

In some prestressed members (such as T-sections with their flanges in the tension zone, and eccentrically compressed sections), a high stress-strength ratio in the concrete of the compression zone induces nonlinear creep strain prior to cracking. Because we have assumed that sections plane before bending remain plane after bending, there appear constraints which prevent nonelastic strains from developing nonuniformly over the depth of the section. As a result, restrained creep is accompanied by the relaxation of stress. The normal stress distribution diagram becomes curved, and the peak stress ordinate moves towards the centre of the section (see Fig. II.2). All this reduces the value of M_{cr}. As has been shown by experiments, the concrete of the compression zone may behave inelastically and M_{cr} may decrease by about 20% already at a medium stress-strength ratio, but with long-time loading.

The value of M_{cr} with allowance for nonlinear creep in the concrete and duration of loading may quite rigorously be determined by a computer, using a discrete physical model consisting of a number of rods subjected to axial compression and axial tension. In the practical design calculation of M_{cr}, the curved normal stress distribution diagram in the concrete of the compression zone is replaced by a rectangular or trapezoidal diagram. Let us examine one of such practical methods using a rectangular normal stress distribution diagram (Fig. VII.2).

We shall assume that the concrete in the compression zone of the section behaves inelastically if the stress calculated for a triangular stress diagram by Eq. (VII.7) is

$$\sigma_{com} \geqslant 0.7R_{prII} \qquad \text{(VII.23)}$$

In this case, the curved normal stress distribution diagram is replaced by a rectangular diagram in either zone of the section where the elastic strain factor is

$$\nu = \nu_{ten} = 0.5 \qquad \text{(VII.24)}$$

Then the stress in the concrete of the compression zone is

$$\sigma_{com} = \varepsilon_{c,ten} x \nu E_c/(h - x) = R_{tenII} x/(h - x) \qquad \text{(VII.25)}$$

Recalling that the compressive force in the web, $N_{c,w}$, is applied at $x/2$ from the edge of the section, we may write for the internal

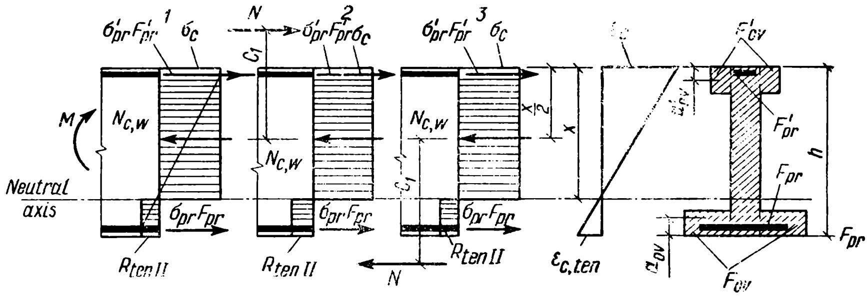


Fig. VII.2. To determining the cracking resistance of prestressed members in (*1*) bending, (*2*) eccentrical compression and (*3*) eccentrical tension in terms of Stage I with the concrete of the compression zone behaving inelastically

moment

$$M_{cr} = R_{tenII} \{bh (h - x)/2 + F_{ov}(h - a_{ov} - x/2) + F'_o x (x/2 - a'_{ov})/(h - x) + F_{pr} (2n + \sigma/R_{tenII}) (h_0 - x/2) - F'_{pr} [\sigma'_0/R_{tenII} - 2nx/(h - x)] (\times x/2 - a')\} \quad \text{(VII.26)}$$

Prior to cracking, the depth of the compression zone is deduced by equating the external force, N, to the forces in the steel and concrete

$$\pm N + R_{tenII} b (h - x) + R_{tenII} F_{ov} + (\sigma'_0 + 2nR_{tenII}) F_{pr} + [\sigma'_0 - 2nR_{tenII} x/(h - x)] F'_{pr} - R_{tenII} x (bx + F'_{ov})/(h - x) = 0 \quad \text{(VII.27)}$$

Here, N is taken with the "plus" sign in compression, and with the "minus" sign in tension; in bending, $N = 0$.

The relative depth of the compression zone is

$$\xi = x/h = 1 - (bh + F'_{ov} + 2nF'_{tr})/[F_{tr} + n(F_{pv} + F_{pr}) + bh + (N_0 \pm N)/R_{ten\,II}] \quad \text{(VII.28)}$$

4. Determining M_{cr} by the Kern Moment Method

Relevant standards recommend to determine M_{cr} approximately by what is known as the kern moment method. The stress-strain state of a section in Stage I before cracking due to the composite action of external loads and prestressing force may be approximated to linear eccentrical compression, on the assumption that all forces act independently. Then, the cracking moment is defined as

$$M_{cr} = R_{ten\,II} W_{cr} + M^k_{pr} \qquad \text{(VII.29)}$$

where M^k_{pr} is the moment due to the prestressing force, N_0, taken about the axis passing through the kern point most distant from the tension zone, that is,

$$M^k_{pr} = N_0 (e_{0pr} + r_k) \qquad \text{(VII.30)}$$

W_{cr} is the elastic-plactic moment of resistance of the tension zone in the reinforced concrete section with zero longitudinal force;

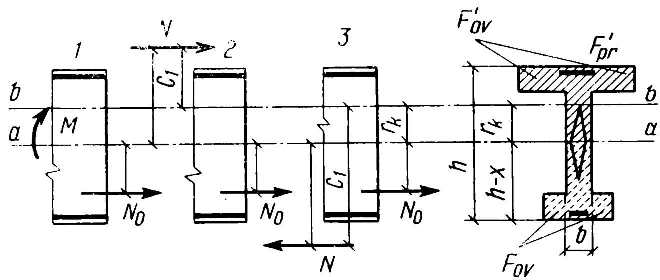


Fig. VII.3. To the cracking-resistance analysis of members in (*1*) bending, (*2*) eccentrical compression and (*3*) eccentrical tension by the kern moment method

a—centroid of the transformed section; *b*—boundary of the kern

e_{0pr} is the eccentricity of the prestressing force, N_0, with respect to the centroid of the transformed section; r_k is the distance from the kern point farthest from the tension zone to the centroid of the transformed section (Fig. VII.3). To allow for inelastic strain in the concrete of the compression zone, the value of r_k is found as follows:

— for prestressed members in bending and eccentrical compression

$$r_k = 0.8 W_0 / F_{tr} \qquad \text{(VII.31)}$$

— for members in eccentrical tension

$$r_k = W_{cr} / [F_{tr} + n (F_{pr} + F'_{pr})] \qquad \text{(VII.32)}$$

provided that

$$e_0 - e_{0pr} \leqslant R_{ten\,II} W_{cr}/N_0 \tag{VII.33}$$

— for nonprestressed members in bending and eccentrical tension

$$r_k = W_0/F_{tr} \tag{VII.34}$$

if the condition (VII.33) is not met, where W_0 is the elastic moment of resistance of the tension zone in the transformed section; F_{tr} is the cross-sectional area of the transformed section; e_0 is the eccentricity of the longitudinal force, N, with respect to the centroid of the transformed section. The value of W_{cr} may be determined by Eqs. (VII.16) and (VII.21), or by the following formula

$$W_{cr} = 2\,(I_{com} + nI_s)/(h - x) + S_{ten} \tag{VII.35}$$

Here, I_{com} and I_s are the moments of inertia of the concrete in compression and the steel in compression and tension about the neutral axis, respectively; S_{ten} is the static moment of the concrete in tension about the same axis; $h - x$ is the distance from the neutral axis to the tensile face. The position of the neutral axis is located from the following condition

$$S_{com} + nS_s = (h - x)\,F_{ten}/2 \tag{VII.36}$$

where S_{com} and S_s are the static moments of the concrete in compression and the steel in compression and tension about the neutral axis, respectively; F_{ten} is the cross-sectional area of the concrete in tension.

The value of W_{cr} may also be determined in terms of the elastic moment of resistance, W_0, using the following formula

$$W_{cr} = \gamma W_0 \tag{VII.37}$$

Here, the factor γ takes care of inelastic strain in the concrete of the tension zone according to the shape of the cross section. For rectangular sections and T-sections with their flanges in the compression zone, $\gamma = 1.75$; for other cross sections the values of γ are given in Appendix X.

The external moment about the axis passing through an assumed kern point is defined by Eq. (VII.4). In the case of eccentrical compression, the lever arm is $c_1 = e_0 - r_k$. Then,

$$M_{ext}^{k} = N\,(e_0 - r_k) \tag{VII.38}$$

For eccentrical tension,

$$M_{ext}^{k} = N\,(e_0 + r_k) \tag{VII.38a}$$

For bending,

$$M_{ext}^{k} = M \tag{VII.38b}$$

In manufacture and erection it may so happen that a zone designed to work in compression under external loads is actually in tension. In such a case,

$$M_{cr} = R_{tenII} W_{cr} - N_0(e_{0pr} - r_k) \tag{VII.39}$$

Here, the values of W_{cr} and r_k are taken for the face subjected to tension due to the prestressing force, N_0; and R_{tenII} is determined depending on the transfer strength of concrete, R_0. In these calculations, the external moment is found for the loads acting in a particular stage (for example, the self-weight of the member).

5. Inclined Cracking Analysis

Members are checked for inclined crack resistance in the zone of principal tensile stresses. This check is carried out at several points along the length of a member, according to variations in the shape of the cross section, shearing force diagram and bending moment diagram. Along the depth of the cross section, members should be checked at the centroid of the transformed section and at the points where the width of the section suddenly changes or where compressed flanges adjoin the rib of a T-section. In structures reinforced with prestressed steel without anchors, the check is applied to the end portions within the transmission length, l_{trans}, with allowance for a reduction in the prestress σ_0, for which purpose, the latter is multiplied by the service factor, m_{s3} (see Chapter II).

In crack resistance analysis, account must be taken not only of principal tensile stresses, but also of principal compressive stresses. Experiments with concrete test specimens have shown that in a biaxial state of stress, compression in one direction reduces the ability of the concrete to resist tension in the other direction. This is especially pronounced in concretes of brand number M-500 and higher.

An inclined section is said to have an adequate crack resistance, if the principal tensile stresses in it meet the following conditions:

$$\sigma_{prin,ten} \leqslant R_{tenII}$$

$$\text{at } \sigma_{prin,com} \leqslant mR_{prII} \tag{VII.40}$$

$$\sigma_{prin,ten} \leqslant nR_{tenII} \left(1 - \sigma_{prin,com}/R_{prII}\right)$$

$$\text{at } \sigma_{prin,com} > mR_{prII} \tag{VII.41}$$

The coefficients m and n depend on the type and brand number of concrete and are given in Table VII.1.

The principal tensile and compressive stresses are given by the following formula

$$\sigma_{\substack{prin,\ ten \\ prin,\ com}} = (\sigma_x + \sigma_y)/2 \pm \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2} \tag{VII.42}$$

TABLE VII.1. Coefficients m and n

Concrete		Coefficient	
heavy	porous-aggregate	m	n
M-400 and lower	M-200 and lower	0.5	2
M-500	M-250	0.375	1.6
M-600	M-300	0.25	1.33

where σ_x is the normal stress in the concrete induced by the external load and prestressing force N_0; σ_y is the compressive stress in the concrete parallel to the longitudinal axis of the member, induced by the support reaction, concentrated load, distributed load, and also prestressing force in the transverse reinforcement; τ_{xy} are the shearing stresses in the concrete induced by the external load and prestressing force in the curved tendons; the stresses σ_x and σ_y are taken with the "plus" sign in tension, and with the "minus" sign in compression.

The normal and shearing stresses are determined on the assumption that the concrete behaves elastically

$$\sigma_x = \pm My/I_{tr} \pm N/F_{tr} \pm N_0 e_{0pr} y/I_{tr} - N_0/F_{tr} \qquad \text{(VII.43)}$$

where N is taken with the "plus" sign in tension, and with the "minus" sign in compression;

$$\sigma_y = \sigma_{y0} + \sigma_{y,loc} \qquad \text{(VII.44)}$$

where σ_{y0} is the stress in the concrete due to the prestressed transverse reinforcement and curved tendons;

$$\sigma_{y0} = \sigma_{0tr} F_{pr,tr}/ub + \sigma_0 F_{pr,curv} \sin\alpha / u_{curv} b \qquad \text{(VII.45)}$$

where $F_{pr,tr}$ is the cross-sectional area of the prestressed stirrups lying in the same plane normal to the longitudinal axis of the member in the portion in question; $F_{pr,curv}$ is the cross-sectional area of the prestressed curved tendons in the portion $u_{curv} = h/2$ long symmetric about the section in question (section 0-0 in Fig. VII.4); σ_{0tr} is the prestress in the stirrups with allowance for all losses; u is the spacing between the stirrups; u_{curv} is the distance between the planes of the curved tendons measured along the normal to them; b is the width of the member at the section in question; σ_0 is the prestress in the curved tendons with allowance for all losses; α is the angle between the longitudinal axis of the member and the tangent to the axis of the prestressed steel at section 0-0; $\sigma_{y,loc}$ is the stress in the concrete due to local compression near the points of application of the support reactions and concentrated loads at

the top face of the beam (Fig. VII.5); if $y \leqslant 0.4h$ and $x \leqslant 2.5h$,

$$\sigma_{y,loc} = -0.4P\,(h/y - 1)\,(1 - 0.4x/y)/bh \qquad \text{(VII.46)}$$

if $y > 0.4h$ and $x \leqslant h$,

$$\sigma_{y,loc} = -P\,(1 - y/h)\,(1 - x/y)/bh \qquad \text{(VII.47)}$$

where x and y are the distances (parallel and normal to the longitudinal axis, respectively) from the point of application of the concentrated load to the point in question; P is the concentrated load

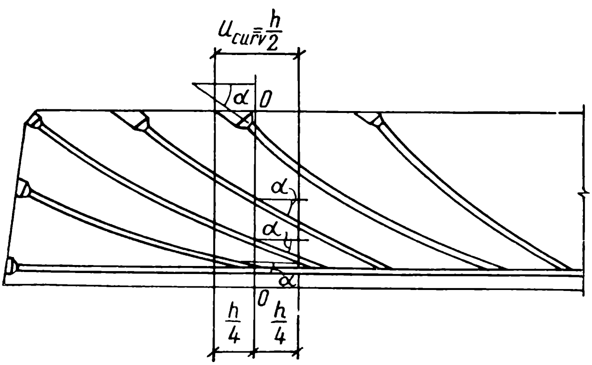


Fig. VII.4. Prestressed curved tendons

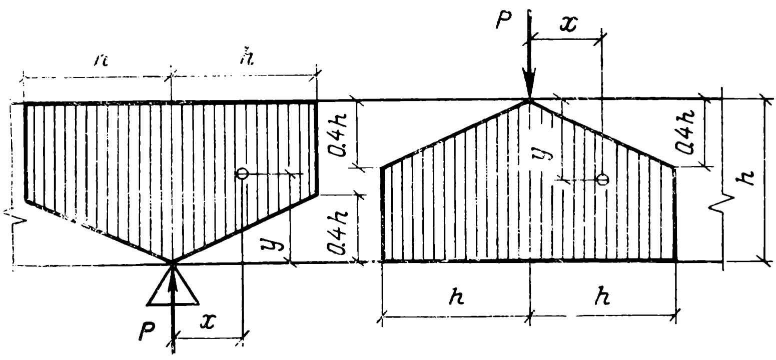


Fig. VII.5. Stress in the concrete induced by local compression

or support reaction; Q is the shearing force due to the external load; S is the static moment of that part of the section which is in shear about the centroid of the transformed section; N_0 is the prestressing force in the curved tendons which end at the support or between the support and the section located within $h/4$ of the section in question (section 0-0 in Fig. VII.4);

$$\tau_{xy} = (Q - \Sigma N_0 \sin\alpha)\,S/bI_{tr} \qquad \text{(VII.48)}$$

VII.3. CRACK-OPENING RESISTANCE. GENERAL

1. Width of Normal Cracks

After the onset of cracking in the tension zones of reinforced concrete members, further loading causes the cracks to open. This is Stage II of the stress-strain state. Experiments show that in tension the spacing between cracks may be larger or smaller than the average value by about 50% due to the nonuniform inner structure of concrete.

The width of cracks normal to the longitudinal axis of a member is defined as the difference between the elongations in the steel and tensile concrete over the spacing l_{cr} between the cracks

$$a_{cr} = \varepsilon_{s,av} l_{cr} - \varepsilon_{c,ten,av} l_{cr}$$

The average strain of the tensile concrete, $\varepsilon_{c,ten,av}$, is much smaller than the average strain of the tensile steel, $\varepsilon_{s,av}$, so it is usually ignored. As a result,

$$a_{cr} = \varepsilon_{s,av} l_{cr}$$

Let us designate the ratio of the average strain of the tensile steel between the cracks to the strain of the steel at the section containing that crack as ψ_s

$$\psi_s = \varepsilon_{s,av}/\varepsilon_s \leqslant 1 \qquad \text{(VII.49)}$$

Then, the crack width at the level of the tensile steel axis will be

$$a_{cr} = \psi_s \varepsilon_s l_{cr} = \psi_s \sigma_s l_{cr}/E_s \qquad \text{(VII.50)}$$

The crack width depends on ψ_s (which, in turn, depends on the bond between the steel and the concrete), the stress in the steel at the section containing the crack, σ_s, and the crack spacing, l_{cr}. The values of the above factors are found by calculation.

According to relevant standards, the width of cracks normal to the longitudinal axis of a member should be determined at the level of the tensile steel axis by the following empirical formula

$$a_{cr} = 20\,(3.5 - 100\mu)\, k\eta c_d \sigma_s \sqrt[3]{d}/E_s \qquad \text{(VII.51)}$$

where $\mu = F_s/bh_0$ is the reinforcement ratio of the section (the rib of a T-section) taken as not more than 0.02 in the design calculations; F_s is the cross-sectional area of the tensile steel; k is taken as 1 for members in bending and eccentrical compression, and 1.2 for members in eccentrical tension; η is taken as 1 for deformed bars, 1.2 for class Bp-I, Bp-II wire and class K-7 strands, 1.3 for hot-rolled plain bars, and 1.4 for class B-I and B-II wire; c_d is the factor taking care of the duration of loading and assumed as 1 for short-time loading and 1.5 for long-time loading; σ_s is the stress or the increment in

the stress after the prestress in the tensile steel at the section containing the crack has been cancelled out; d is the tensile steel bar diameter in mm; if the bars differ in diameter, use should be made of an average bar diameter.

The short-time crack width in members whose crack resistance should meet the requirements of Category Two is determined for the short-time action of all loads. For members whose crack resistance should be in compliance with the requirements of Category Three, it is determined, using a nonlinear relationship, as the sum of the increment in the crack width ($a_{cr1} - a_{cr2}$) caused by a short-time increase in load up to the total load value, and the crack width (a_{cr3}) due to dead and long-time live loads

$$a_{cr} = a_{cr1} - a_{cr2} + a_{cr3} \qquad \text{(VII.52)}$$

For the limiting crack width and the effect of the duration of loading, see Chapter II.

2. Inclined Crack Width

In members subjected to bending, the inclined crack width is determined by the following empirical formula

$$a_{cr} = (h_0 + 30d_{max})\, k\eta c_d t^2/\mu_{tr}E_s^2 \qquad \text{(VII.53)}$$

where $t = Q/bh_0 - 0.25N_0/F$ is the spalling stress in the concrete due to the shearing force, Q, and the prestressing force, N_0; $\mu_{tr} = \mu_{st} + \mu_b$ is the transverse reinforcement ratio; $\mu_{st} = F_{st}/bu$ is the stirrup (transverse bar) reinforcement ratio; $\mu_b = F_b/bu_b$ is the bent bar reinforcement ratio; $k = (20\text{-}1\,200\mu_{tr})\,10^3$ but not less than 8×10^3; d_{max} is the maximum transverse and diagonal bar diameter; η and c_d have the same meaning as with normal cracks.

Crack-opening analysis is applied to sections within h_0 or more of the support.

VII.4. CRACK-OPENING RESISTANCE OF MEMBERS IN AXIAL TENSION

1. Coefficient ψ_s

The strain and stress in the tensile steel are nonuniformly distributed between cracks. At a section containing a crack, the strain is equal to ε_s, and the stress to σ_s. As the distance from the crack edges increases, the stress in the steel decreases, and that in the concrete increases owing to the bond between the concrete and steel (Fig. VII.6). The average strain, $\varepsilon_{s,av}$, becomes smaller than ε_s, and

the average stress, $\sigma_{s,av}$, smaller than σ_s. In design, the fact that the concrete between the cracks is in tension and that the respective strain and stress in the steel are nonuniform is taken care of by the coefficient ψ_s

$$\varepsilon_{s,av} = \psi_s\varepsilon_s \quad \text{and} \quad \sigma_{s,av} = \psi_s\sigma_s \tag{VII.54}$$

The stress-strain curve plotted for reinforcement in bond with concrete in tension markedly differs from that for free steel (Fig.

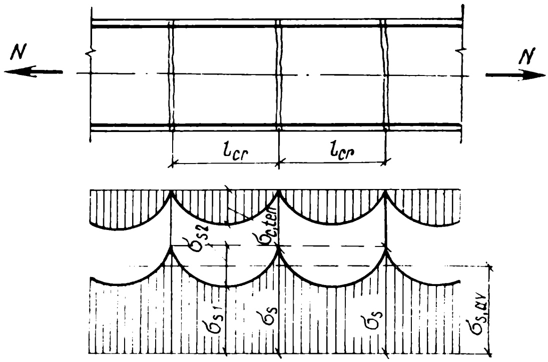


Fig. VII.6. To determining ψ_s in the case of axial tension

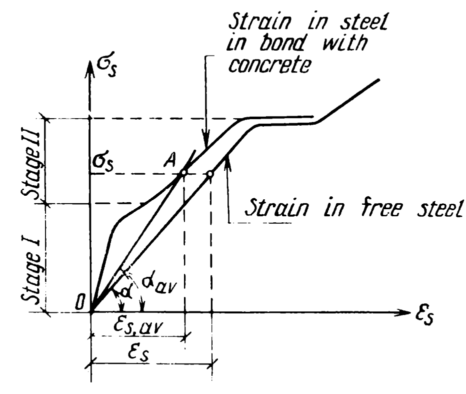


Fig. VII.7. Stress-strain diagram for tensile steel

VII.7). Reinforcing steel bonded to concrete has a higher modulus of elasticity whose average value is defined as the slope of the chord at the point corresponding to the specified stress

$$E_{s,av} = \text{tg}\ \alpha_{ch} = \sigma_s/\varepsilon_{s,av}$$
$$= \sigma_s/\psi_s\varepsilon_s = E_s/\psi_s \tag{VII.55}$$

Graphically, the coefficient ψ_s may be defined as the ratio between the area of the steel stress diagram over the length l_{cr} and the total area of the steel stress diagram with the ordinate σ_s (see Fig. VII.6), that is,

$$\begin{aligned}\psi_s &= (\sigma_s l_{cr} - \omega_{ten}\sigma_{s2} l_{cr})/\sigma_s l_{cr} \\ &= 1 - \omega_{ten}\sigma_{s2}/\sigma_s\end{aligned} \qquad \text{(VII.56)}$$

Here, σ_{s2} is the reduction in stress in the steel due to bond and the effect of concrete working in tension between the cracks; and ω_{ten} is the factor taking care of the shape of the steel stress diagram over the length l_{cr}.

If we assume that the concrete at a section between the cracks carries a tensile force equal to $\chi N_{c,cr}$, where

$$N_{c,cr} = R_{tenII} F \qquad \text{(VII.57)}$$

then the ratio σ_{s2}/σ_s may be found by assuming that the forces acting at the crack and between the cracks are the same (and equal to N), that is,

$$N = \sigma_s F_s = (\sigma_s - \sigma_{s2}) F_s + \chi N_{c,cr}$$

Hence,

$$\sigma_{s2} = \chi N_{c,cr}/F_s \qquad \text{(VII.58)}$$

As a consequence, the stress ratio is

$$\sigma_{s2}/\sigma_s = \chi N_{c,cr} F_s/F_s N = \chi N_{c,cr}/N \qquad \text{(VII.59)}$$

Substituting the ratio σ_{s2}/σ_s into Eq. (VII.56) gives

$$\psi_s = 1 - \omega_{ten}\chi N_{c,cr}/N \qquad \text{(VII.60)}$$

On the basis of experimental data, the product $\omega_{ten}\chi$ is taken as 0.7 for short-time loading, and 0.35 for long-time loading. Thus, for short-time loading

$$\psi_s = 1 - 0.7 N_{c,cr}/N \qquad \text{(VII.61)}$$

and for long-time loading

$$\psi_s = 1 - 0.35 N_{c,cr}/N \qquad \text{(VII.62)}$$

In prestressed members, the concrete begins to work in tension only after the applied force, N, has exceeded the prestressing force, N_0, so, in such members, ψ_s is

$$\psi_s = 1 - 0.7 N_{c,cr}/(N - N_0) \qquad \text{(VII.63)}$$

$$\psi_s = 1 - 0.35 N_{c,cr}/(N - N_0) \qquad \text{(VII.64)}$$

If the ratios

$$N_{c,cr}/N > 1 \quad \text{or} \quad N_{c,cr}/(N - N_0) > 1$$

then in design calculations they are taken equal to unity.

2. Stress in Tensile Steel

After the force due to the external load, N, has exceeded the prestressing force, N_0, the increment in the stress in the tensile steel at a crack is

$$\sigma_s = (N - N_0)/F_{pr} \tag{VII.65}$$

and the stress in the steel of a nonprestressed member at a crack is

$$\sigma_s = N/F_s \tag{VII.66}$$

These values of σ_s are substituted into the design formulas when determining the crack width.

3. Crack Spacing

Because the strength of a member is not the same along its length, the first cracks form at the weakest points (Fig. VII.8). On moving

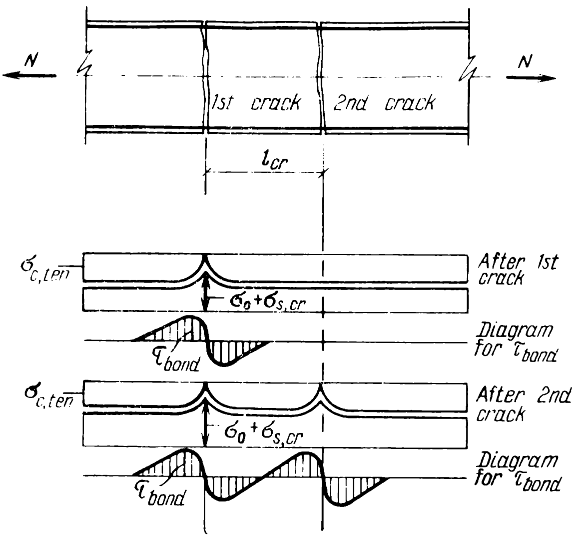


Fig. VII.8. Stress distribution after the advent of cracking in a member subjected to axial tension

away from the edges of the crack, the tensile stress in the concrete grows, and another crack appears at the point where it reaches R_{tenII}. The spacing between these cracks will be l_{cr}.

After the prestress in the concrete has been cancelled out, that is, immediately after the first crack has formed, the increment in the tensile steel stress, $\sigma_{s,cr}$, is induced by the transfer of an additional force from the cracked concrete to the steel. Because the

tensile force in a section is the same ($N = N_{cr}$) in Stage I and Stage II, we may, on recalling Eqs. (VII.2) and (VII.65), write that

$$\sigma_{s,cr} = (N - N_0)/F_{pr}$$

$$= R_{tenII}F/F_{pr} + 2nR_{tenII} \quad \text{(VII.67)}$$

The crack spacing, l_{cr}, can be found by equating the difference in stress between the tensile steel at the cracks and between the cracks to the bond between the steel and the concrete. Then,

$$(\sigma_0 + \sigma_{s,cr}) F_{pr} - (\sigma_0 + 2nR_{tenII}) F_{pr}$$

$$= \tau_{bond} s l_{cr} \omega \quad \text{(VII.68)}$$

where τ_{bond} is the maximum bond stress between the steel and the concrete; s is the perimeter of the steel cross section; and ω is the factor taking care of the shape of the bond stress diagram.

Substituting $\sigma_{s,cr}$ as defined by Eq. (VII.67) into Eq. (VII.68) gives

$$R_{tenII}F = \tau_{bond} s l_{cr} \omega$$

Hence, the crack spacing is

$$l_{cr} = R_{tenII}F_c/\tau_{bond}s\omega \quad \text{(VII.69)}$$

On designating

$$R_{tenII}/\tau_{bond}\omega = \eta_{sh}; \quad F_{pr}/s = u;$$

$$F_{pr}/F_c = \mu_1$$

we finally get

$$l_{cr} = u\eta_{sh}/\mu_1 \quad \text{(VII.70)}$$

According to experimental data, the factor η_{sh} taking care of the type and shape of the steel is assumed as 0.7 for deformed bars, 0.9 for class Bp-I and Bp-II deformed wire and wire strands, 1 for plain bars, and 1.25 for class B-I and B-II plain wire. In nonprestressed members, the crack spacing, l_{cr}, is defined by the same Eq. (VII.70), but the cross-sectional area of the prestressed steel F_{pr} is replaced by F_s.

VII.5. CRACK-OPENING RESISTANCE OF MEMBERS IN BENDING, ECCENTRICAL COMPRESSION AND ECCENTRICAL TENSION

1. Coefficient ψ_s

After the advent of cracking, members in bending, eccentrical compression and eccentrical tension whose sections are simultaneously subjected to tension and compression are in Stage II of the stress-strain state (Fig. VII.9). Let us designate the total force due to the external load and prestress as

$$N_{tot} = \pm N + N_0 \qquad \text{(VII.71)}$$

where N is taken with the "minus" sign in the case of eccentrical tension.

To get a better insight into the shape of the stress distribution diagram in eccentrical tension, we determine $e_{0,tot}$, that is, the

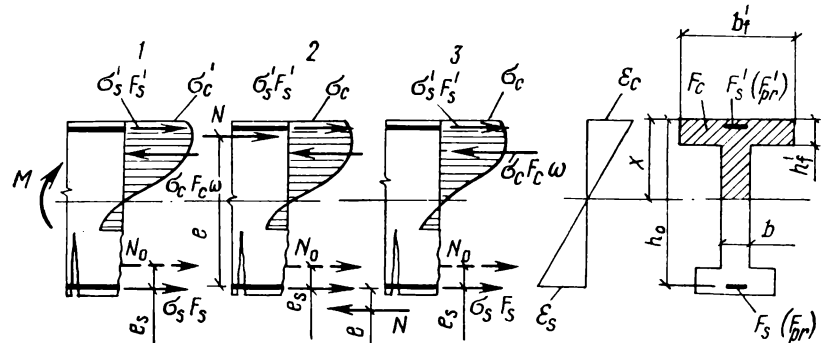


Fig. VII.9. Stress distribution after the advent of cracking in members
(1) bending, (2) eccentrical compression; and (3) eccentrical tension

distance from the centroid of the transformed section to the point of application of the total force, N_{tot}. If $e_{0,tot} \geqslant 0.8h_0$, the section is partly in compression and partly in tension; if $e_{0,tot} < 0.8h_0$, the section is entirely in tension.

At cracks, the depth of the compression zone decreases, whereas between cracks it increases. As a result, the neutral axis undulates along the length of a member (Fig. VII.10). In the zone of pure bending and in the zone of maximum bending moments in single-span beams subjected to a distributed load, cracks are spaced approximately equally apart. In other zones, shearing forces have a certain effect on the crack spacing. As in axial tension, the strain and stress

in the tensile steel are nonuniform between cracks. As the distance from the crack edges increases, the stress in the steel falls, and that in the concrete rises.

By analogy with members in axial tension [see Eq. (VII.60)], the value of ψ_s for members in bending may be found by assuming

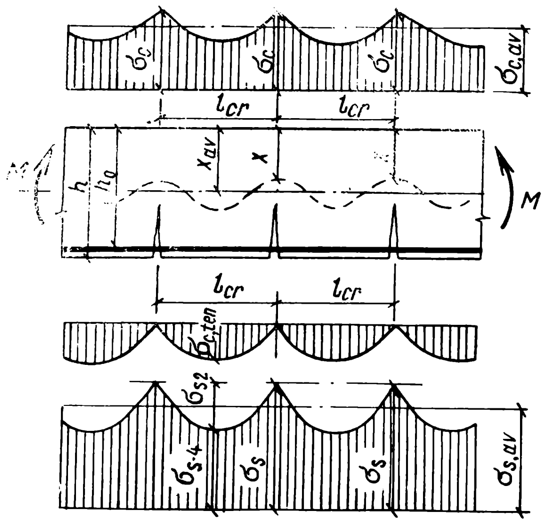


Fig. VII.10. Neutral axis of a cracked bending reinforced concrete member

that the external bending moments at the crack and between cracks are the same (and equal to M)

$$\psi_s = 1 - \omega_{ten}\chi M_{c,cr}/M \tag{VII.72}$$

where $M_{c,cr} = R_{ten\,II} W_{c,cr}$ is the moment taken up by a plain concrete section prior to cracking.

In prestressed members in bending, the concrete begins to work in tension only after the external moment, M, has exceeded the prestressing moment, M^k_{pr}. So,

$$\psi_s = 1 - \omega_{ten}\chi M_{c,cr}/(M - M^k_{pr}) \tag{VII.73}$$

Experiments have shown that the product $\omega_{ten}\chi$ may be taken as 0.8 for short-time loading, and 0.4 for long-time loading. The coefficient ψ_s may range from 0.3 or 0.5 to about 1. It has also been proved that the creep of the concrete in the tension zone raises the value of ψ_s. If a member is subjected to repeated and dynamic loading, ψ_s tends to unity.

According to appropriate standards, ψ_s for bending and eccentrically loaded members should be determined by the following empirical formula

$$\psi_s = 1.25 - sm - (1 - m^2)/(3.5 - 1.8m)\, e_{s,tot}/h_0 \tag{VII.74}$$

but in design calculations it is assumed to be not more than 1. Here, s is the factor taking care of the duration of loading and type of steel and assumed to be 1.1 for deformed bars, 1 for plain bars and wire under short-time loading, and 0.8 for long-time loading irrespective of the steel type; $e_{s,tot}$ is the distance from the centroid of the tensile steel to the point of application of the total force, N_{tot};

$$m = R_{ten\,II} W_{cr}/(M_{ext}^{h} - M_{pr}^{h}) \leqslant 1 \tag{VII.75}$$

and W_{cr} is as defined by Eq. (VII.37).

For nonprestressed members in bending, the last term on the right-hand side of Eq. (VII.74) is taken as zero, so

$$\psi_s = 1.25 - sR_{ten\,II} W_{cr}/M \tag{VII.76}$$

but not more than 1.

2. Coefficient ψ_c

In Stage II, the marginal strain in the concrete of the compression zone is also nonuniformly distributed along a member, being a maximum at a crack, and decreasing on moving away from crack edges. The fact that the marginal strain in the concrete of the compression zone is nonuniformly distributed along a member is reflected by the coefficient ψ_c which is defined as the ratio of the average strain, $\varepsilon_{c,av}$, to the strain at a crack, ε_c

$$\psi_c = \varepsilon_{c,av}/\varepsilon_c = \sigma_{c,av}/\sigma_c \leqslant 1 \tag{VII.77}$$

Experiments have shown that ψ_c may range between 0.75 and 1. Relevant standards recommend that ψ_c should approximately be taken as 0.9 for all cases under long- and short-time loading.

3. Stresses in Concrete and Steel at Cracks

Let us examine an I-section member in bending after the onset of cracking (Fig. VII.11). The concrete of the tensile flange at a crack carries no load. We shall start with an analysis of the stress-strain state in the absence of prestress, based on the following assumptions:

(1) in the zone of pure bending, the inner sections between cracks, subjected to symmetric forces from the left and right, remain plane after bending;

(2) the depth of the compression zone at a crack, x, is empirically related to the average depth of the compression zone as

$$\varphi = x/x_{av} = 1 - 0.7/(100\mu + 1) \tag{VII.78}$$

(3) the concrete of the tensile zone above a crack is not taken into account in design calculations; the effect of this concrete is sometimes significant, but it has yet to be evaluated.

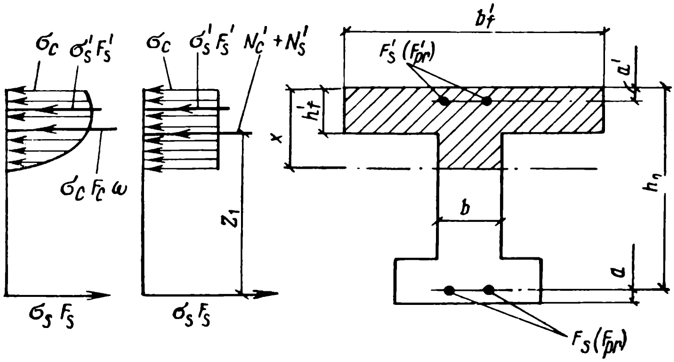


Fig. VII.11. To determining the stresses in the concrete and steel of an I-section member with the compressed flange

Now, we shall express the stress in the concrete and steel of the tensile zone at a crack in terms of the stress in the tensile steel, σ_s, and find the depth of the compression zone. The strain in the concrete at the section edge is

$$\varepsilon_c = \varepsilon_{c,av}/\psi_c = x_{av}\varepsilon_{s,av}/(h_0 - x_{av})\,\psi_c$$
$$= x\psi_s\varepsilon_s/(\varphi h_0 - x)\,\psi_c \tag{VII.79}$$

the stress in the concrete at the section edge is

$$\sigma_c = E'_c\varepsilon_c = x\nu\psi_s\sigma_s/(\varphi h_0 - x)\,n\psi_c \tag{VII.80}$$

and the stress in the compressive steel within a' of the section edge is

$$\sigma'_s = E_s\varepsilon_c\,(x_{av} - a')\,/x_{av}$$
$$= (x - \varphi a')\,\psi_s\sigma_s/(\varphi h_0 - x)\,\psi_c \tag{VII.81}$$

The equilibrium between the internal forces at a crack is described by the following equation

$$\sigma_s F_s - \sigma_c F_c \omega - \sigma'_s F'_s = 0 \tag{VII.82}$$

where

$$F_c = bx + (b'_f - b)\,h'_f \tag{VII.83}$$

ω is the factor taking care of the shape of the stress diagram in the concrete of the compression zone; b'_f is the width of the compressed flange (described in Chapter III).

Substituting the values of σ_c and σ'_s found by Eqs. (VII.80) and (VII.81) into Eq. (VII.82) gives

$$\sigma_s F_s - x\omega\nu\psi_s\sigma_s F_c/(\varphi h_0 - x)\, n\psi_c - (x - \varphi a')\, \psi_s\sigma_s F'_s/(\varphi h_0 - x)\, \psi_c = 0 \tag{VII.84}$$

After multiplication by $(\varphi h_0 - x)/\sigma_s b h_0$, substituting F_c and transformation, Eq. (VII.84) reduces to a quadratic equation for the depth of the compression zone

$$x^2 + (\alpha + \gamma')\, h_0 x - (1 + \mu' a'/\mu h_0)\, \alpha\varphi h_0^2 = 0 \tag{VII.85}$$

where

$$\alpha = \mu n\psi_c/\omega\nu\psi_s \tag{VII.86}$$

$$\gamma' = [(b'_f - b)\, h'_f + nF'_s/\nu]/bh_0 \tag{VII.87}$$

Dividing Eq. (VII.85) through by h_0^2 and neglecting $\mu' a'/\mu h_0$ in the absolute term in comparison with unity, we get

$$\xi^2 - (\alpha + \gamma')\, \xi - \alpha\varphi = 0 \tag{VII.88}$$

Hence, the relative depth of the compression zone at a crack is

$$\xi = x/h_0 = -(\alpha + \gamma')/2 + \sqrt{(\alpha + \gamma')^2/4 + \alpha\varphi} \tag{VII.89}$$

If the depth of the compression zone turns out to be less than h'_f, it should be found again as for a rectangular section of width b'_f.

It should be mentioned that under short-time loading the product $\omega\nu$ in Eq. (VII.86) is insignificantly affected by the shape of the normal stress diagram in the concrete of the compression zone. For example, with a rectangular diagram, $\omega = 1$, and, since such a diagram is due to nonelastic strain, $\nu = 0.5$; so, $\omega\nu = 0.5$. With a triangular diagram and a rectangular section, $\omega = 0.5$ and $\nu = 1$; again we have $\omega\nu = 0.5$. In view of this, the depth of the compression zone is determined for a rectangular stress distribution diagram (see Fig. VII.11).

Creep in the concrete of the compression zone arising under long-time loading causes the neutral axis to move so that the depth of the compression zone increases.

Relevant standards recommend that the depth of the compression zone at a crack should be taken approximately the same for short- and long-time loading. For members (both prestressed and nonprestressed) in bending and eccentrical compression and tension, the

relative depth of the compression zone is empirically defined as

$$\xi = 1/\{1.8 + [1 + 5(L + T)]/10\mu n\} \pm (1.5 + \gamma')/(11.5e_{s,tot}/h_0 \mp 5) \quad \text{(VII.90)}$$

but not more than 1. For the second term on the right-hand side of Eq. (VII.90), the upper signs apply when N_{tot} is compressive, and the lower, when N_{tot} is tensile.

In Eq. (VII.90), γ' is determined by Eq. (VII.87); for prestressed members, F'_s is replaced by F'_{pr}; ν is taken for short-time loading (according to appropriate standards, $\nu = 0.45$);

$$T = \gamma' (1 - h'_f/2h_0) \quad \text{(VII.91)}$$

where, for rectangular sections, h'_f is replaced by $2a'$;

$$L = M_{eq}/bh_0^2 R_{tenII} \quad \text{(VII.92)}$$

where M_{eq} is the equivalent moment, that is, the moment about the centroid of the tensile steel due to the external forces and the prestressing force, N_0;

— for members in bending, it is defined as

$$M_{eq} = M + N_0 e_{s,pr} \quad \text{(VII.93)}$$

— for members in eccentrical compression and tension, it is defined as

$$M_{eq} = Ne + N_0 e_{s,pr} \quad \text{(VII.94)}$$

Here, $e_{s,pr}$ is the distance from the point of application of the prestressing force, N_0, to the centroid of the tensile steel; e is the distance from the point of application of the force induced by the external loads, N, to the centroid of the tensile steel.

The above formulas give an approximate depth of the compression zone at a crack; in most cases, however, this does not markedly affect the results of crack-width, curvature or sagging analyses.

For a T-section having a rectangular stress distribution diagram in the concrete of the compression zone, the arm of the internal couple, z_1, is equal to the distance from the force in the tensile steel to the resultant of the forces in the concrete and steel of the compression zone (see Fig. VII. 11). It may be defined as the ratio of the static moment of the transformed compression zone section, S_{tr}, about the centroid of the tensile steel to the transformed area

$$z_1 = S_{tr}/F_{tr} = [S_c + nF'_{tr}(h_0 - a')/\nu]/(\gamma' + \xi) bh_0$$

After transformation

$$z_1 = h_0 [1 - (h'_f\gamma'/h_0 + \xi^2)/2(\gamma' + \xi)] \quad \text{(VII.95)}$$

The stress in the concrete of the compression zone at a crack can be found by equating the equivalent moment and the internal moment about the centroid of the tensile steel

$$M_{eq} = \sigma_c (\gamma' + \xi)\, bh_0 z_1 \qquad \text{(VII.96)}$$

Hence,

$$\sigma_c = M_{eq}/(\gamma' + \xi)\, bh_0 z_1 = M_{eq}/W_{com} \qquad \text{(VII.97)}$$

The denominator in Eq. (VII.97) is the elastic-plastic moment of resistance of the compression zone after the advent of cracking; it is defined as

$$W_{com} = (\gamma' + \xi)\, bh_0 z_1 \qquad \text{(VII.98)}$$

After the external moment has exceeded the prestressing moment, the incremental stress in the tensile steel can be found from the equation for the moments at a crack. The equivalent moment about the axis passing through the point of application of the resultant force in the concrete and steel of the compression zone is equal to the internal moment

$$M_{eq} - N_{tot} z_1 = \sigma_s F_{pr} z_1 \qquad \text{(VII.99)}$$

Hence,

$$\sigma_s = (M_{eq} - N_{tot} z_1)/F_{pr} z_1 \qquad \text{(VII.100)}$$

The denominator in Eq. (VII.100) is the elastic-plastic moment of resistance of the tension zone after the advent of cracking defined as

$$W_s = F_{pr} z_1 \quad \text{or} \quad W_s = F_s z_1 \qquad \text{(VII.101)}$$

Substituting for M_{eq} from Eqs. (VII.93) and (VII.94) and for N_{tot} from Eq. (VII.71), we may re-write Eq. (VII.100) thus:

— for members in bending

$$\sigma_s = [M - N_0(z_1 - e_{s,pr})]/W_s \qquad \text{(VII.102)}$$

— for members in eccentrical compression

$$\sigma_s = [N\,(e - z_1) - N_0\,(z_1 - e_{s,pr})]/W_s \qquad \text{(VII.103)}$$

— for members in eccentrical tension

$$\sigma_s = [N\,(e + z_1) - N_0(z_1 - e_{s,pr})]/W_s \qquad \text{(VII.104)}$$

For members in eccentrical tension in which $e_{s,tot} < 0.8h_0$, the value of σ_s is found by Eq. (VII.104) where z_1 is taken as z_s, that is, the distance between the centroids of the tensile and compressive steel.

For nonprestressed members in bending

$$\sigma_c = M/W_{com} \tag{VII.105}$$

$$\sigma_s = M/W_s \tag{VII.106}$$

The above value of σ_s is substituted in the design formulas for the crack width.

4. Crack Spacing

After the external moment, M_{ext}^h, has exceeded the prestressing moment, M_{pr}, the incremental stress in the tensile steel at a crack, $\sigma_{s,cr}$ (immediately after cracking), can be found by equating the

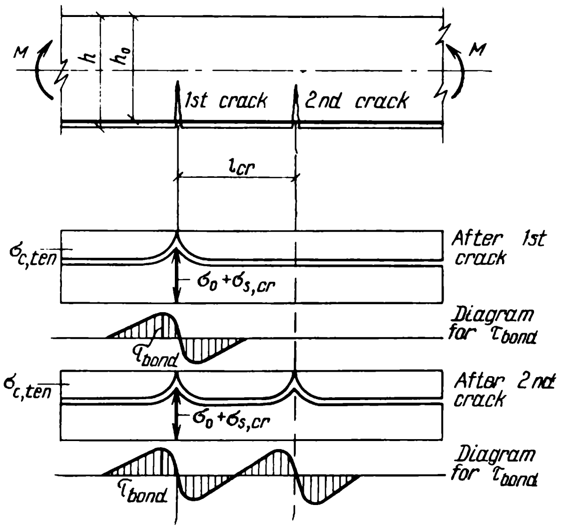


Fig. VII.12. Stress distribution in a cracked bending member

moments acting on the section in Stage I and Stage II, $M_{ext}^h = M_{cr}$. Then, recalling Eq. (VII.29), we may write

$$\sigma_{s,cr} = (M_{cr} - M_{pr}^h)/W_s = R_{ten\,II} W_{cr}/W_s \tag{VII.107}$$

The crack spacing, l_{cr}, in the zone of pure bending may be found as with axial tension, by equating the difference in force between the tensile steel at a crack and between cracks to the bond between the steel and the concrete (Fig. VII.12). Then, according to Eq. (VII.68),

$$(\sigma_0 + \sigma_{s,cr})\,F_{pr} - (\sigma_0 + 2n\,R_{ten\,II})\,F_{pr} = \tau_{bond}\,s l_{cr}\,\omega$$

Substituting for $\sigma_{s,cr}$ from Eq. (VII.107) gives

$$(W_{cr}/W_s - 2n)\,R_{ten\,II} F_{pr} = \tau_{bond} s l_{cr} \omega$$

Hence, the crack spacing is

$$l_{cr} = (W_{cr}/nW_s - 2)\ nuR_{tenII}/\omega\tau_{bond} \tag{VII.108}$$

or

$$l_{cr} = k_1 nu\eta_{sh} \tag{VII.109}$$

where

$$k_1 = W_{cr}/nW_s - 2 \tag{VII.110}$$

and u and η_{sh} have the same meaning as in Eq. (VII.70) for axial tension.

The crack spacing in nonprestressed members is also found by Eq. (VII.109).

5. Crack Closure

In prestressed members designed to meet the requirements of Category Two, normal and inclined cracks should safely close after the load has been reduced to a certain value. This is important because long-time opening of cracks is most dangerous for reinforcing steel as it may affect its corrosion resistance. If the total loading, constituted by short- and long-time loads, results in cracking, then, after it has been reduced to the long-time load only, the cracks will not close unless the steel behaves elastically and no permanent set takes place.

Normal cracks will safely close if:

(1) $\sigma_0 + \sigma_s \leqslant kR_{sII}$

where σ_0 is the prestress in the steel with allowance for all losses; σ_s is the increment in the stress in the steel due to the external loads; and k is taken as 0.65 for high-strength wire, and 0.8 for reinforcing bars;

(2) under dead and long-time live loads, the tensile face of the member at a section containing a crack in the tension zone remains in compression at a normal stress $\sigma_c \geqslant 1$ MPa.

For an elastic transformed section, the compressive stress, σ_c, is determined with allowance for the external loads and the prestressing force, N_0.

Inclined cracks will safely close if, at the level of the centroid of the transformed section, $\sigma_{prin,ten} = \sigma_{prin,com}$, they both are compressive and are at least equal to 1 MPa. For this purpose, it may prove necessary to induce a biaxial prestress (by using prestressed stirrups or bent bars).

VII.6. CURVATURE OF THE DEFLECTED AXIS AND STIFFNESS OF REINFORCED CONCRETE MEMBERS IN BENDING

1. General

The analysis of reinforced concrete members for deflection (linear and angular) involves determining the curvature of the axis or stiffness (or flexural rigidity) of members in bending.

Depending on the type of loading and the state of stress, the tension zone of a reinforced concrete member may or may not have portions free from cracks (or portions where cracks are closed). For the purpose of our discussion, we shall take members or their portions as crack-free in the tension zone if dead, long- and short-time loads do not cause cracking when taken with the overload factor $n = 1$ applied.

2. Curvature of the Deflected Axis and Stiffness of Reinforced Concrete Members Based on Crack-Free Portions

The curvature of the deflected axis of bending and eccentrically loaded members based on crack-free portions is determined as for a continuous transformed section in Stage I of the stress-strain state, using the following formula

$$1/\rho = cM/B \qquad \text{(VII.111)}$$

where M is the bending moment due to the loads for which the curvature of the deflected axis is being found; B is the stiffness of the transformed section which, for heavy and coarse-aggregate quartz-sand concrete and short-time loading, is

$$B = 0.85\ E_c I_{tr} \qquad \text{(VII.112)}$$

Here, the factor 0.85 takes care of the reduction in stiffness caused by plastic strain in the concrete of the tension zone; c is the factor taking care of the reduction in stiffness (increase in curvature) due to creep in the concrete of the compression zone under long-time loading, taken as 2 at an average relative humidity of more than 40%, and 3 at an average relative humidity of 40% and less.

The curvature, $1/\rho_{pr}$, due to the short-time prestressing force is also found by Eq. (VII.111) at the bending moment equal to

$$M = N_0 e_{0pr} \qquad \text{(VII.113)}$$

The curvature due to creep in the concrete produced by the prestressing force is taken equal to the slope of the strain distribution

diagram

$$1/\rho_{creep} = (\varepsilon_{creep} - \varepsilon'_{creep})/h_0 \qquad \text{(VII.114)}$$

where ε_{creep} and ε'_{creep} are the strains in the concrete due to creep at the centroid of the tensile steel and the extreme compressive fibre in the concrete:

$$\varepsilon_{creep} = \sigma_{creep}/E_s \text{ and } \varepsilon'_{creep} = \sigma'_{creep}/E_s \qquad \text{(VII.115)}$$

Here, the losses are equal to $\sigma_{creep} = \sigma_6 + \sigma_9$ and $\sigma'_{creep} = \sigma'_6 + + \sigma'_9$.

If normal cracks in the tension zone remain closed under the load in question, the curvatures defined by Eq. (VII.111) may be increased by 20%.

3. Curvature of the Deflected Axis and Stiffness of Reinforced Concrete Members Based on Cracks

In portions where normal cracks form in Stage II, the general stress-strain state is described in terms of the average strain in the tensile steel, $\varepsilon_{s,av}$, the average strain in the concrete of the compression zone, $\varepsilon_{c,av}$, and the average position of the neutral axis with

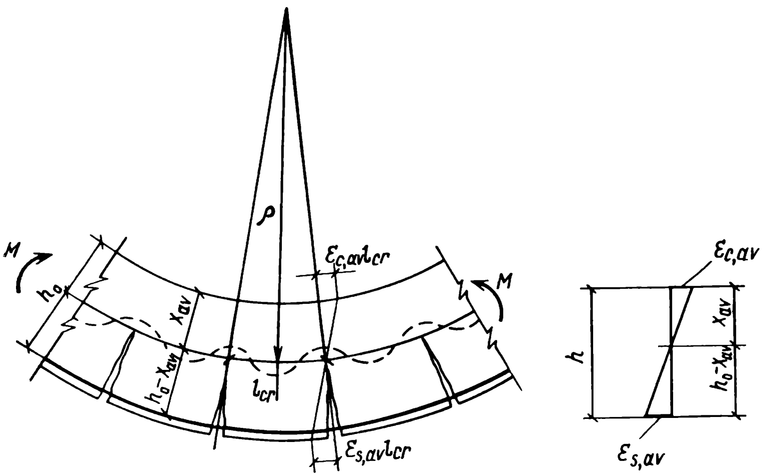


Fig. VII.13. To determining the curvature of the deflected axis

the radius of curvature ρ (Fig. VII.13). Let us examine the pure bending zone of a reinforced concrete member. The curvature of the deflected axis and the average strain in the steel and concrete are related as follows

$$l_{cr}/\rho = \varepsilon_{s,av} l_{cr}/(h_0 - x_{av})$$
$$= \varepsilon_{c,av} l_{cr}/x_{av} = (\varepsilon_{s,av} + \varepsilon_{c,av})\, l_{cr}/h_0$$

After dividing the above expression through by l_{cr}, the curvature of the axis can be expressed as the slope of the average strain distribution diagram

$$1/\rho = \varepsilon_{s,av}/(h_0 - x_{av})$$
$$= \varepsilon_{c,av}/x_{av} = (\varepsilon_{s,av} + \varepsilon_{c,av})/h_0 \quad \text{(VII.116)}$$

Recalling that

$$\varepsilon_{s,av} = \psi_s\sigma_s/E_s \text{ and } \varepsilon_{c,av} = \psi_c\sigma_c/\nu E_c$$

we may define the curvature of the axis as

$$1/\rho = \psi_s\sigma_s/E_s\,(h_0 - x_{av})$$
$$= \psi_c\sigma_c/\nu E_c x_{av} = \psi_s\sigma_s/E_s h_0 + \psi_c\sigma_c/\nu E_c h_0 \quad \text{(VII.117)}$$

On substituting the stresses in the steel and concrete

$$\sigma_s = M/W_s \text{ and } \sigma_c = M/W_{av}$$

into Eq. (VII.117), we get the following expression for the curvature

$$1/\rho = M\psi_s/E_s W_s\,(h_0 - x_{av}) = M\psi_c/\nu E_c W_{av} x_{av}$$
$$= M\,(\psi_s/E_s W_s + \psi_c/\nu E_c W_{av})/h_0 \quad \text{(VII.118)}$$

The denominator in Eq. (VII.118) is the stiffness of the reinforced concrete section. For the tension zone, it is

$$B = E_s W_s(h_0 - x_{av})/\psi_s \quad \text{(VII.119)}$$

for the compression zone,

$$B = \nu E_c W_{av} x_{av}/\psi_c \quad \text{(VII.120)}$$

and for both zones,

$$B = h_0/(\psi_s/E_s W_s + \psi_c/\nu E_c W_{av}) \quad \text{(VII.121)}$$

Introducing an allowance for the values of the elastic-plastic moments of resistance, W_s and W_{av}, the expressions for curvature and stiffness may be re-written as follows

$$1/\rho = M\,[\psi_s/E_s F_s + \psi_c/(\gamma' + \xi)\,\nu E_c b h_0]/h_0 z_1 \quad \text{(VII.122)}$$

$$B = h_0 z_1/[\psi_s/E_s F_s + \psi_c/(\gamma' + \xi)\,\nu E_c b h_0] \quad \text{(VII.123)}$$

In the general case, for prestressed members in bending, eccentrical compression and eccentrical tension with $e_{0av} \geqslant 0.8\ h_0$, the external loads and the prestressing force are replaced by the equivalent moment, M_{eq}, and the total longitudinal force, N_{tot}. Then, in view of Eq. (VII.97), the stress in the concrete of the compression zone is

$$\sigma_c = M_{eq}/(\gamma' + \xi)\,b h_0 z_1$$

and, in view of Eq. (VII.100), the stress in the tensile steel is

$$\sigma_s = M_{eq}/F_{pr}z_1 - N_{tot}/F_{pr} \tag{VII.124}$$

On substituting for σ_c and σ_s, the general expression for the curvature of the deflected axis takes the following form

$$\begin{aligned} 1/\rho = {} & M_{eq}[\psi_s/E_sF_{pr} \\ & + \psi_c/(\gamma' + \xi)\, \nu E_c b h_0]/h_0 z_1 \\ & - N_{tot}\psi_s/h_0E_sF_{pr} \end{aligned} \tag{VII.125}$$

The curvature of the deflected axis, $1/\rho$, and stiffness, B, in cracked portions vary with time; so, in design, they are multiplied by various factors, such as ψ_s taking care of the tensile behaviour of the concrete between cracks, ψ_c taking care of the fact that the strain in the concrete of the compression zone is nonuniform between cracks, and ν taking care of the nonelastic strain in the concrete of the compression zone. The values of ψ_s and ν are determined according to the duration of loading.

Relevant standards specify the values of ν for heavy and porous-aggregate concrete according to the type of loading and service conditions of the structure. With short-time loading, $\nu = 0.45$; with long-time loading, $\nu = 0.15$ for an average relative humidity of more than 40%, and $\nu = 0.1$ for an average relative humidity of 40% and less. It should be noted that the standards specify the values of the product $\omega\nu$ rather than ν. This is done because with the rectangular stress diagram adopted for the concrete of the compression zone in Stage II and with $\omega = 1$ the product is numerically equal to ν.

4. Deflections of Reinforced Concrete Members

Sag of Reinforced Concrete Members Based on Crack-Free Tension Zones. This is determined in terms of the stiffness B of the transformed section with allowance for the factor c adjusted for long-time loading. The total sag is

$$f = f_{sh} + f_l - f_h - f_{h,cr} \tag{VII.126}$$

where f_{sh} is the sag due to short-time loading; f_l is the sag due to dead and long-time live loads; f_h is the hog due to the short-time prestressing force, N_0, with allowance for all losses; and $f_{h,cr}$ is the hog due to the creep in the concrete induced by prestressing.

In prestressed members of constant depth, the hog due to eccentrical prestressing is

$$f_h = N_0 e_{0pr} l^2/8B \tag{VII.127}$$

and the hog due to the creep in the concrete induced by prestressing is

$$f_{h,cr} = l^2/8\rho_{h,cr} \tag{VII.128}$$

Sag of Reinforced Concrete Members Based on Cracked Tension Zones. This is determined in terms of the curvature of the deflected axis

$$f = \int_0^l \overline{M} \frac{1}{\rho}(x)\,dx \tag{VII.129}$$

where $\overline{M}$ is the bending moment at the xth section due to the unit force applied in the direction of the deflection in question; $\frac{1}{\rho}(x)$ is defined by Eq. (VII.125).

When determining deflections in constant-section members, it will be a good plan to determine the curvature for the most stressed section within each portion where the bending moment does not change sign, and to assume it changing in proportion to the bending moment diagram elsewhere. This is equivalent to finding B for the most stressed section and assuming it to be constant elsewhere.

For prestressed members designed to meet the crack-resistance requirements of Categories Two and Three, the above assumptions may sometimes result in overestimated sags, because cracked portions in the tension zone may be of a limited length. In such cases, the sag is

$$f = \sum \int \overline{M} \frac{1}{\rho}(x)\,dx \tag{VII.130}$$

Here, the curvature diagram, $\frac{1}{\rho}(x)$, is divided into several portions along the span as a piecewise-linear function, and the deflection integral is evaluated by multiplying the diagrams according to Vereshchagin's rule. In each crack-free and cracked portion, the curvature, $\frac{1}{\rho}(x)$, is found by Eqs. (VII.111) and (VII.125).

The angular deflection of reinforced concrete members is likewise found by Eq. (VII.129) or (VII.130), but the integration is performed with respect to the moment $\overline{M}$ at the xth section due to the unit moment.

In the simplest cases, the sag of nonprestressed bending members (slabs, panels, beams and so on) due to distributed loads is

$$f = 5ql^4/384B$$

The sag of single-span beams and cantilevers induced by various loads is determined in terms of the curvature or stiffness at the section where the bending moment is a maximum, using the following

general formula

$$f = sl^2/\rho \text{ or } f = sl^2M/B \tag{VII.131}$$

where s depends on the type and disposition of loads. For a simply supported beam: $s = 5/48$ with a distributed load; $s = 1/12$ with a concentrated load applied at the midspan; and $s = 1/8$ with two equal moments applied at the ends. For a cantilever beam:

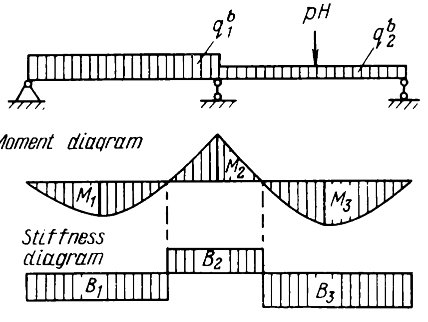


Fig. VII.14. Bending moment diagram and stiffness diagram of a two-span beam

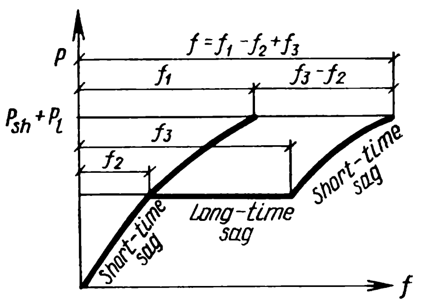


Fig. VII.15. Sag of a bending reinforced concrete member under short- and long-time loading

$s = 1/4$ with a distributed load; $s = 1/3$ with a concentrated load applied at the cantilever arm; and $s = 1/2$ with a moment applied at the cantilever arm.

For multispan beams whose sections are in both compression and tension, the stiffness of each portion is assumed to be the same and equal to that at the section where the moment is a maximum (Fig. VII.14).

The sag of short bending members with the ratio $l/h < 10$ (such as crane beams, hammer beams, and the like) should be determined with allowance for shearing forces. In this case, the total sag is equal to the sum of sags due to bending strain and shear strain, f_Q. The sag is defined as

$$f_Q = \int_0^l 1.5\bar{Q}Qc\beta(x)\, dx/GF$$

where $\bar{Q}$ is the shearing force at the xth section due to the unit force applied in the direction of the deflection in question; c is the loading-duration factor; $\beta(x)$ is the factor taking care of the effect that cracks have on the shear strain, and assumed as 1 for portions free from normal and inclined cracks, 4.8 for portions containing

inclined cracks only, and

$$\beta(x) = 3B/B_{crack} \quad \text{or} \quad \beta(x) = \frac{3B}{M}\frac{1}{\rho}(x)$$

for portions containing only normal cracks or both normal and inclined cracks; B_{crack} is the stiffness of the section upon cracking.

The total sag with allowance for the duration of loading is

$$f = f_1 - f_2 + f_3 - f_{h,cr} \tag{VII.132}$$

where f_1 is the sag due to all loads applied for a short time; f_2 is the sag due to dead and long-time live loads applied for a short time; f_3 is the sag due to dead and long-time live loads applied for a long time; and $f_{h,cr}$ is the hog due to prestressing and creep in the concrete caused by prestressing.

The values of f_1 and f_2 are calculated with ψ_s and ν taken for short-time loading, and that of f_3, with ψ_s and ν taken for long-time loading.

The physical meaning of Eq. (VII.132) is illustrated by the P-f diagram shown in Fig. VII.15.

The total sag of prestressed members with allowance for the duration of loading is defined as the total curvature

$$1/\rho = 1/\rho_1 - 1/\rho_2 + 1/\rho_3 - 1/\rho_{h,cr} \tag{VII.133}$$

5. Average Stiffness of Reinforced Concrete Members Based on Cracked Tension Zones

When designing statically indeterminate reinforced concrete structures (for example, multi-storey framed skeletons), we need to know the stiffness of the individual members, or their ratios. For eccentrically compressed members with their sections in both compression and tension and containing crack-free or cracked portions in the tension zone, it is necessary to determine the average stiffness.

Let us examine a nonprestressed eccentrically compressed strut rectangular in cross section and containing reinforcing steel of area $F_s = F_s'$ (Fig. VII.16). Here, the longitudinal compressive force is taken as $N = M/e_0$, and the equivalent moment as $M_{eq} = Me/e_0$. From Eq. (VII.125) for the curvature of the axis, the stiffness for cracked portions is found to be

$$B = M\Big/\frac{1}{\rho} = e_0 h_0 z_1/[\psi_s(e - z_1)/E_sF_s + \psi_c/(\gamma' + \xi)\, bh_0\nu E_c] \tag{VII.134}$$

The stiffness varies along the strut because of the varying eccentricities e_0 and e, and some other factors. In crack-free portions, the stiffness is constant and defined by Eq. (VII.112).

Varying stiffness, however, is inconvenient in the practical design of structures (for example, in the design of statically indeterminate frames). So, in practice, use is made of the average stiffness which is constant along the member and determined by deeming the deflection to be the same throughout.

In an eccentrically compressed strut containing crack-free and cracked portions along its length, the angular deflection due to the end moments and the longitudinal force is defined as

$$\theta = \sum \int \bar{M} \frac{1}{\rho} x \, (dx)$$

The same angular deflection at a section near the support of a strut having an average stiffness along its length is

$$\theta = Ml/6B_{av}$$

The average stiffness of an eccentrically compressed strut is determined by equating the above two expressions for the angular deflections of the section near the support. Omitting intermediate calculations, we may write the final result which can be used in practical computation as follows

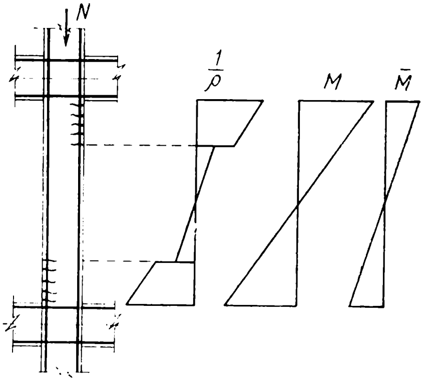


Fig. VII.16. To determining the average stiffness, B_{av}, of struts in eccentrical compression with allowance for the variable eccentricity of the longitudinal force and cracking at the ends

$$B_{av} = k_0 E_c I_c \tag{VII.135}$$

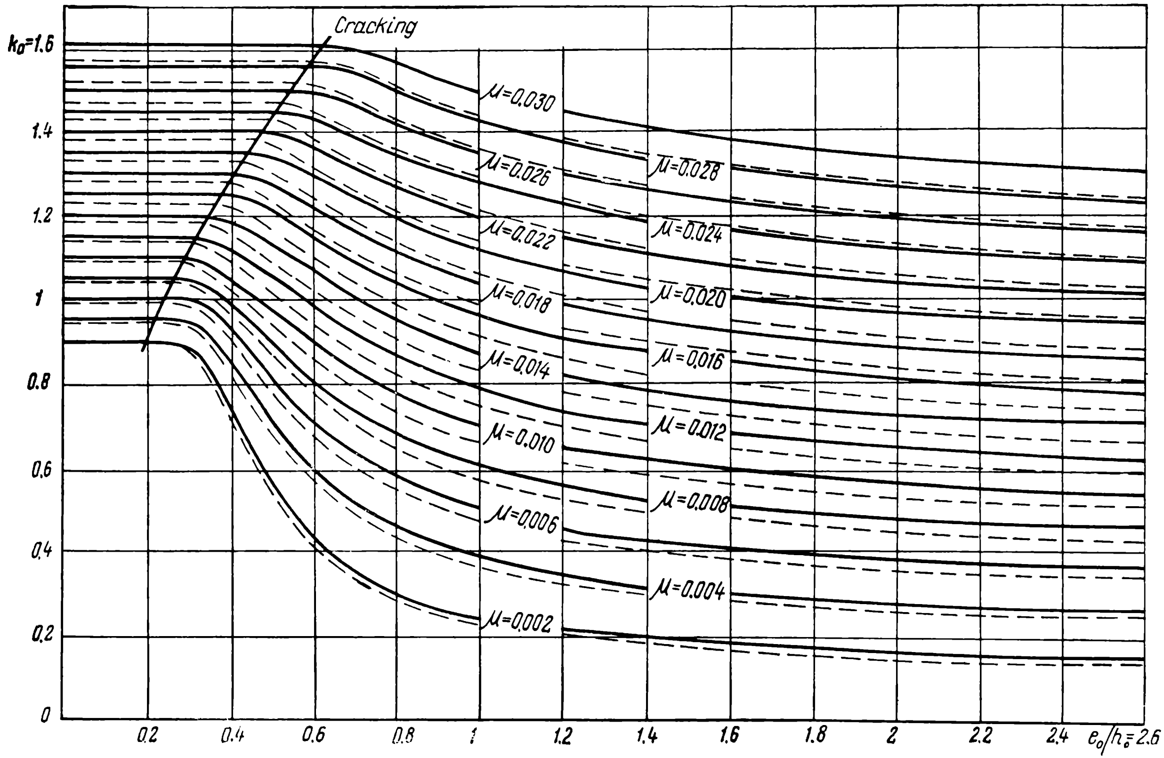
where I_c is the moment of inertia of the concrete area of the strut and k_0 is chosen in accordance with the relative eccentricity e_0/h_0, reinforcement ratio $\mu = E_s/bh_0$, concrete brand, and class of reinforcing steel from the curves of Fig. VII.17.

VII.7. EFFECT OF INCIPIENT CRACKING IN CONCRETE OF COMPRESSION ZONE OF PRESTRESSED MEMBERS

When designing prestressed members in terms of the second group of limit states, it is necessary to take into account cracks which may form in the zone subjected to compression in service. Such cracks may form in manufacture, prestressing, transportation and erection. They reduce the crack resistance and stiffness of members.

(a) In the cracking analysis of members containing incipient cracks, the value of M_{cr} should be reduced by $\Delta M_{cr} = \theta M_{cr}$.

(a)


Cracking
k_o=1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
e_o/h_o=2.6
μ=0.030
μ=0.028
μ=0.026
μ=0.024
μ=0.022
μ=0.020
μ=0.018
μ=0.016
μ=0.014
μ=0.012
μ=0.010
μ=0.008
μ=0.006
μ=0.004
μ=0.002

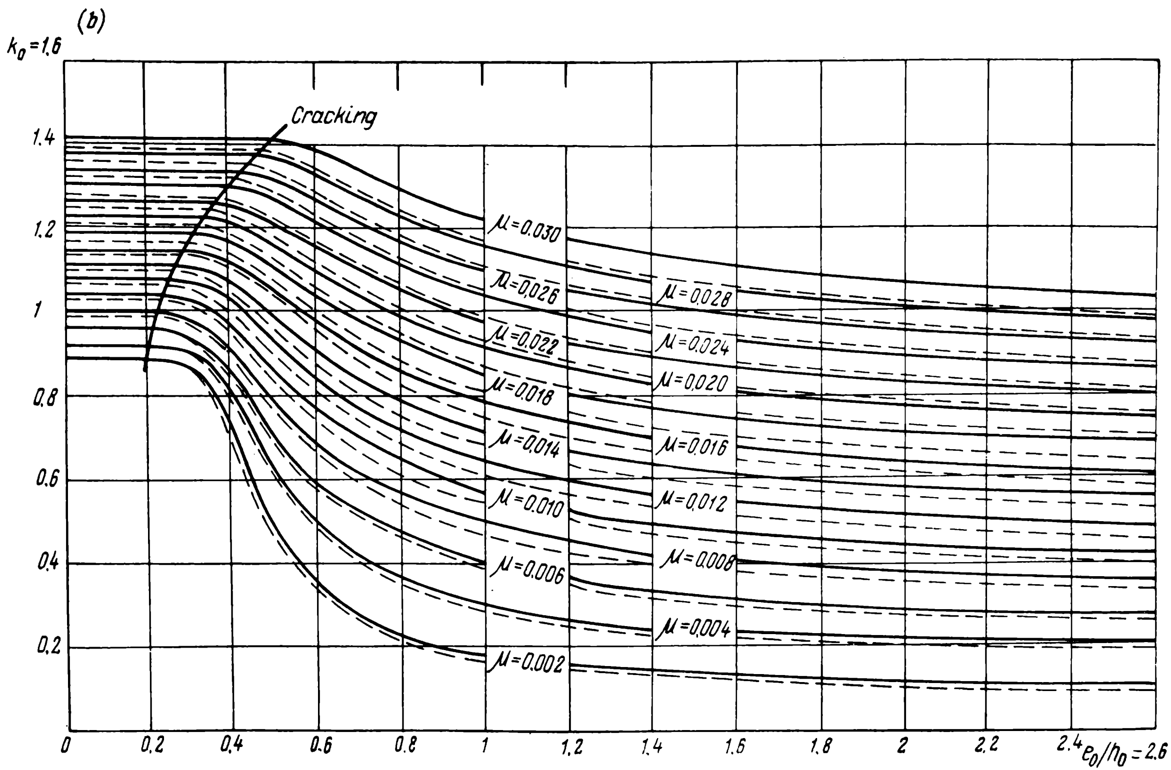


Fig. VII.17. Coefficient k_0 for the average stiffness of eccentrically compressed struts

(a) M-200 concrete; (b) M-400 concrete; solid lines — class A-II steel; dashed lines — class A-III steel

Experiments show that

$$\theta = (1.5 - 0.9/c_{cr})(1 - m) \qquad \text{(VII.136)}$$

where

$$c_{cr} = yF_{pr}/(h - y)(F_{pr} + F'_{pr}) \leqslant 1.4$$

which should be reduced by 15% for structures reinforced with wire; y is the distance from the centroid of the transformed section to the tensile face; m is defined by Eq. (VII.75), but its value should not be less than 0.45.

(b) In the crack-width analysis of members with incipient cracking, the value of N_0 should be taken reduced by ΔN_0:

$$\Delta N_0 = \theta N_0 \qquad \text{(VII.137)}$$

In addition, it is necessary to check the depth of incipient cracking, h_{cr}:

$$h_{cr} = h - (1.2 + m)\xi h_0 \leqslant 0.5h$$

where ξ is the depth of the zone in compression due to the external load and prestressing force, defined by Eq. (VII.90); m is defined by Eq. (VII.75).

(c) In the crack-closure analysis of members with incipient cracking in the compression zones, the value of N_0 is multiplied by the reduction factor $1.1(1 - \theta)$ which must not exceed 1; the value of θ is defined by Eq. (VII.136).

(d) In the deflection analysis of reinforced concrete members with incipient cracking in the compression zones, the curvatures should be increased by 15%; the value of $1/\rho_{h,cr}$ is increased by 25%; and the value of $1/\rho$ in cracked portions is determined using the force N_0 reduced by ΔN_0.

Example VII.1. Given: a rectangular beam; $b = 20$ cm; $h = 50$ cm; design span $l = 5.8$ m; concrete: M-200 heavy heat-cured concrete; axial compressive strength $R_{prII} = 11.5$ MPa; $m_{c1} = 1$; axial tensile strength $R_{ten\,II} = 1.15$ MPa; tangent modulus of elasticity $E_c = 2.15 \times 10^4$ MPa; reinforcing steel: two class A-III bars 20 mm in diameter; $F_s = 6.2$ cm²; $E_s = 2 \times 10^5$ MPa; basic dead and long-time live load including the self-weight of the beam $q^b = 11.8$ kN/m; average relative humidity is above 40%.

To find: the sag due to long-time loading.

Solution. Determine the bending moment due to the load multiplied by the overload factor $n = 1$

$$M = 11.8 \times 5.8^2/8 = 49.5 \text{ kN m}$$

Find the effective depth of the section

$$h_0 = h - a = 50 - 3.5 = 46.5 \text{ cm}$$

Compute the reinforcement ratio

$$\mu = F_s/bh_0 = 6.28/(20 \times 46.5) = 0.0068$$

$$\mu_1 = F_s/bh = 6.28/(20 \times 50) = 0.0063$$

Calculate the modular ratio

$n = E_s/E_c = (2 \times 10^5)/(2.2 \times 10^4) = 9.1$

Using Eq. (VII.21), find the elastic-plastic moment of resistance for the tension zone of the singly reinforced rectangular section

$$W_{cr} = (0.292 + 1.5\mu_1 n)\, bh^2$$
$$= (0.292 + 1.5 \times 0.0063 \times 9.1)\, 20 \times 50^2 = 18\,900 \text{ cm}^3$$

Using Eq. (VII.76), find ψ_s for long-time loading ($s = 0.8$)

$$\psi_s = 1.25 - sR_{tenII} W_{cr}/M$$
$$= 1.25 - 0.8\,(1.15 \times 18\,900)\, 10^{-1}/4\,950 = 0.89$$

Take the value of ψ_c equal to 0.9.

Using Eq. (VII.90), determine the depth of the compression zone at a crack for the singly reinforced rectangular member in bending

$$\xi = 1/[1.8 + (1 + 5L)/10\mu n]$$
$$= 1/[1.8 + (1 + 5 \times 0.101)/10 \times 0.0068 \times 9.1] = 0.24$$

where

$$L = M/R_{tenII} bh_0^2 = 4\,950/[(1.15 \times 20 \times 46.5^2)\, 10^{-1}] = 0.101$$

Compute the arm of the internal couple

$$z_1 = h_0\,(1 - \xi/2) = 46.5\,(1 - 0.24/2) = 41.1 \text{ cm}$$

Using Eq. (VII.123), determine the stiffness of the beam in the portion containing cracks in the tension zone at $\nu = 0.15$ (with long-time loading and an ambient humidity of more than 40%)

$$B = h_0 z_1/(\psi_s/E_s F_s + \psi_c/\xi\nu E_c bh_0)$$
$$= (46.5 \times 41.1)/[0.89/(2 \times 10^5 \times 6.28) + 0.9/(0.24$$
$$\times 0.15 \times 2.15 \times 10^4 \times 20 \times 46.5)] = 9.6 \times 10^8 \text{ MPa cm}^4$$

Recalling that 1 MPa cm^4 is equivalent to 100 N, find the sag of the beam

$$f = 5q^b\, l/384B = (5 \times 0.118 \times 580^4)/[384\,(9.6 \times 10^8)\, 10^{-1}] = 1.8 \text{ cm}$$

Unless otherwise specified, the limiting sag of the beam for a span of $5 < l = 5.8 < 10$ m is, according to Table II.4, 2.5 cm. As is seen, $f = 1.8 \text{ cm} < 2.5$ cm.

Example VII.2. Given: a prestressed I-beam (Fig. VII.18); concrete: M-400 heavy heat-cured concrete; axial compressive strength $R_{prII} = 22.5$ MPa; $m_c = 1$; axial tensile strength $R_{tenII} = 1.8$ MPa; $E_c = 3 \times 10^4$ MPa; prestressed steel: 13 class K-7 wire strands 15 mm in diameter ($F_{pr} = 18.04$ cm^2) in the tension zone, two class K-7 strands 15 mm in diameter ($F'_{pr} = 2.75$ cm^2) in the compression zone; $E_s = 1.8 \times 10^5$ MPa. The prestress in the steel with allowance for all losses and with the tensioning accuracy factor $m_{ac} = 0.9$ applied is $\sigma_0 = 640$ MPa; $\sigma'_0 = 740$ MPa. The distance from the tensile face to the centroid of the tensile steel is $a = 12.5$ cm; the distance from the compressive face to the centroid of the compressive steel is $a' = 5$ cm.

To find: the cracking moment M_{cr}.

Solution. (a) Determine the section characteristics: the average depth of the compressed flange, $h'_f = 24$ cm; the average depth of the flange in tension, $h_f = 25$ cm; the cross-sectional area of the compressed flange overhangs, $F'_{ov} =$

$= 28 \times 24 = 670$ cm²; the distance to the centroids of the top flange overhangs, $a'_{ov} = 12$ cm; the cross-sectional area of the bottom flange overhangs, $F_{ov} = 20 \times 25 = 500$ cm²; the distance to the centroids of the bottom flange over-

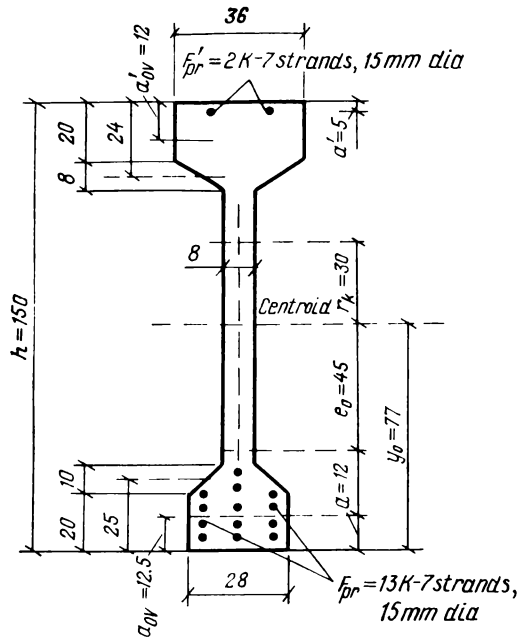


Fig. VII.18. To Example VII.2

hangs, $a_{ov} = 12.5$ cm; and the cross-sectional area of the transformed section by Eq. (VII.14)

$$F_{tr} = bh + F'_{ov} + F_{ov} + n\,(F_{pr} + F'_{pr})$$

$$= 8 \times 150 + 670 + 500 + 6\,(18.04 + 2.75) = 2\,490 \text{ cm}^2$$

Compute the modular ratio

$$n = E_s/E_c = (1.8 \times 10^5)/(3 \times 10^4) = 6$$

Find the static moment of the transformed section about the tensile face by Eq. (II.29)

$$S_{tr} = \Sigma F_i y_i = (8 \times 150^2)/2 + 670\,(150 - 12)$$

$$+ 500 \times 12.5 + 6\,(18.04 \times 12.5 + 2.75 \times 144) = 190\,200 \text{ cm}^3$$

Calculate the distance from the centroid of the transformed section to the tensile face

$$y_0 = S_{tr}/F_{tr} = 190\,200/2\,490 = 77 \text{ cm}$$

Determine the moment of inertia of the transformed section by Eq. (II.31)

$$\begin{aligned} I_{tr} &= \Sigma\,[I_{0i} + F_i\,(y_0 - y_i)^2] \\ &= (8 \times 150^3)/12 + 8 \times 149\,(150/12 - 77)^2 \\ &+ (28 \times 24^3)/12 + 670\,(77 - 138)^2 \\ &+ (20 \times 25^3)/12 + 500\,(77 - 12.5)^2 \\ &+ 6 \times 18.04\,(77 - 12.5)^2 + 6 \times 2.75\,(77 - 145)^2 = 72.8 \times 10^5 \quad \text{cm}^4 \end{aligned}$$

Calculate the moment of resistance of the tension zone in the transformed section with the concrete behaving elastically

$$W_0 = I_{tr}/y_0 = (72.8 \times 10^5)/77 = 94\,500 \text{ cm}^3$$

Determine the elastic-plastic moment of resistance of the tension zone in the transformed section at $b_f'/b = 36/8 = 4.5$, $b_f/b = 28/8 = 3.5$ and $\gamma = 1.5$ (according to Appendix X)

$$W_{cr} = \gamma W_0 = 1.5 \times 94\,500 = 142\,000 \text{ cm}^3$$

(b) Using Eqs. (II.26) and (II.27), find the prestressing force, N_0, and the eccentricity, e_{0pr}, about the centroid of the transformed section

$$\begin{aligned} N_0 &= \sigma_0 F_{pr} + \sigma'_{pr} F'_{pr} = 640 \times 18.04 + 740 \times 2.75 \\ &= 13\,650 \text{ MPa cm}^2 \quad (\text{or } 1\,365 \text{ kN}) \end{aligned}$$

$$\begin{aligned} e_{0pr} &= (\sigma_0 F_{pr} y_{pr} - \sigma_0' F'_{pr} y'_{pr})/N_0 = [640 \times 18.04\,(77 - 12.5) \\ &- 740 \times 2.75\,(150 - 77 - 5)]/13\,650 = 45 \text{ cm} \end{aligned}$$

(c) Now, determine the cracking moment M_{cr}. First, find the distance from the kern point to the centroid of the transformed section

$$r_k = 0.8 W_0/F_{tr} = (0.8 \times 93\,000)/2\,490 = 30 \text{ cm}$$

Then, using Eq. (VII.31), find

$$\begin{aligned} M_{cr} &= R_{tenII} W_{cr} + N_0\,(e_{0pr} + r_k) \\ &= 1.8 \times 140\,000 + 13\,650\,(45 + 30) \\ &= 127.7 \times 10^4 \text{ N m (MPa cm}^3) \end{aligned}$$

That is, $M_{cr} = 1\,277$ kN m.

Find the stress-strength ratio in the concrete of the compression zone before the advent of cracking; for this purpose, determine

$$\delta'_{ov} = a'_{ov}/h = 12/150 = 0.08 \quad \text{and}$$

$$\delta' = a'/h = 5/150 = 0.034$$

Compute the relative depth of the compression zone before the advent of cracking by Eq. (VII.13)

$$\begin{aligned} \xi &= 1 - [bh + 2\,(1 - \delta'_{ov})\,F'_{ov} + 2nF'_f\,(1 - \delta')]/(2F_f - F_{ov} + N_0/R_{tenII}) \\ &= 1 - (8 \times 150 + 2 \times 0.92 \times 670 + 2 \times 6 \times 2.75 \\ &\times 0.966)/(2 \times 2\,490 - 500 + 13\,650/1.8) = 0.8 \end{aligned}$$

$$x = \xi h = 0.8 \times 150 = 120 \text{ cm}$$

Using Eq. (VII.7), find the stress in the concrete of the compression zone

$$\sigma_c = 2R_{tenII}x/(h - x) = (2 \times 1.8 \times 120)/(150 - 120) = 14.4 \text{ MPa}$$

Finally, determine the stress-strength ratio

$$k = \sigma_c/R_{prII} = 14.4/22.5 = 0.64$$

CHAPTER VIII

DYNAMIC RESISTANCE OF REINFORCED CONCRETE

VIII.1. VIBRATION OF MEMBERS

1. Dynamic Loads

In accordance with their purpose, members of reinforced concrete structures may be subjected to dynamic as well as static loads.

Dynamic loads are many and diverse. They may be caused by stationary equipment installed on building floors, such as machines with rotating parts (electric motors, fans, lathes, etc.), mechanisms with their parts in reciprocating motion (looms, printing machines, and so on), and impact or impulse machines.

Dynamic live loads imposed by travelling overhead cranes on structural members produce impact effects (impacts of wheels against rail joints), vibration (due to unbalanced running gear) and so on.

Wind loads (due to gusts and pulsation) cause multi-storey buildings and elevated structures (such as stacks, towers and poles) to vibrate.

Earthquakes produce impact effects on structures.

Impact and impulse short-time loads developing and diminishing at a high rate are known as explosion loads.

Dynamic loads may vary in kind (force or moment), form (vibration, periodic loads, or impacts), manner of application (dead or live), and direction (vertical or horizontal).

According to duration and periodicity, dynamic loads may be divided into repeated or single (nonrepeated). Repeated loads include those arising from continuously working machines and equipment, and also repeated impacts and impulses. Single loads include single impacts and impulses, short-time overloads brought about when machines are started and stopped, and the like.

Standard dynamic loads due to machines and equipment, live dynamic loads, wind loads and earthquake loads are given in relevant standards and specifications.

2. Free Vibrations of Members with Allowance for Inelastic Behaviour of Reinforced Concrete

Free vibrations of members having one degree of freedom are described by the following equation for harmonic motion (Fig. VIII.1)

$$y = A \sin (\omega t + \varepsilon) \tag{VIII.1}$$

where A is the amplitude of vibration; ω is the angular frequency or number of vibrations per 2π (s), related to the period of vibrations, T, and frequency, n (Hz), thus

$$\omega = 2\pi/T = 2\pi n \tag{VIII.2}$$

ε is the initial phase or epoch which defines the phase of motion of the point at the initial moment of time ($t = 0$), and determines its initial displacement $A \sin \varepsilon$.

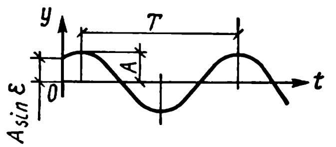


Fig. VIII.1. Plot of free vibrations of a system

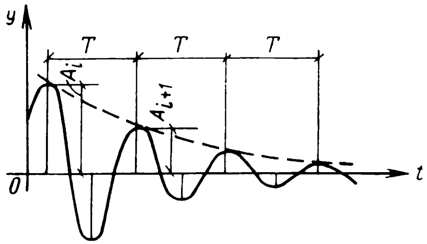


Fig. VIII.2. Plot of damped free vibrations of a system

The free vibrations of members actually observed are damped harmonic vibrations which have a progressively decreasing amplitude and which can be described by the following equation

$$y = A \exp (-\delta t/T) \sin (\omega t + \varepsilon) \tag{VIII.3}$$

where $\exp(-\delta t/T)$ is an exponentially decaying time function.

In structural members, the successive amplitudes of damped free vibrations decrease in geometric progression, so that the ratio A_i/A_{i+1} remains constant (Fig. VIII.2). The logarithm to the base e of this ratio $\delta = \ln A_i/A_{i+1}$ is called the logarithmic decrement or, simply, the decrement: it characterizes the rate of damping. As $\delta \to 0$, the vibrations change to undamped free vibrations.

During each cycle of free vibrations, some energy is irreversibly expended to overcome resistance in the system. This resistance is composed of both internal and external resistance. The internal resistance is mainly offered by inelastic strain in the concrete, whereas the external resistance owes its existence to friction at supports,

and the ambient atmosphere. In reinforced concrete structures, the external resistance is usually small as compared with the internal resistance.

Experiments have shown that the relation between the external force, P, and displacement, y, over a vibration cycle can be described by the force-deformation curve shown in Fig. VIII.3. The diagram is in the form of a hysteresis loop whose area gives the energy,

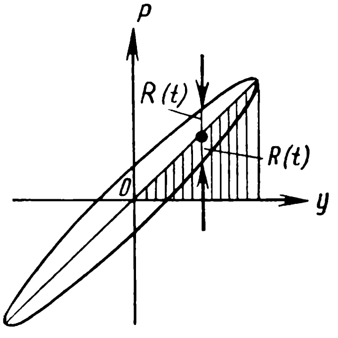


Fig. VIII.3. Force-deformation curve plotted for one cycle, a hysteresis loop

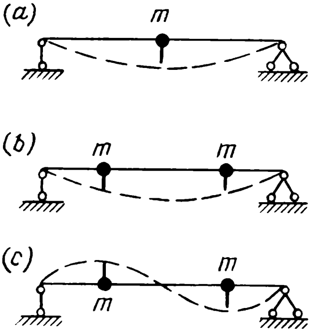


Fig. VIII.4. Vibrations of a system with (a) one degree of freedom, (b) two degrees of freedom (symmetric mode) and (c) two degrees of freedom (antisymmetric mode)

Δw, irreversibly expended during one vibration cycle and dissipated as heat. The rate of damping depends on the energy absorption factor

$$\psi = \Delta W / W \tag{VIII.4}$$

where W is the work done by the elastic forces of the system during a quarter of a cycle; it is equal to the area of the shaded triangle.

As is known from experience, the energy absorption factor for reinforced concrete depends on the rigidity of joints between members, and composite action of slabs, panels, beams and other members in vibration. Tests under field conditions have shown that there may be a certain spread in ψ, depending on the type of structure and the test procedure used. Some experimentally found values of ψ are given in Table VIII.1.

The energy absorption factor, ψ, is equal to twice the logarithmic decrement for free vibrations

$$\psi = 2\delta \tag{VIII.5}$$

TABLE VIII.1. Energy Absorption Factor for Reinforced Concrete

Structure	ψ		
	min	max	average
Large-panel floor			
with unconcreted joints	0.2	0.24	0.22
with concreted joints	0.44	0.6	0.52
Crane beam			
with unconcreted joints	0.24	0.4	0.32
with concreted joints	0.38	0.56	0.47
In-situ ribbed-slab floor	0.39	0.78	0.58

Dynamic design calculations use the inelastic resistance factor of reinforced concrete. It is defined as

$$\gamma = \psi/2\pi = \delta/\pi \tag{VIII.6}$$

and may range in value from 0.05 to 0.1, depending on the dynamic category of the machine.

If a vibrating system requires n independent coordinates to specify its position, the system is said to have n degrees of freedom. A simply supported beam with one concentrated weight, $m = P/g$, in the span is a system with one degree of freedom (the self-weight of the beam is negligibly small as compared with the concentrated weight, so it may be neglected). The same beam with two concentrated weights is a system with two degrees of freedom (Fig. VIII.4). A simply supported beam carrying a distributed load is regarded as a system with an infinite number of degrees of freedom.

The number of frequencies and modes of free vibrations in a system is equal to the number of its degrees of freedom. In each kth harmonic, where $k = 1, 2, \ldots, n$, a system with n degrees of freedom is in independent free vibration described as

$$y_k = A_k \exp\left(-\delta t/T\right) \sin\left(\omega_k t + \varepsilon_k\right) \tag{VIII.7}$$

where ω_k is the frequency of free vibrations of the kth harmonic; A_k and ε_k are the initial amplitude and epoch of the kth harmonic; and δ is the decrement, the same for all harmonics.

The frequencies of free vibrations are arranged in increasing order of magnitude

$$0 < \omega_1 < \omega_2 \cdots < \omega_n$$

and make up the frequency spectrum of free vibrations. Each frequency corresponds to a certain unique mode of free vibrations. As a rule, reinforced concrete structures are statically indeterminate systems with a large (or infinite) number of degrees of freedom. So,

for practical determination of frequencies and modes of free vibrations, structures are approximately divided into separate elements. For example, reinforced concrete floors are arbitrarily separated into slabs and beams.

The frequencies of free vibrations, ω_k, are the same for damped and undamped systems. Damping has a significant effect only around the resonance frequency of forced vibrations.

3. Forced Vibrations of Members

A disturbing (driving) force $P(t)$ applied to a system gives rise to forced vibrations. For a system with one degree of freedom, the displacement will be caused by the inertia of its mass, md^2y/dt^2, and the disturbing force, that is,

$$y = -\delta_{11} md^2y/dt^2 + \delta_{11} P(t) \tag{VIII.8}$$

On the basis of the above expression, we may write a differential equation of motion for forced vibrations

$$md^2y/dt^2 + y/\delta_{11} = P(t) \tag{VIII.9}$$

If the disturbing force varies harmonically, $P(t) = P \sin \theta t$, the solution of Eq. (VIII.9) may be written as follows

$$y = A(\sin \theta t - \sin \omega t) \tag{VIII.10}$$

Here, the amplitude of forced vibrations is

$$A = P/m(\omega^2 - \theta^2) \tag{VIII.11}$$

Using the expression for the frequency of vibrations, we may transform the expression for the amplitude of forced vibrations as

$$\omega^2 = 1/\delta_{11} m \tag{VIII.12}$$

Then

$$A = \omega^2 \delta_{11} P/(\omega^2 - \theta^2) = f/(1 - \theta^2/\omega^2)$$
$$= \beta f \tag{VIII.13}$$

where $f = \delta_{11} P$ is the static sag due to P; and

$$\beta = 1/(1 - \theta^2/\omega^2) \tag{VIII.14}$$

is the dynamic factor defined as the ratio of the dynamic to static sag.

With the dynamic factor β known, we can carry out the dynamic analysis of a beam statically. The point is that a β-fold increase in the sag caused by the dynamic load (with the same shape of the deflected axis) produces an equal increase in all internal forces and strains.

Including an allowance for damped free vibrations, the dynamic factor is

$$\beta = 1/\sqrt{(1-\theta^2/\omega^2)^2+\gamma^2} \tag{VIII.15}$$

The epoch of vibrations, ε, with allowance for damping, is

$$\operatorname{tg}\varepsilon = \gamma/(1-\theta^2/\omega^2) \tag{VIII.16}$$

As is seen from Eq. (VIII.15), the dynamic factor depends on the ratio of the squared frequency of the disturbing force to the squared frequency of free vibrations, θ^2/ω^2, and the inelastic resistance factor, γ. When the frequency of the disturbing force, θ, is the same as the frequency of free vibrations, ω, the system is said to be at resonance; in the circumstances, the amplitude of forced vibrations reaches its maximum.

At resonance, the dynamic factor for reinforced concrete may be as high as 10 to 20.

Figure VIII.5 shows the resonance curves for different values of the inelastic resistance factor and the frequency ratio, θ/ω. From comparison of these curves we can see that the effect of the inelastic resistance of reinforced concrete on the amplitude of forced vibrations is most pronounced at resonance where $\theta/\omega = 1$, whereas away from resonance it is insignificant. It should be noted that prior to resonance, $\beta \geqslant 1$ always, and past resonance β may be less than 1.

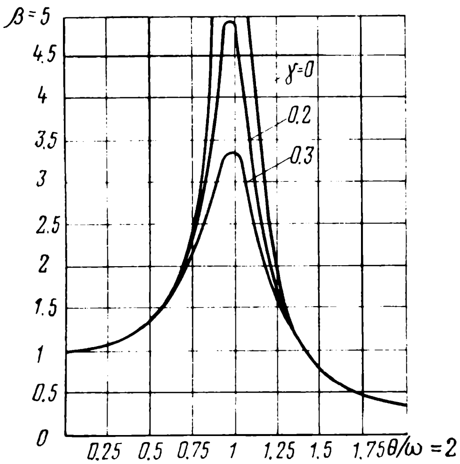


Fig. VIII.5. Resonance curves for different values of the inelastic resistance factor

In an ideal elastic system, the amplitude of forced vibrations at resonance increases without bound and tends to infinity. By contrast, in real reinforced structures the amplitudes of forced vibrations at resonance cannot exceed a certain limit which decreases with increasing inelastic strain factor.

The ability of reinforced concrete (as well as other construction materials) to absorb energy irreversibly improves the dynamic resistance of structures.

For systems with a large number of degrees of freedom, the dynamic factor β should be determined at that frequency of free vibrations, ω_k, which is the nearest to the frequency of the disturbing

force, θ. The static sag, f, should be calculated for the kth mode of vibrations and according to the position of the disturbing force in the loading system.

4. Dynamic Stiffness of Reinforced Concrete Members

Experience with real vibrating reinforced concrete structures shows that the deflections caused by static loads are usually many times those due to dynamic loads, so the stresses change their signs very seldom under conditions of vibration.

When reinforced concrete members vibrate so that their stresses change from zero to their maximum during a short time, the dynamic modulus of elasticity of concrete may practically be considered constant and equal to its tangent modulus of elasticity.

The stiffness of reinforced concrete structures carrying dynamic service loads is determined in the same manner as with static loads. If a member contains cracks in the tension zone, the stiffness, B, is determined on assuming that $\psi_s = \psi_c = 1$ (see Chapter VII). Repeated vibrating loads result in the accumulation of residual deflections (brought about by shrinkage in the compression zone of a member caused by vibration), and the member begins to vibrate about the line of steady-state sags, that is, as an elastic system. In view of this, B is determined, with the factor ν taken as for short-time loading.

Estimates of the frequencies of vibrations and deflection amplitudes should be based on the average stiffness most probable under real conditions of manufacture. It should be remembered that dynamic deflections do not vary linearly with stiffness: when the stiffness of a member decreases, dynamic deflections may either decrease or increase, depending on the new frequency of free vibrations.

VIII.2. DYNAMIC LIMIT-STATE ANALYSIS OF STRUCTURAL MEMBERS

General. In the dynamic analysis of reinforced concrete structures, it is necessary to allow for the possibility that the free and forced vibrations caused by pulsating and vibrating loads may have the same frequency, thereby giving rise to resonance accompanied by an increase in the swing of vibrations. Here, three factors are of importance: (1) vibration has a destructive effect on structures because fatigue reduces the strength of the concrete and steel; (2) vibration is harmful for the people working in a vibrating building (it may reduce their productivity and, sometimes, cause the vibration disease); (3) vibration disturbs the normal operation of machines, tools, and precision instruments.

To carry out the dynamic analysis of a structure, we should first determine the amplitudes of dynamic forces and analyze the load-bearing capacity of structural members with allowance for the forces induced by static loads. Then, we should find the amplitudes of forced vibrations and check to see if they are permissible as regards their effect on people and machines, that is, make sure whether the structural members are fit for normal service.

The frequencies and modes of free vibrations and the amplitudes of dynamic forces may be looked up in relevant handbooks, specifications and guides.

Static and dynamic loads acting together produce respective forces and deflections in structures. The load-bearing capacity of members should be checked by strength and fatigue analysis on the basis of the first group of limit states, and their fitness for normal service by crack-resistance and deflection analyses on the basis of the second group of limit states.

First Group of Limit States. The strength of bending members is said to be adequate if the sum of moments due to the design static loads, M_{st}, and dynamic loads, M_d, multiplied by appropriate combination factors does not exceed the moment, M_{sec}, taken by a section, with the service factors for the concrete and steel applied:

$$M_{st} + M_d \leqslant M_{sec} \tag{VIII.17}$$

Here, M_{sec} is determined for Stage III of the stress-strain state (see Chapter III).

The fatigue strength is said to be adequate if the stresses produced by the design static and repeated dynamic loads in the concrete of the compression zone and tensile steel do not exceed the design strengths multiplied by the appropriate service factors:

$$\sigma_{c,\,\max} \leqslant R_{pr} m_{c2} \tag{VIII.18}$$

$$\sigma_{s,\,\max} \leqslant R_s m_{s1} \tag{VIII.19}$$

Compressive steel is not analyzed for fatigue.

Fatigue analysis is carried out for Stage I of the stress-strain state. The basic points of the analysis are as follows: (1) the stresses in the concrete and steel are determined as for an elastic material on the basis of the transformed section (see Chapter II); they are assumed to be induced by the design static and dynamic loads and the prestressing force, N_0, with allowance for all losses; (2) inelastic strain in the compression zone is taken care of by an appropriate reduction in the modulus of elasticity of the concrete; the values of $n' = \nu E_c/E_s$ are taken according to the brand number of concrete from Table VIII.2; (3) the area of the concrete in the tension zone is taken into account only if the maximum normal stress in the concrete of the tension zone is

$$\sigma_{c,\,ten,\,\max} \leqslant R_{ten} m_{c2} \tag{VIII.20}$$

TABLE VIII.2. Coefficient n'

Concrete brand number	200	300	400	500 and higher
n'	25	20	15	10

In fatigue analysis, it is assumed that no cracks will form during manufacture, transit or erection in the zone of members which is expected to be compressed by external loading.

The service factors for the concrete, m_{c2}, and tensile steel, m_{s1}, take care of the reduction in the strength of the materials to the respective fatigue limits brought about by repeated loading (see Chapter I). The service factor, m_{c2}, depends on the ratio of the minimum to maximum normal stress in the concrete, that is, the cycle characteristic $\rho_c = \sigma_{c,\min}/\sigma_{c,\max}$, type of concrete and its moisture content. The fatigue strength of porous-aggregate concrete is smaller than that of heavy concrete, and it decreases with increasing moisture content. The values of m_{c2} are given in Table VIII.3.

TABLE VIII.3. Service Factor m_{c2} for Concrete under Repeated Loading

Concrete	Moisture content	Cycle characteristic $\rho_c = \sigma_{c,\min}/\sigma_{c,\max}$						
		0-0.1	0.2	0.3	0.4	0.5	0.6	0.7
Heavy	Natural	0.75	0.8	0.85	0.9	0.95	1	1
	High	0.5	0.6	0.7	0.8	0.9	0.95	1
Porous-aggregate	Natural	0.6	0.7	0.8	0.85	0.9	0.95	1
	High	0.45	0.55	0.65	0.75	0.85	0.95	1

During a loading cycle, no tensile stresses are allowed in the zone assumed to be in compression, so $\rho_c \geqslant 0$.

The service factor for the tensile steel, m_{s1}, depends on the ratio of the minimum to maximum stress in the steel, $\rho_s = \sigma_{s,\min}/\sigma_{s,\max}$, type and class of steel. The values of m_{s1} are given in Table VIII.4.

The fatigue strength of tensile steel joined by welding between members, at intersections in bar mats and fabric, and elsewhere, is assumed to be lower, because welded joints act as concentrators for stresses induced by repeated loading. At welded joints, the design strength of the tensile steel should be multiplied by the service factor, m_{s2}, given in Table VIII.5.

The service factor, m_{s2}, depends on the cycle characteristic, ρ_s, bar diameter and group of welded joints. All welded joints are divided into three groups according to the manner in which they are

TABLE VIII.4. Service Factor m_{s1} for Tensile Steel under Repeated Loading

Type and class of steel	Cycle characteristic						
	0	0.2	0.4	0.7	0.8	0.9	1
Hot-rolled deformed bars, class A-III	0.45	0.5	0.6	0.9	1	1	1
Same, class A-IV	—	—	0.4	0.75	0.95	1	1
High-strength deformed wire, class Bp-II	—	—	—	0.7	0.85	0.95	1
High-strength plain wire, class B-II	—	—	—	0.8	1	1	1
Class K-7 wire strands:							
4.5 to 9 mm in diameter	—	—	—	0.8	0.95	1	1
12 and 15 mm in diameter	—	—	—	0.65	0.8	1	1
Ordinary plain and deformed wire, class B-I and Bp-I	0.6	0.75	0.9	1	1	1	1

made: Group I covers resistance butt welded joints mechanically finished to remove rough spots; Group II covers resistance butt welded joints in the "as-welded" condition, resistance spot-welded cross-bar joints, and the like; Group III includes butt welded joints with splice bars arc-welded by fillet welds, butt welded joints with steel pads welded in a pool of molten metal within a reusable mould by multi-electrode technique, and so on.

TABLE VIII.5. Service Factor m_{s2} for Tensile Steel with Welded Joints under Repeated Loading

Type and class	Group of welded joints	Cycle characteristic						
		0	0.2	0.4	0.7	0.8	0.9	1
Class A-III hot-rolled deformed bars not more than 20 mm in diameter	I	0.9	0.95	1	1	1	1	1
	II	0.6	0.65	0.65	0.7	0.75	0.85	1
	III	0.2	0.25	0.3	0.45	0.6	0.8	1

Note. The values of m_{s2} should be reduced by 5% for bars more than 20 mm in diameter, and by 10% for bars more than 32 mm in diameter.

Members are analyzed for fatigue strength at inclined sections on the assumption that all of the resultant force due to the principal tensile stresses applied at the centroid of the transformed section is taken by the transverse steel and that the stress in the latter is equal to the design strength, R_s, multiplied by the service factors, m_{s1} and m_{s2}. If a member is reinforced with stirrups or transverse bars,

then

$$\sigma_{prin,ten,\max} ub \leqslant F_{tr} R_s m_{s1} m_{s2} \tag{VIII.21}$$

where m_{s1} is the service factor for the steel, determined according to the cycle characteristic $\rho = \sigma_{prin,ten,\min}/\sigma_{prin,ten,\max}$; F_{tr} is the cross-sectional area of stirrups or transverse bars lying in the same plane; u is the spacing between the stirrups or transverse bars; and b is the width of the member.

Members having no transverse reinforcement should meet the condition similar to that adopted in inclined cracking analysis (see Chapter VII), but with the design strength of the concrete taken for the first group of limit states (R_{ten} and R_{pr}) and multiplied by m_{c2}.

Second Group of Limit States. Under repeated loading, the normal cracking analysis of members is carried out on the same basis as fatigue analysis (excluding the limitations concerning the cross-sectional area of the concrete in the tension zone), but with the axial tensile strength of the concrete assumed for the second group of limit states:

$$\sigma_{c,ten,\max} \leqslant R_{tenII} m_{c2} \tag{VIII.22}$$

The inclined cracking analysis of members is carried out on the assumption that repeated loading causes inclined cracks which may eventually destroy the member. Here, the design strength of the concrete, R_{tenII} and R_{prII}, is taken with the service factor, m_{c2}, applied.

The amplitudes of dynamic deflections should meet the following condition

$$A \leqslant a_0 \tag{VIII.23}$$

where A is the amplitude of forced vibrations determined in dynamic analysis; a_0 is the maximum amplitude of forced vibrations unharmful for people, machines, instruments, etc:

$$a_0 = W_0/4\pi^2 n^2 \tag{VIII.24}$$

or

$$a_0 = V_0/2\pi n \tag{VIII.25}$$

Here, n is the frequency of forced vibrations in Hz; W_0 is the peak acceleration for harmonic vibrations in mm/s²; and V_0 is the peak velocity for harmonic vibrations in mm/s.

The averaged peak values of the above quantities are $W_0 = 150$ mm/s² at $n < 10$ Hz, and $V_0 = 2.4$ mm/s at $n \geqslant 10$ Hz. Detailed information about the amplitudes of forced vibrations, velocities, and accelerations can be found in appropriate specifications.

If the condition (VIII.23) is not satisfied, additional measures should be taken to reduce the amplitudes of forced vibrations. Should an unfavourable result be obtained in this case, this is an indication that the frequency of free vibrations, ω, is very close to that of the disturbing force, θ.

The effect of vibration may be reduced by moving the source of vibration to some other place, balancing a vibrating machine, or changing the frequency of free vibrations of members. The frequency of free vibrations may be altered by changing the stiffness of a member, its design, or span. If it is necessary to raise the frequency of free vibrations, the stiffness of a member should be increased. This will reduce the dynamic factor, β, and static sag. The replacement of a simply supported beam by a beam with its both ends elastically built-in will nearly double the frequency of free vibrations; adding new braces and increasing the static indeterminacy of a structure always affects the frequency of free vibrations similarly to an increased stiffness. Any reduction in span will also increase the frequency of free vibrations.

The shock-mounting of machines is one of the most effective measures to minimize vibration. This may be active or passive. The former consists in isolating sources of vibration and in reducing the dynamic loads imposed on a structure by machines. The latter refers to the protection of sensitive equipment against floor vibration. Most commonly, shock-mounting involves the use of hanging rods, springs, rubber pads, and so on. Shock-mounting should be designed according to relevant standards and specifications. Improperly designed or poorly chosen shock-mounting may increase rather than reduce vibration.

CHAPTER IX

DESIGN OF REINFORCED CONCRETE PRODUCTS FOR MINIMUM COST

IX.1. COST RELATIONSHIPS FOR REINFORCED CONCRETE PRODUCTS

Economically, the key criterion in the Soviet Union for comparable civil-engineering products (here "comparable" is used to mean items equal in terms of compliance with service requirements, service life, and fire resistance) is the minimum equivalent cost which is the sum of what we call the current expenditures C_i (cost of civil-engineering work) and the lump-sum expenditures K_i (investment or operating costs) referred to an annual basis by applying a standard investment utilization factor E_u, or symbolically

$$C_i + E_u K_i = \text{minimum} \tag{IX.1}$$

where E_u, as adopted in civil engineering, is 0.12.

The estimated equivalent cost of a product is based on its "end" cost, that is, its cost in a finished building or structure (with adjustment for the reduction, if there is any, in the predetermined or standard overhead expense owing to a cut-down in time and/or labour requirements), quoted as of the beginning of the project's first year in service, and also on the investment costs, that is, the capital invested in the project, and the maintenance cost.

From among the likely alternate designs differing in size, amount of reinforcement, class of reinforcing steel, brand number of concrete, manufacturing process, etc., the costwise-optimal product (say, an r.c. beam, roof slab, truss, column, etc.) can, in a first approximation, be recognized as such only on the basis of the "end" cost. The final decision may markedly be influenced by variations in the cost of associated members of a structure (walls, supports, foundations) and also in heating and ventilation costs, which may be brought about by changes in ceiling height or overall dimensions. This may, however, be neglected where the comparison is run among

identical products or the products are to be used in a large-area building.

In the course of design, the estimated "end" cost of a product, $C_{p,e}$, is found as the sum of the total cost of production (factory cost), C_p, the transportation expense, C_t, incurred in delivering the product from maker to site, erection (and/or assembly) cost, C_e, and the variable part of the overhead expense, ΔH. The cost thus obtained is then multiplied by 1.02 (an averaged factor) to account for the cost of procurement and warehousing, and by another factor, k_w, which takes into account the higher wage rates paid to labour in winter (for concreting not exceeding 15% of the total work involved in the manufacture of prefabricated products, this factor is taken as 1.025). To sum up,

$$C_{p,e} = (C_p + C_t)\,1.02 + (C_e + C_{p\text{-}a})\,k_w + \Delta H \tag{IX.2}$$

In Eq. (IX.2), the value of C_e is taken from a relevant costing reference source; C_{p-a} is the cost of preassembly (here, preassembly refers to putting together the basic elements into a larger unit which is then delivered to the job); C_t refers to the cost of delivery by truck or rail, including the unloading cost and the cost of all the necessary accessories. For the purpose of estimating C_t, the volume of the products is found from their geometrical dimensions minus voids. The total estimated factory cost of a product is

$$C_p = C_{p,f}1.145k_{reg} \tag{IX.3}$$

where $C_{p,f}$ is the first cost of the product; 1.145 is the factor applied to account for variations in effectiveness in the industry and for indirect charges; and k_{reg} is the coefficient applied to allow for regional variations in the cost of materials and finished product.

The variable part of the overhead expense, ΔH, is defined as

$$\Delta H = 0.6\ (MD) + 0.15W_e + 0.082C_{p,d} \tag{IX.4}$$

where 0.6 is the total overhead expense per man-day on the site; MD is the man-days expended in assembly and erection work on the site; 0.15 is the total overhead expense per money unit of the basic wage rate of labour involved in assembly and erection; W_e is the basic wage rate for labour involved in erection work; 0.082 is the percentage of the predetermined or standard part of the overhead expense on the job (50% of overhead is assumed to amount to 16.5% of the total expenses); $C_{p,d}$ is the direct part of the "end" cost of a product, which is an intermediate result yielded by Eq. (IX.2).

For the purpose of design, the estimated first cost of a product should be found as the sum of several terms, namely

$$\begin{aligned} C_{p,f} &= C_{cm} + C_s + C_r + C_{pr} + C_{ep} + C_{inst} \\ &\quad + C_{ten} + C_{pl} + C_{fm} + C_{ht} + C_{ind} \end{aligned} \tag{IX.5}$$

where C_{cm} is the total cost of the concrete mix; C_s is the total cost of all steel used in a product, including embedded parts; C_r is the total cost of nonprestressed reinforcement (mats, cages, bars, lifting lugs); C_{pr} is the total cost of prestressed reinforcement (bars, cables, etc.); C_{ep} is the cost of embedded parts including those made from structural shapes, sheet, and rod steel; C_{inst} is the cost of installing nonprestressed reinforcement and embedded parts in forms; C_{ten} is the total cost of tensioning the reinforcement; C_{pl} is the cost of placing concrete in forms; C_{fm} is the cost of form maintenance; C_{ht} is the cost of heat treatment (steam-curing); and C_{ind} is the cost of industrialization (that is, the cost of "wet" and other operations carried out at the factory rather than on the site, such as assembly of basic units into larger units, finishing, etc.).

Knowledge of the estimated factory cost of a product is essential for an approximate technical and economic evaluation of the selected product design or for the choice of a product design that would minimize the cost of the product.

IX.2. REFERENCE DATA

In Eq. (IX.5), all terms except the last one are found as follows:

$$\left.\begin{aligned}
C_{cm} &= V_c k_{cm} P_{cm}\\
C_s &= \Sigma M_{r(pr,\,d)} k_s P_s/1\,000\\
C_r &= \Sigma M_r\,(1)\,P_r/1\,000\\
C_{pr} &= M_{pr} P_{pr}/1\,000\\
C_{ep} &= \Sigma M_{ep}\,(1)\,P_{ep}/1\,000\\
C_{inst} &= (M_r + M_{ep})\,P_{inst}/1\,000\\
C_{ten} &= M_{pr} P_{ten}/1\,000\\
C_{pl} &= V_c P_{pl}\\
C_{fm} &= V_c P_{fm}\\
C_{ht} &= V_c P_{ht}
\end{aligned}\right\} \qquad (IX.6)$$

where V_c is the volume of concrete in the finished product, m^3; $M_{r(pr,d)}$ is the mass, in kg, of steel of a given nominal diameter, kind, and dimensions stated in working drawings; $M_r(1)$ is the mass, in kg, of each nonprestressed reinforcing product of a given kind (mats, cages, lifting eyes) and a given mass group; M_{pr} is the mass, in kg, of prestressed reinforcement in a given class; $M_{ep}(1)$ is the mass of each kind of embedded parts, kg; M_r is the total mass of nonprestressed reinforcement in a product, kg; M_{ep} is the total mass of embedded parts in a product, kg; P_{cm} is the price of 1 m^3 of concrete mix (according to concrete grade and brand number, aggregate

grain size, mix workability and final strength of the product); P_s is the price of 1 tonne of steel as delivered to the prefab concrete maker's stockyard (depending on class, grade, diameter, shape and purpose); P_r is the price of 1 tonne of non-prestressed reinforcing products (according to kind and mass group); P_{pr} is the price of 1 tonne of prestressed reinforcing products; P_{ep} is the price of 1 tonne of embedded parts; P_{inst} is the price of installing 1 tonne of nonprestressed reinforcing and embedded parts in forms; P_{ten} is the price of tensioning 1 tonne of prestressed reinforcement according to class, kind, and tensioning method; P_{pl} is the cost of placing 1 m^3 of concrete (in terms of a finished product) in forms; P_{fm} is the price of maintaining the forms (on the basis of 1 m^3 of concrete in a finished product); P_{ht} is the price of steam used to cure 1 m^3 of concrete (in a finished product); k_{cm} is the concrete-mix utilization factor applied to account for the displacement of some concrete by steel, loss and wastage of concrete mix in the course of placement; and k_s is the steel utilization factor applied to account for the loss and waste of steel in the fabrication of reinforcing products and embedded parts.

As already noted, the last term in Eq. (IX.5), that is, C_{ind}, refers to the cost of all "wet" and other operations transferred from the site to the factory to make the entire fabrication process more industrialized. It covers the preassembly of basic elements into larger units, installation of thermal insulation, vapour barriers, surface finishing, and the like.

IX.3. DESIGN OF R. C. MEMBERS FOR MINIMUM COST

Referring to Eqs. (IX.5) and (IX.6), we can conclude that the estimated factory cost of a product is a function of several factors, namely the geometrical data G (which include the depth of a member, web or rib width, flange width, cross-sectional area of reinforcement); main-reinforcement data A (steel class, grade and kind); concrete type and brand number R; element length L; concrete and steel price data P; the process(es) used in the manufacture, T_m; and the process(es) used in assembly and erection (including delivery to the job), T_e; that is,

$$C_{p,\,f}^{(m)} = \varphi\,(G,\ A,\ R,\ L,\ P,\ T_m,\ T_e) \qquad \text{(IX.7)}$$

The factors L and P may be treated as nonmodifiable. The factors T_m and T_e are, in the general case, modifiable, but since the number of likely alternatives is usually limited, they, too, may be taken as specified in advance on the basis of previous experience, according to the type and brand number of concrete and the class

of reinforcement used, and also depending on the type of the basic member or the assembly of members. Where there exists a choice of several alternatives for T_m and T_e, the value of $C_{p,f}$ must be established for each alternative process of manufacturing or erection (this does not refer to the choice of an optimal manufacturing or erection process; this is done separately).

The factors G, A and R are modifiable, but the type of concrete (heavy or lightweight) must be specified in advance. If, on the other hand, the designer has the right to choose between heavy and lightweight concrete, $C_{p,f}$ must be found separately for either case. Existing standards require that the values of G, A and R should be chosen such that no limit states will ever arise in the member during manufacture, erection and service. More specifically, to avoid the first group of limit states, the following conditions must be satisfied:

$$M_q \leqslant M_{lim}; \; Q_q \leqslant Q_{lim}; \; N_q \leqslant N_{lim} \tag{IX.8}$$

To avoid the second group of limit states, the following conditions must be satisfied:

$$\left.\begin{array}{l} M_q \leqslant M_{cr}; \quad N_q \leqslant N_{cr}; \quad \sigma^q_{prin,\,ten} \leqslant R_{ten} \\ f_q/l \leqslant [f/l]; \quad a^q_{cr} \leqslant [a_{cr}] \end{array}\right\} \tag{IX.9}$$

In the above equations, the terms on the left-hand side are determined by the design or standard (basic) values of forces, member length, and type of fixation. The terms on the right-hand side are load-bearing capacity indices (in terms of moment, longitudinal force, and shearing force) or cracking-resistance indices, or the relative sag and crack width as limited by applicable standards.

Experience has suggested several more design constraints, namely:

$$\left.\begin{array}{l} x/h_0 \leqslant \xi_R; \quad Q \leqslant 0.35 R_{pr} b h_0 \,(100) \\ \mu_{\min} \leqslant \mu \leqslant \mu_{\max} \end{array}\right\} \tag{IX.10}$$

and also constraints on the amount of prestress in reinforcement and concrete:

$$\sigma_{0,\min} \leqslant \sigma_0 \leqslant \sigma_{0,\max}; \; \sigma_c \leqslant \sigma'_{c,\max} \tag{IX.11}$$

Substitution of Eqs. (IX.8) and (IX.9) into Eq. (IX.7) yields an unwieldy expression which cannot practically be minimized for the independent variables in the general case. The problem of minimizing the cost of a member can be solved by an analysis of the cost given by Eqs. (IX.7) through (IX.9), subject to Eqs. (IX.10) and (IX.11), assuming discrete values for the independent variables.

From inspection of Eq. (IX.7), subject to Eqs. (IX.8) through (IX.11), for constant-section I-beams in bending under conditions of

a specified amount of loading and loading configuration, the following relationship is obtained, in which the cost appears to be a function of only four independent variables:

$$C^{(m)}_{p,f} = \varphi (h, b'_f, A, R) \tag{IX.12}$$

It is an easy matter to show that the remaining parameters of an I-beam (h'_f, h_f, b, b_f, F_{pr}, and σ_0) are functionally related to the four variables by Eqs. (IX.8) and (IX.9). For example, the main longitudinal reinforcement of prestressed members does not practically depend on the width and depth of a section. Within a cross section (Fig. IX.1), it may be neglected as a constant which cannot affect an optimal solution.

The strength analysis of I-section bending members at a normal section (see Fig. IX.1) will confirm that the most economical section is that in which the depth of the compressed flange, h'_f, is equal to the depth of the compression zone. On substituting $m_{s4}R_sF_{pr}$ for R_sF_s in Eq. (III.27), we find the depth of the compressed flange to be

$$h'_f = x = m_{s4}R_sF_{pr}/b'_fR_{pr} \tag{IX.13}$$

Obviously, it cannot be less than the value practically permissible in a member.

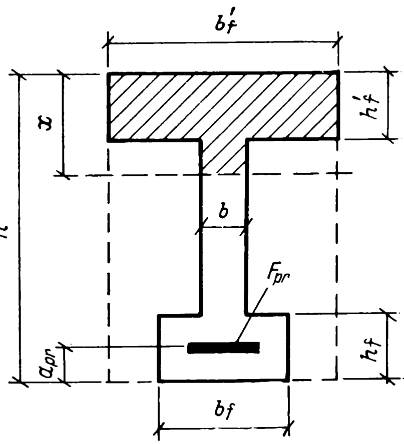


Fig. IX.1. Cross section of a bending prestressed I-beam

In view of the fact that $h'_f = x$, the required cross-sectional area of prestressed reinforcement for the effective section depth equal to $h_0 = h - a_{pr}$ may be written as

$$F_{pr} = M/m_{s4}R_s (h - a_{pr} - 0.5h'_f) \tag{IX.14}$$

The amount of prestress, σ_0, applied to the steel is chosen so as to meet the requirement for the cracking resistance of the member; it accounts for a certain share of R_s.

The web thickness, b, is usually taken as 6 to 8 cm, which is the minimum value permissible from the view-point of manufacture. According to Eq. (III.67), the shear-strength condition (that is, in terms of Q), relates b to h_0, R, and the amount of transverse reinforcement.

Thus, from the strength condition for a member, we can see that the parameters h'_f, h_f, b, b_f, F_{pr} and σ_0 do depend on the variables h, b'_f, A, and R.

Analysis (omitted here to save space) of a member for cracking resistance and stiffness, that is, for compliance with Eq. (IX.9), would show that in this case, too, h, b'_f, A and R remain independent variables, whereas all the other terms are related to them functionally.

Allowance for the strength and stiffness of a member during transportation and erection will not affect the above conclusion.

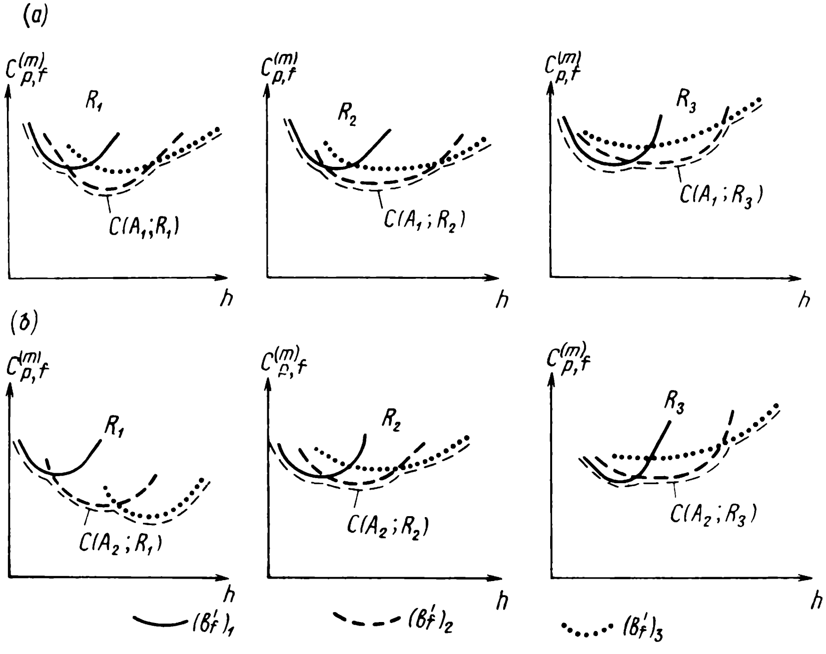


Fig. IX.2. Cost of a member as a function of overall dimensions, h and b'_f, and also strength, deformation and cost of concrete brands R_1, R_2 and R_3
(a) for reinforcement alternative A_1; (b) for reinforcement alternative A_2

In optimizing the cost of a member in terms of Eq. (IX.12), the first step is to assume the set data for the first reinforcement alternative, A_1 (class, grade, method of tensioning, strength, deformation, cost). Then, the set of data is assumed in turn for each of the concrete brands available, R_1, R_2, R_3, ..., and an I-beam is engineered to satisfy the above conditions. This done, its estimated cost $C^{(m)}_{p,f}$ is found, depending on the value of h, for several fixed widths of the compressed flange, that is, b'_{f1}, b'_{f2}, b'_{f3}, ... In doing this,

the trial values of h and b'_f are taken in a particular sequence and at a particular interval, until the procedure leads to a minimum cost. This procedure can be depicted graphically as shown in Fig. IX.2*a*, where the envelopes C (A_1, R_1), C (A_1, R_2) and C (A_1, R_3) are formed by the portions of the curves plotted for the various values of b'_{f1}, b'_{f2}, b'_{f3}, ...

The next step is to assume a second set of data for a second reinforcement alternative, A_2, and to proceed as already explained above. This will yield the envelopes C (A_2, R_1), C (A_2, R_2), C $(A_2,$

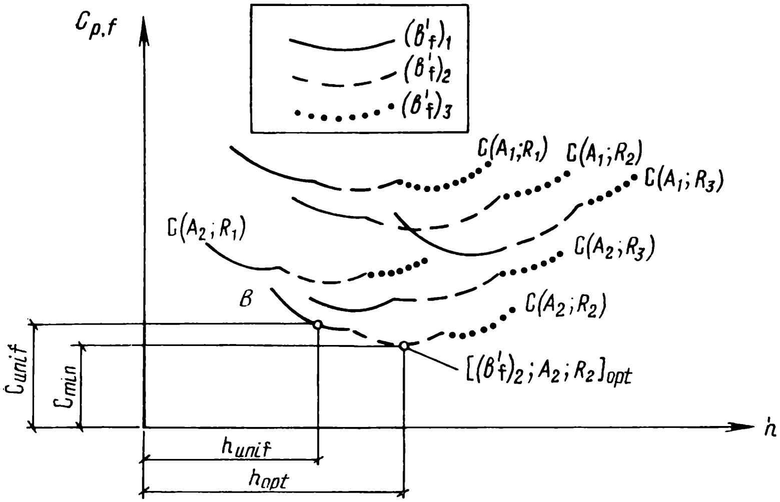


Fig. IX.3. Determining the optimal values of h, b'_f, A_i and R_i by the least cost of a member

$R_3)$, etc., as shown in Fig. IX.2*b*. Proceeding as before, similar envelopes can be obtained for a third reinforcement alternative, A_3, etc.

From a comparison of the envelopes (Fig. IX.3), we can readily pin-point the alternative offering a member of the least estimated first cost and the optimal values of the respective parameters: h_{opt}, $b'_{f,opt}$, A_{opt}, and R_{opt}.

If the optimal alternative thus found has to be abandoned in practice for some reason, the increase over the least cost can be found from the plot of Fig. IX.3. As an example, suppose that the final choice has fallen on the reinforcement alternative, A_2, and the concrete alternative, R_2, and that the section dimensions, subject to unification, are b'_{f1} and h_{unif}. On the envelope plots (see Fig. IX.3), these data correspond to point B whose ordinate is C_{unif}. The rise in the cost of the member is $\Delta C_{unif} = C_{unif} - C_{min}$.

Because r.c. products are expected to meet quite a number of building-code requirements, the search for an optimal solution is a fairly complex problem. However, it can readily be handled by a digital computer. At present, there are computer algorithms and programs to solve this problem without any simplifications whatsoever and in compliance with all the building-code requirements, namely as regards strength, cracking resistance, stiffness under loads likely to be encountered during manufacture, erection and service, and also design requirements defined by Eqs. (IX.10) and (IX.11).

There are also simplified algorithms and computer programs which yield solutions complying with only some of the building-code requirements.

If a member needs only to be optimized in terms of the most vital of the building-code requirements (strength at normal sections, stiffness, and cracking resistance), the problem can well be handled by hand. In such cases, it will be well advised to use ready formulas derived analytically, and to adjust the design of the member to suit the building-code requirements not included in the solution.

A similar approach is valid in optimizing members for some other criteria, such as labour requirements, mass, and consumption of scarce materials.

The above technique applies to both compression and tension members.

We have described an analytical alternative-testing method which involves a limited number of independent variables and therefore makes it possible to save computer time and simplifies analysis of the results.

The minimum estimated factory cost of basic members, $C^{(m)}_{\min}$ (see Fig. IX.3), can be utilized to find the minimum estimated factory cost of a larger product assembled from such members, $C_{p,f}$, with allowance for unification and likely departures from $C^{(m)}_{\min}$, using the minimum total rise in the cost of all the members in the larger product as the criterion. After that, one can find the estimated "end" cost of the product, $C_{p,e}$, by Eq. (IX.2).

Depending on the conditions of the problem, C_t in Eq. (IX.2) may or may not be modifiable. In the former case, it may be modified independently, to suit the distance from the maker to the job and the form of carrier used (by road, by rail, or in some other way). If several makers offer comparable manufacturing alternatives [different T_m in Eq. (IX.7)], C_t and C_p must be modified so as to preserve a proper relation between them.

The term C_e in Eq. (IX.2) may likewise be modified independently. If, however, the choice of assembly and erection methods markedly affects T_e in Eq. (IX.7), C_e must be modified together with C_p.

The terms C_t and C_e may be treated as independent of each other.

Where a particular product design is used on a limited scale, the minimum cost of the associated basic elements may be established to comply with a limited number of building code requirements (with a check for compliance with the other requirements and suitable adjustments, if necessary). Where a particular product design is used repeatedly on a large scale, the minimum cost must be determined on the basis of all the building code requirements.

Mir Publishers welcome your comments on the content, translation, and design of the book.

We would also be pleased to receive any suggestions you care to make about our future publications.

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Moscow, I-110, GSP,
Pervy Rizhsky Pereulok, 2,
Mir Publishers

Printed in the Union of Soviet Socialist Republics

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